AN ANALYSIS OF THE LOWEST ORDER MODE IS PROPAGATING IN THICK-WALLED PARALLEL PLATE WAVEGUIDE

Yakup Hameş  İsmail Hakkı Tayyar

e-mail: yakuphames@hotmail.com  e-mail: tayyar@gyte.edu.tr

Yüzüncü Yil University, Erciş Vocational College, 65400, Erciş, Van, Turkey
Gebze Institute of Technology, Department of Electronics Engineering, P.O. Box 141, Gebze, 41400, Kocaeli, Turkey

Key words: Propagating, mode, thick-wall, parallel-plate, waveguide

ABSTRACT

In the radiation problem, because of the lowest order mode don't propagate under whatever condition, to determine the lowest order mode is propagating, which is very important for engineering and mathematics applications. Sometimes, it is very compicate to obtain the necessary condition to ensure that the lowest order mode is propagating. In this case, graphically solution is applied. In this study, the lowest order mode is propagating in dielectric loaded thick-walled parallel plate waveguide, which was investigated through graphically solution.

I. INTRODUCTION

Radiation from open-ended parallel-plate waveguide has been subjected to numerous investigations. In the present work, we consider the lowest order mode is propagating in dielectric filled impedance waveguide junction with a thick dielectric half-plane. In the case, where thick half-plane and inside region of waveguide has a different nonmagnetic dielectric material.

In this context, the problem of radiation from two parallel semi-infinite plates of zero thickness has been considered [1-3]. In other hands one was investigated the radiation from an open waveguide with reactive walls, which is a canonical model simulating an impedance loaded horn and horn type surface wave launchers [4]. Later, this work was generalized [5]. In all of these works, the lowest order mode is propagating, which was investigated with different methods under different conditions.

II. MATHEMATICAL ANALYSIS

We consider the radiation of the dominant TE mode wave which is incident from the left in the parallel plate region formed by two semi-infinite impedance plates defined by the impedance half plane

\[ S_1 = \{(x, y, z); x \in (-\infty, 0), y \in (a, b), z \in (-\infty, \infty)\} \]

and

\[ S_2 = \{(x, y, z); x \in (-\infty, 0), y \in (-b, -a), z \in (-\infty, \infty)\} \]

respectively, as depicted in Fig. 1.

![Figure 1. Geometry of radiation problem](image)

The surface impedances of the horizontal walls \( y = \pm b, x < 0 \), and \( y = \pm a, x < 0 \) are denoted by \( Z_1 = \eta_1 Z_0 \) and \( Z_2 = \eta_2 Z_0 \) respectively, while the impedance of the vertical walls \( x = 0, y \in (a, b) \) and \( x = 0, y \in (-b, -a) \) is \( Z_3 = \eta_3 Z_0 \), with \( Z_0 \) being the characteristic impedance of the free space.

Here, \( k_0 \) is the free space wave number which is assumed to have a small positive imaginary part and denoted by \( k_0 = \omega \sqrt{\varepsilon_0 \mu_0} \) with \( \varepsilon_0 \) and \( \mu_0 \) being the dielectric permittivity and the magnetic permeability of the free space. The lossless case can then be obtained by making \( \Im m(k_0) \rightarrow 0 \) at the end of the analysis. \( k_1 \) is the wave numbers denoted by \( k_1 = \omega \sqrt{\varepsilon_1 \mu_0} \) with \( \varepsilon_1 \) being the dielectric permittivity in the regions \( x < 0, \ y \in (-a, a) \).

Incident wave that satisfy the boundary conditions:

\[ E'_1 = u'(x, y) = \left[A_0 \sin(\xi_0 y) + B_0 \cos(\xi_0 y)e^{ik_0 x}\right] e^{ik_0 x} \] (1a)

With

\[ A_0 = \left(\cos\xi_0 a + \frac{\eta_2}{ik_1} \sin\xi_0 a\right), \quad B_0 = \left(\sin\xi_0 a - \frac{\eta_2}{ik_1} \cos\xi_0 a\right) \] (1b)

The configuration is two dimensional and the assumed incident field, only three field components namely,
\[ E_z^i = u^i(x, y), \]  
\[ H_x = -\frac{i}{\omega \mu} \frac{\partial}{\partial y} u^i(x, y), \]  
\[ H_y = -\frac{i}{\omega \mu} \frac{\partial}{\partial x} u^i(x, y) \]

are nonzero.

\[ \beta_0 \] in the incident wave equation is the lowest real root of the characteristic equation

\[ L(\alpha) = \left( \cos[K_0] + \frac{\eta_2}{ik_1} K_1 \sin[K_0] \left( \sin[K_0] - \frac{\eta_2}{ik_1} K_1 \cos[K_0] \right) \right) = 0 \]  
(2a)

where

\[ K_1(\alpha) = \sqrt{k_1^2 - \alpha^2} \]  
(2b)

and \( \tilde{\xi}_0 = K_1(\beta_0) \) is defined by

\[ \tilde{\xi}_0 = K_1(\beta_0) \]  
(2c)

The square-root function is defined in the complex \( \eta \) plane cut along \( \alpha = k_1 \) to \( \alpha = k_1 + i\infty \) and \( \alpha = -k_1 \) to \( \alpha = -k_1 - i\infty \) such that \( K_1(0) = k_1 \). Note that equation (2a) admits an infinite of symmetric roots denoted by \( \pm \beta_n \) satisfying

\[ L(\beta_n) = 0, \ \Im(\beta_n) > \Im(k_1) \]  
(2d)

with

\[ \tilde{\xi}_n = K_1(\beta_n) \]  
(2e)

Equation (2a) has two multiplier which we suppose \( \beta_n^e \) satisfying

\[ \left( \cos[K_1] + \frac{\eta_2}{ik_1} K_1 \sin[K_1] \right) = 0 \]  
(2f)

and \( \beta_n^o \) satisfying

\[ \left( \sin[K_1] - \frac{\eta_2}{ik_1} K_1 \cos[K_1] \right) = 0 \]  
(2g)

In these two equation, we can write \( K_1a \) as \( x \), \( \eta_2 \) as \( i \eta_2 \) and \( \frac{\eta_2}{k_1} \) as \( A \) for mathematical easiness. Thus the equations (2f) and (2g) are converted to

\[ \cot x = -Ax \]  
(2h)

and

\[ \tan x = Ax \]  
(2i)

respectively. \( \beta_n \) can be written as \( \beta_n = \sqrt{[\eta_2/a]^2 - \tilde{\xi}_n^2} \), \( n = 0, 1, 2, \ldots \). The necessary condition to ensure that the lowest order mode is propagating in the waveguide one can take into account the slopes, \( -A \) and \( A \), of the equations (2h) and (2i) respectively.

Figure 2. Graphically solution of \( \tan x = Ax \), \( \cot x = -Ax \) equations.

There are three states for \( A \).

I) If \( A > 1 \) : First root of \( \tan x = Ax \) is in the interval \( x \in (0, \frac{\pi}{2}) \). First root of \( \cot x = -Ax \) is in the interval \( x \in (\frac{\pi}{2}, \pi) \). If \( k_1a < \frac{\pi}{2} \), there is not real root for \( \cot x = -Ax \). There is only one real root is root for \( \tan x = Ax \) in the interval \( x \in (0, \frac{\pi}{2}) \) and \( x_n^e < k_1a < \frac{\pi}{2} \). If \( A < 1 \) : There is not any root for \( \tan x = Ax \) in the interval \( x \in (0, \frac{\pi}{2}) \). Its first root is belong to the interval \( x \in (\frac{\pi}{2}, \pi) \). Coefficient \( B_0 \) has first root in the interval \( x \in (0, \frac{\pi}{2}) \). If \( x_n^o < k_1a < \pi \), first and one real root is \( \beta_n^o \) else if \( x_n^o < k_1a < \frac{\pi}{2} \), there are two real root which satisfy \( \beta_n^o < \beta_n^e \).

III) If \( A < 0 \) : First root of the equations \( \tan x = Ax \) and \( \cot x = -Ax \) is in the intervals \( x \in (\frac{\pi}{2}, \pi) \) and \( x \in (0, \frac{\pi}{2}) \), respectively. If \( x_n^o < k_1a < \frac{\pi}{2} \), there is only one real root named \( \beta_n^o \) else if \( x_n^o < k_1a < \frac{\pi}{2} \), there are two real roots satisfy \( \beta_n^o < \beta_n^e \).

To satisfy dominant mode wave propagation in the waveguide one must take into account both the conditions \( \eta_1 > k_1a \) and \( x_n^o < k_1a < \frac{\pi}{2} \). In these conditions one can obtain dominant mode propagation constant \( \beta_n^0 \) which belongs to equation (2g). Coefficient \( B_0 \) in equation (1b) is zero.

After all, consequently, the incident field expression \( u^i(x, y) \) can be written as follows:

\[ E_z^i = u^i(x, y) \left( \cos[\xi_0] + \frac{\eta_2}{ik_1} \sin[\xi_0] \right) e^{i\beta_n} \]  
(2j)
III. CONCLUSION

In this study, the lowest order mode is propagating in dielectric loaded thick-walled parallel plate waveguide, which was investigated through graphically solution. Since Verification of the truthness of this solution, radiation of the end of waveguide was investigated. Existing of the mode is propagating in dielectric loaded thick-walled parallel plate waveguide, which was observed, for different values of the $\eta_2$. For the values except for the necessary condition to ensure that the lowest order mode is propagating in the waveguide, one was observed that any mode is propagating. This is guaranteed that only one and the lowest order mode is exist.

Figure 10. Radiated field amplitude versus the observation angle for different values of the $\eta_2$

REFERENCES