A Controversial Issue: Power Components in Nonsinusoidal Single-Phase Systems

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Abstract

This paper presents a numerical comparison and a discussion on apparent power decompositions in single phase circuits with sinusoidal/nonsinusoidal voltages and/or currents for the purpose of understanding the debate on the interpretation of power components. The scope of this study is limited to the apparent power decompositions of IEEE Std. 1459-2010 and C. I. Budeanu who is the owner of the first three dimensional approach. The points which are in common and separated from each other are given by numerical analysis. In addition, the content of IEEE's harmonic distortion power (D_H) is introduced in terms of power components.

1. Introduction

The physical meaning and interpretation of electric power quantities have been a great matter of debate since 1880s when the first electrical inventions developed by the brightest electricians of the time [1]. A lot of effort has been put in place for a long time in order to clarify the definitions for the measurement of power quantities under nonsinusoidal conditions. In 2010, Power System Instrumentation and Measurement Committee of the IEEE Power and Energy Society published IEEE Std. 1459 [2] to provide criteria for designing and using metering instrumentation for electrical energy and power quantification under sinusoidal, nonsinusoidal, balanced or unbalanced conditions.

From the very beginning, the interaction between voltage and current waveforms has been analyzed for the purpose of determining a concept for electrical energy flow by interpreting power components. All power components have same significance since each one of them affects others. In sinusoidal case, the classical definitions of active, reactive and apparent power are well-known and universally accepted. However, these definitions become inadequate, when the voltage and current waveforms are nonsinusoidal.

Constantin I. Budeanu made the first attempt [3] in 1927 by discovering a nonactive power different than reactive power, since he defined a new component as distortion power and proposed a three dimensional system for nonsinusoidal conditions [4]. After that, several approaches were suggested in order to identify apparent power components by different researchers. The main difference among these decompositions is the resolution level of grouping and classification of the apparent power components. In another words, suggested approaches are mainly based on the three dimensional system; eventually, active, reactive and distortion power.

Therefore, this study is focused only on Budeanu's decomposition in conjunction with IEEE's latest revised and reconfirmed standardized definitions to determine the main motivation and purpose of the researches. In this context, the apparent power components in single phase sinusoidal and nonsinusoidal case have been analyzed and the points which are in common and separated from each other emphasized by means of quantitative comparison. Finally, in order to understand the contents of harmonic distortion power (D_H), an analysis is performed in terms of power components.

2. Single phase power definitions under sinusoidal conditions

The well-known and universally accepted concepts for this case are explicitly given in the IEEE Standard. Let us assume that the voltage and current signals in a linear single phase system are given by;

$$v = \sqrt{2}V sin(\omega t) \text{ and } i = \sqrt{2} I sin(\omega t - \theta) \tag{1}$$

where V and I are the rms values of voltage and current, respectively. θ represents the phase angle between the current and the voltage.

The instantaneous power p is the multiplication of instantaneous voltage and current signals which is divided into two components, namely; instantaneous active power p_a and instantaneous reactive power p_q.

$$p = vi = p_a + p_q \tag{2}$$

Instantaneous active power is the rate of unidirectional flow of the energy from the source to the load. Its steady state rate of flow is not negative and consists of active power P and intrinsic power $-P\cos(\omega t)$. In the IEEE Std. 1459-2010, it's stated that the intrinsic power is always present when net energy is transferred to the load; however, this oscillating component does not cause power loss in the supplying lines.

$$p_a = VI\, cos(\theta)[1 - cos(2\omega t)] = P[1 - cos(2\omega t)] \tag{3}$$

Active power, which is also called real power, is the average value of the instantaneous power p.

$$P = \frac{1}{kT}\int_{\tau}^{\tau+kT} p dt = \frac{1}{kT}\int_{\tau}^{\tau+kT} p_a dt = VI\, cos\, \theta \tag{4}$$

Instantaneous reactive power is produced by the reactive component of the current and the related energy component oscillates between the source and load where the net transfer of energy to the load is nil. However, these power oscillations cause power loss in the conductors.

$$p_q = -VI\sin\theta\sin(2\omega t) = -Q\sin(2\omega t) \tag{5}$$

The magnitude of the reactive power Q is equal to the amplitude of the oscillating instantaneous reactive power p_q. Due to the phase shift between voltage and current, if the load is inductive Q is positive and if the load is capacitive Q is negative.

$$Q = VI\sin\theta \tag{6}$$

The apparent power S is equal to the product of the rms voltage and the rms current, which is interpreted as the maximum active power that can be transmitted through the same line while keeping load rms voltage V and rms current I constant.

$$S = VI = \sqrt{P^2 + Q^2} \tag{7}$$

3. Single phase power definitions under nonsinusoidal conditions

In sinusoidal case, the deviation from the optimal case is in the responsibility of the load. But in nonsinusoidal case, the existence of the harmonics introduces the nonlinearity of the supply voltage in addition to the nonlinearity of the load as the source of the distortion. This situation causes the long-standing dispute on the generalization of the classical concepts [5].

The nonsinusoidal single phase periodic voltage and current waveforms have two distinct components: the power system frequency components v_1, i_1 and the remaining terms; harmonic components v_H and i_H.

$$v = v_1 + v_H \text{ and } i = i_1 + i_H \tag{8}$$

$$v_1 = \sqrt{2}V_1\sin(\omega t - \alpha_1) \tag{9}$$

$$i_1 = \sqrt{2}I_1\sin(\omega t - \beta_1) \tag{10}$$

$$v_H = V_0 + \sqrt{2}\sum_{h\neq 1} V_h\sin(h\omega t - \alpha_h) \tag{11}$$

$$i_H = I_0 + \sqrt{2}\sum_{h\neq 1} I_h\sin(h\omega t - \beta_h) \tag{12}$$

3.1. Budeanu's decomposition

The first attempt to solve the problem of defining power components under nonsinusoidal conditions is credited to Budeanu [1] who introduced a frequency domain based approach [6]. Using Lagrange's identity (13), Budeanu separated apparent power into three components; active, reactive and distortion powers.

$$\sum_{h=1}^{v} A_h^2 \sum_{h=1}^{v} B_h^2 = \left(\sum_{h=1}^{v} A_h B_h\right)^2 + \sum_{m=1}^{v-1}\sum_{n=m+1}^{v}(A_m B_n - A_n B_m)^2 \tag{13}$$

He asserts that apparent power consists of two orthogonal components which are active and deactive powers. Active power P is the sum of all individual harmonic active powers in frequency domain.

$$P = \frac{1}{T}\int_0^T v(t)\cdot i(t)dt = \sum_{n\in N^+} V_n I_n\cos\varphi_n \tag{14}$$

Deactive power has two components which are named Budeanu's reactive and distortion powers. Then, he described reactive power as the sum of all individual harmonic reactive powers:

$$Q_b = \sum_{n\in N^+} V_n I_n\sin\varphi_n \tag{15}$$

Afterwards he introduced a new quantity which is called as distortion power D_b.

$$D_b = \sqrt{S^2 - P^2 - Q_b^2} \tag{16}$$

$$= \sqrt{\sum_{m=1}^{v-1}\sum_{n=m+1}^{v}[(V_m I_n)^2 + (V_n I_m)^2 - 2V_m V_n I_m I_n\cos(\varphi_m - \varphi_n)]} \tag{17}$$

It is calculated by cross product of different harmonic voltages and currents. Despite Budeanu's reactive power can be entirely compensated by a simple capacitor, this is not valid for distortion power [7]. Besides, Czarnecki criticizes Budeanu's reactive power as the definition has no physical meaning and it provides useless information for power factor improvement [8]. Furthermore, in the recent revision of the IEEE Standard, Budeanu's reactive power has been removed and the usage of varmeters under distorted waveforms is reviewed in Annex A.2.

3.2. IEEE's decomposition

In the IEEE Standard, for nonsinusoidal situation, as given in the equations (8 - 12), voltage and current is divided into two components, namely; fundamental and harmonic parts. For both parts, rms values are calculated. The corresponding squared rms values are as follows;

$$V^2 = \frac{1}{kT}\int_{\tau}^{\tau+kT} v^2 dt = V_1^2 + V_H^2 \tag{18}$$

$$I^2 = \frac{1}{kT}\int_{\tau}^{\tau+kT} i^2 dt = I_1^2 + I_H^2 \tag{19}$$

$$V_H^2 = V_0^2 + \sum_{h\neq 1} V_h^2 = V^2 - V_1^2 \tag{20}$$

$$I_H^2 = I_0^2 + \sum_{h\neq 1} I_h^2 = I^2 - I_1^2 \tag{21}$$

Due to harmonic components, total harmonic distortion for voltage (THD_V) and current (THD_I) is defined as the ratio between rms values of harmonics to fundamental component.

$$THD_V = \frac{V_H}{V_1} = \sqrt{\left(\frac{V}{V_1}\right)^2 - 1} \tag{22}$$

$$THD_I = \frac{I_H}{I_1} = \sqrt{\left(\frac{I}{I_1}\right)^2 - 1} \tag{23}$$

Active power P definition is same in both sinusoidal and nonsinusoidal case.

$$P = \frac{1}{kT}\int_{\tau}^{\tau+kT} p dt = P_1 + P_H \tag{24}$$

$$P_{11} = V_1 I_1 \cos\theta_1 \tag{25}$$

$$P_H = P - P_1 = \sum_{h \neq 1} V_h I_h \cos\theta_h \tag{26}$$

The most controversial part of the standard is about the definitions and physical interpretations of reactive power and distortion power. Only fundamental reactive power definition is given. Accordingly, distortion powers individually for voltage, current and harmonics are defined by using THD values. But there is not any physical interpretation and also a definition for total distortion power.

Fundamental reactive power:

$$Q_{11} = V_1 I_1 \sin\theta_1 \tag{27}$$

Fundamental apparent power:

$$S_{11} = V_1 I_1 = \sqrt{P_{11}^2 + Q_{11}^2} \tag{28}$$

Current distortion power:

$$D_I = V_1 I_H = S_1(THD_I) \tag{29}$$

Voltage distortion power:

$$D_V = V_H I_1 = S_1(THD_V) \tag{30}$$

Harmonic apparent power:

$$S_H = V_H I_H = S_1(THD_I)(THD_V) \tag{31}$$

Harmonic distortion power:

$$D_H = \sqrt{S_H^2 - P_H^2} \tag{32}$$

Finally, apparent power becomes as;

$$S^2 = (VI)^2 = S_{11}^2 + D_I^2 + D_V^2 + S_H^2 \tag{33}$$

Nonfundamental apparent power:

$$S_N^2 = S^2 - S_1^2 = D_I^2 + D_V^2 + S_H^2 \tag{34}$$

Nonactive power:

$$N = \sqrt{S^2 - P^2} \tag{35}$$

4. Case studies

The approaches of IEEE and Budeanu are analyzed for four different cases by using numerical examples. The fundamental frequency is $f_1 = 50Hz$. For the sake of simplicity, the degree of harmonic components of nonsinusoidal waveforms is limited to the fundamental, third, fifth and seventh. For all cases, Table 1 shows the rms values and phase angles of the voltage and current signals. The values of all harmonic power components are presented in Table 2. Table 3 lists the values of Budeanu's power components. The results of the IEEE's power decomposition are shown in Table 4. As the unit system is provided by IEEE, for active powers watts (W), for apparent powers volt-amperes (VA) and for the entire nonactive powers volt-ampere-reactive (var) is used. The first subscript of the power components refers the harmonic order of voltage and the second one refers current. The cross products of voltage and current harmonics are defined as distortion powers and denoted by D_{mn} where $(m \neq n)$.

4.1. Case 1: Sinusoidal voltage and current

The first example is the sinusoidal case, where the voltage and current waveforms include only fundamental component.

$$v(t) = 100\sqrt{2}\sin(\omega_1 t) \tag{36}$$

$$i(t) = 60\sqrt{2}\sin(\omega_1 t - 30°) \tag{37}$$

Due to the nonexistence of voltage and current harmonics, apparent power includes only the fundamental component of active power and reactive power as stated in (7). As expected, both active powers of IEEE and Budeanu are equal to the fundamental active power $(P = P_{11} = 5196.2\,W)$. Budeanu's reactive power and IEEE's nonactive power is equal to the $(Q_b = N = Q_{11} = 3000\,var)$ fundamental component of reactive power. In sinusoidal conditions, the total harmonic distortions of voltage and current are zero $(THD_V = THD_I = 0)$. Therefore the results of the IEEE's nonfundamental powers $(S_H = S_N = P_H = D_I = D_V = D_H = 0)$ are nil and similarly Budeanu's distortion power is zero $(D_b = 0)$.

4.2. Case 2: Sinusoidal voltage nonsinusoidal current

The second example considers the hypothetical case of sinusoidal voltage and nonsinusoidal waveforms as follows;

$$v(t) = 100\sqrt{2}\sin(\omega_1 t) \tag{38}$$

$$\begin{aligned} i(t) = \sqrt{2}[&60\sin(\omega_1 t - 30°) + 15sin(\omega_3 t - 165°) \\ &+12sin(\omega_5 t + 285°) + 10sin(\omega_7 t + 310°)] \end{aligned} \tag{39}$$

In addition to Case 1, only current harmonics are added to the current waveform in order to approach the general case step by step. In this condition, only $\sum_{h=3,5,7} V_1 I_h$ components appear in addition to the fundamental components of active and reactive powers. The active powers of IEEE and Budeanu, the same as in the Case 1, are equal to the fundamental component of active power $(P = P_1 = 5196.2\,W)$. Budeanu's reactive power is equal to the fundamental reactive power $(Q_b = Q_1 = 3000\,var)$ and Budeanu's distortion power is equal to the IEEE's current distortion power and nonfundamental apparent power $(D_b = D_I = S_N = 2165.6\,var)$. As it can be seen in Table 4, there is an increase in the apparent and nonactive powers $(S = 6378.9VA,\ N = 3700\,var)$ and in the current total harmonic distortion $(THD_I = 0.3609)$, due to the existence of current harmonics.

4.3. Case 3: Nonsinusoidal voltage sinusoidal current

The third example illustrates another hypothetical situation while there is nonsinusoidal voltage and sinusoidal current waveforms.

Table 1. RMS values and phase angles of the signal harmonics

	Case 1	Case 2	Case 3	Case 4
V_1	100	100	100	100
V_3	0	0	20	20
V_5	0	0	25	25
V_7	0	0	10	10
V	100	100	105.4751	105.4751
V_h	0	0	33.5410	33.5410
I_1	60	60	60	60
I_3	0	15	0	15
I_5	0	12	0	12
I_7	0	10	0	10
I	60	63.7887	60	63.7887
I_h	0	21.6564	0	21.6564
α_1	0°	0°	0°	0°
α_3	0°	0°	70°	70°
α_5	0°	0°	-140°	-140°
α_7	0°	0°	-210°	-210°
β_1	30°	30°	30°	30°
β_3	0°	165°	0°	165°
β_5	0°	-285°	0°	-285°
β_7	0°	-310°	0°	-310°

$$v(t) = \sqrt{2}[100\sin(\omega_1 t) + 20sin(\omega_3 t - 70°) \\ +25sin(\omega_5 t + 140°) + 10sin(\omega_7 t + 210°)] \tag{40}$$

$$i(t) = 60\sqrt{2}\sin(\omega_1 t - 30°) \tag{41}$$

It is obvious that the only difference than Case 2 is the existence of voltage distortion power D_V instead of current distortion power D_I. For this new situation, the values of voltage distortion power ($D_b = D_V = S_N = 2012.5\ var$) and voltage total harmonic distortion ($THD_V = 0.3554$) can be seen in Table 4. Due to the magnitudes of voltage harmonic components, nonactive power ($N = 3612.5\ var$) and apparent power ($S = 6328.5\ VA$) values are updated.

4.4. Case 4: Nonsinusoidal voltage and current

The last example considers the general case; nonsinusoidal voltage and current signals as follows;

$$v(t) = \sqrt{2}[100\sin(\omega_1 t) + 20sin(\omega_3 t - 70°) \\ +25sin(\omega_5 t + 140°) + 10sin(\omega_7 t + 210°)] \tag{42}$$

$$i(t) = \sqrt{2}[60\sin(\omega_1 t - 30°) + 15sin(\omega_3 t - 165°) \\ +12sin(\omega_5 t + 285°) + 10sin(\omega_7 t + 310°)] \tag{43}$$

As it can be seen in Table 2, all cross and common products of the harmonic power components appear. The active power of Budeanu and IEEE is the arithmetic sum of all harmonic active powers ($P = P_{11} + P_{33} + P_{55} + P_{77} = 4906.9W$). The reason behind the decrease in the active power in this case regarding to the previous cases is the negative sign of nonfundamental harmonic active power components which can be seen in detail in Table 2. This negative flow is a result of the phase difference between related harmonic voltages and currents. Identically, Budeanu's reactive power (15) is calculated as the arithmetic sum of individual harmonic reactive powers and its value is dependent to the sign of the individual harmonic reactive power components which explains the increase ($Q_b = Q_{11} + Q_{33} + Q_{55} + Q_{77} = 3028.3\ var$). In addition, there is an increase in the distortion power of Budeanu ($D_b = 3466.9\ var$).

Harmonic apparent power ($S_H = 726.378\ VA$), harmonic active power ($P_H = -289.257\ W$) and harmonic distortion power ($D_H = 666.2997\ var$) appears in addition to current distortion power ($D_I = 2165.6\ var$) and voltage distortion power ($D_V = 2012.5\ var$) which are identical to Case 2 and 3, respectively. Besides, apparent power ($S = 6728.1\ VA$) and nonfundamental power ($N = 4603.3\ var$) are increased with respect to the harmonic voltage and current components. Unlike the previous cases, in this condition ($S_N = 3044.3VA \neq D_b$).

Table 2. Harmonic power components

	Case 1	Case 2	Case 3	Case 4
S_{11}	6000	6000	6000	6000
S_{33}	0	0	0	300
S_{55}	0	0	0	300
S_{77}	0	0	0	100
D_{13}	0	1500	0	1500
D_{15}	0	1200	0	1200
D_{17}	0	1000	0	1000
D_{31}	0	0	1200	1200
D_{51}	0	0	1500	1500
D_{71}	0	0	600	600
D_{35}	0	0	0	240
D_{37}	0	0	0	200
D_{53}	0	0	0	375
D_{57}	0	0	0	250
D_{73}	0	0	0	150
D_{75}	0	0	0	120
P_{11}	5196.2	5196.2	5196.2	5196.2
P_{33}	0	0	0	-26.1467
P_{55}	0	0	0	-245.745
P_{77}	0	0	0	-17.3648
Q_{11}	3000	3000	3000	3000
Q_{33}	0	0	0	298.8584
Q_{55}	0	0	0	-172.072
Q_{77}	0	0	0	-98.4808

Table 3. Budeanu's power definitions

	Case 1	Case 2	Case 3	Case 4
S	6000	6378.9	6328.5	6728.1
P	5196.2	5196.2	5196.2	4906.9
Q_b	3000	3000	3000	3028.3
D_b	0	2165.6	2012.5	3466.9

Table 4. IEEE's power definitions

	Case 1	Case 2	Case 3	Case 4
S	6000	6378.9	6328.5	6728.1
S_{11}	6000	6000	6000	6000
S_H	0	0	0	726.378
S_N	0	2165.6	2012.5	3044.3
P	5196.2	5196.2	5196.2	4906.9
P_{11}	5196.2	5196.2	5196.2	5196.2
P_H	0	0	0	-289.257
Q_{11}	3000	3000	3000	3000
D_I	0	2165.6	0	2165.6
D_V	0	0	2012.5	2012.5
D_H	0	0	0	666.2997
N	3000	3700	3612.5	4603.3
THD_V	0	0	0.3554	0.3354
THD_I	0	0.3609	0	0.3609

5. Conclusions

As it is well-known that Budeanu's reactive power definition has no physical meaning, Budeanu's distortion power also has same problem since it is calculated with reference to the reactive power. Furthermore, in Cases 1-3 the definitions of Budeanu are meaningful compared to IEEE. But in Case 4, there is not any relation for reactive and distortion powers between the two concepts.

From the case studies 2 and 3, it is obvious that current harmonics are responsible for the D_I and voltage harmonics are responsible for D_V. But it is not easy to interpret D_H, because of the existence of both voltage and current harmonics.

Generally, (32) is used for the calculation of harmonic distortion power, thus the contents of D_H are unclear. Using Lagrange's identity (13) in the definition of S_H (31);

$$S_H^2 = \sum_{h\neq 1}^{v} V_h^2 \left[\sum_{h\neq 1}^{v} (I_h cos\theta_h)^2\right] + \sum_{h\neq 1}^{v} V_h^2 \left[\sum_{h\neq 1}^{v} (I_h sin\theta_h)^2\right] \tag{44}$$

Bearing in mind the definition of P_H (26), harmonic distortion power D_H becomes;

$$\begin{aligned} D_H^2 &= \left(\sum_{h\neq 1}^{v} V_h I_h sin\theta_h\right)^2 \\ &+ \sum_{m\neq 1}^{v-1} \sum_{n=m+1}^{v} [(V_m I_n)^2 + (V_n I_m)^2 - 2V_m V_n I_m I_n \cos(\theta_m - \theta_n)] \end{aligned} \tag{45}$$

Using (45) and the trigonometric identity for the cosine term $(\cos(\alpha - \beta) = cos\alpha cos\beta + sin\alpha sin\beta)$ in the Case 4, the components of D_H becomes apparent; such as

$$\begin{aligned} D_H^2 = &(Q_{33} + Q_{55} + Q_{77})^2 \\ &+D_{35}^2 + D_{37}^2 + D_{53}^2 + D_{57}^2 + D_{73}^2 + D_{75}^2 \\ &-2P_{33}P_{55} - 2P_{33}P_{77} - 2P_{55}P_{77} \\ &-2Q_{33}Q_{55} - 2Q_{33}Q_{77} - 2Q_{55}Q_{77} \end{aligned} \tag{46}$$

And one step further, it becomes;

$$\begin{aligned} D_H^2 = &Q_{33}^2 + Q_{55}^2 + Q_{77}^2 \\ &+D_{35}^2 + D_{37}^2 + D_{53}^2 + D_{57}^2 + D_{73}^2 + D_{75}^2 \\ &-2P_{33}P_{55} - 2P_{33}P_{77} - 2P_{55}P_{77} \end{aligned} \tag{47}$$

From (47), it can be seen that D_H is a combination of nonfundemantal harmonic reactive powers $(Q_{33},\ Q_{55},\ Q_{77})$, cross products of nonfundamental voltage and current harmonics (D_{mn}) and also nonfundamental harmonic active powers $(P_{33},\ P_{55},\ P_{77})$. If the values of related power components from Table 2 are used in (47), same result can be obtained $(D_H = 666.2997\ var)$.

6. References

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