DYNAMICALLY EQUIVALENT MANIPULATOR OF A TWO-LINK PLANAR, FREE-FLOATING SPACE MANIPULATOR

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ABSTRACT:

In this paper, we derive the dynamics of a free-floating, two-link planar space manipulator in two different approaches. In the first approach, we use Virtual Manipulator (VM) concept. The second approach is called Dynamically Equivalent Manipulator (DEM) approach. The DEM concept allows us to write the dynamics as a fixed-base manipulator. This paper presents the dynamics of the manipulator using the VM and the DEM approaches and simulation results are given to show the equivalence of the dynamics developed using both approaches.

INTRODUCTION :

Space robots will play important roles in construction and maintanence of space stations [1]. In order to use space manipulators efficiently, we must control them precisely. The main difference between a conventional fixed-base manipulator and space manipulator is the interaction between the manipulator and the base at which the manipulator is mounted. Therefore, obtaining the dynamic model of a space manipulator is more complicated than obtaining the model of a conventional fixed-base manipulator. One of the proposed methods in the past is Virtual Manipulator (VM) method [2]. The VM is a fixed-base robot whose first joint is a passive one representing the free-floating nature of the space manipulator's base. Another method is proposed by Liang et al, [3] called Dynamically Equivalent Manipulator (DEM). The DEM is also a fixed-base robot with a passive first joint. The main difference between the VM and the DEM is that the VM is an idealized massless kinematic chain and can only be simulated in a computer program; but the DEM is a real fixed-base robot which can be physically built and experimentally used for studying the dynamic behavior of the space manipulator. In this paper, dynamics of a two-link, planar, free-floating space manipulator is derived using both the VM and the DEM approaches. Simulations are given to show the equivalence of the dynamics developed by both methods.

I.Dynamic Model Using The VM Approach:

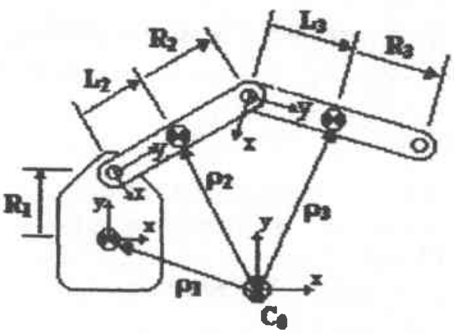

Fig. 1 The space manipulator system

A two-link planar robot manipulator mounted on a free-floating base is shown in Fig-1. The combination of the manipulator and its base forms the space manipulator system. We assume that no external forces or torques are applied on the space manipulator. Therefore, the center of mass C_0 remains fixed in inertial space and can be selected as the origin of the inertial coordinate frame.

The kinetic energy of the space manipulator is given as

$$T=\sum_{i=1}^{3}\left[\frac{1}{2}m_i(\dot{\rho}_i)^T\dot{\rho}_i+\frac{1}{2}\omega_i^{\ T}R_i^0 I_i(R_i^0)^T\omega_i\right] \quad (1)$$

Where m_i =Mass of link i.

$\dot{\rho}_i$ = Linear velocity of the CM of link i with respect to the inertial frame.

ω_i =Angular velocity of link i.

R^i_j =The rotation matrix that describes the coordinate frame j with respect to frame i.

I_i = Inertia tensor of link i

Linear and angular velocities of each link are given in the following equations:

$$\dot{\rho}_1 = \begin{bmatrix} -\dot{\alpha} r_1 \cos\alpha - \dot{\alpha}_1 A \cos\alpha_1 - \dot{\alpha}_{12} B \cos\alpha_{12} \\ -\dot{\alpha} r_1 \sin\alpha - \dot{\alpha}_1 A \sin\alpha_1 - \dot{\alpha}_{12} B \sin\alpha_{12} \\ 0 \end{bmatrix} \quad (2)$$

$$\dot{\rho}_2 = \begin{bmatrix} -\dot{\alpha} r_1 \cos\alpha - \dot{\alpha}_1 C \cos\alpha_1 - \dot{\alpha}_{12} B \cos\alpha_{12} \\ -\dot{\alpha} r_1 \sin\alpha - \dot{\alpha}_1 C \sin\alpha_1 - \dot{\alpha}_{12} B \sin\alpha_{12} \\ 0 \end{bmatrix} \quad (3)$$

$$\dot{\rho}_3 = \begin{bmatrix} -\dot{\alpha} r_1 \cos\alpha - \dot{\alpha}_1 D \cos\alpha_1 - \dot{\alpha}_{12} E \cos\alpha_{12} \\ -\dot{\alpha} r_1 \sin\alpha - \dot{\alpha}_1 D \sin\alpha_1 - \dot{\alpha}_{12} E \sin\alpha_{12} \\ 0 \end{bmatrix} \quad (4)$$

where $\alpha_1 = \alpha + \theta_1$ and $\alpha_{12} = \alpha + \theta_1 + \theta_2$

α = Rotation angle of the base relative to the inertial frame

θ_i = Joint angles of the manipulator.

$$\omega_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{\alpha} \end{bmatrix}, \ \omega_2 = \begin{bmatrix} 0 \\ 0 \\ \dot{\alpha}_1 \end{bmatrix}, \ \omega_3 = \begin{bmatrix} 0 \\ 0 \\ \dot{\alpha}_{12} \end{bmatrix} \quad (5)$$

where constants A-E are

$A = r_2 + l_2 - R_2 - L_2$, $B = r_3 + l_3 - R_3 - L_3$

$C = r_2 + l_2 - R_2$, $D = r_2 + l_2$, $E = r_3 + l_3 - R_3$

R_i = Vector connecting the CM of link i to joint i+1

L_i = Vector connecting joint i to the CM of link i

$$r_i = R_i \left(\sum_{k=1}^{i} m_k \Big/ \sum_{k=1}^{3} m_k\right) \quad (6)$$

$$l_i = L_i \left(\sum_{k=1}^{i-1} m_k \Big/ \sum_{k=1}^{3} m_k\right) \quad (7)$$

Since the free-floating space manipulator system is not acted upon by the gravitational forces, the Lagrangian of the system is equal to its kinetic energy. Assuming $q = [\alpha \quad \theta_1 \quad \theta_2]^T$ as the vector of the generalized coordinates Lagrangian's equation can be written as

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = Q_i \qquad i = 1,2,3 \quad (8)$$

where Q_i is the generalized force corresponding to the generalized coordinate q_i. Since no forces is applied to the base, Q_1=0. $Q_2 = \tau_1$ and $Q_3 = \tau_2$ are the torques applied to the joints 1 and 2 of the manipulator, respectively. Performing derivatives in equation (8) yields the dynamics of the space manipulator system as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = \begin{bmatrix} 0 \\ \tau_1 \\ \tau_2 \end{bmatrix} \quad (9)$$

Where $M(q)$ is the inertia matrix and $C(q, \dot{q})$ represents the Coriolis and centrifugal forces of the system. They are 3×3 matrices and given in Appendix A.

II. Dynamic Model Using The DEM Approach:

The DEM is a fixed-base manipulator whose first joint is a passive joint. The base of the system is located at the CM of the space manipulator system and the parameters of the DEM satisfy the following equations:

$$m_i' = m_i \left(\sum_{k=1}^{3} m_k \Big/ \left(\sum_{k=1}^{i-1} m_k \sum_{k=1}^{i} m_k\right)\right)$$

$$I_i' = I_i, \quad W_i = r_i + l_i, \quad l_{ci} = L_i \left(\sum_{k=1}^{i-1} m_k \Big/ \sum_{k=1}^{3} m_k\right) \quad (10)$$

The vectors W_i represent the link lengths of the DEM, m_i' is the mass of the ith link of the DEM, I_i' is the inertia tensor, and l_{ci} is the vector from the DEM's ith joint to the CM of the ith link, respectively. Fig. 2 shows space manipulator and its corresponding DEM.

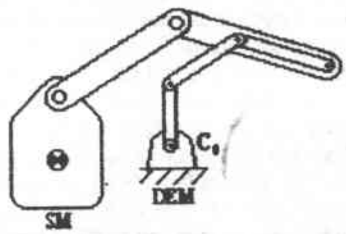

Fig. 2 The space manipulator and its corresponding DEM

Using above parameters and conventional Lagrangian Dynamics approach for fixed-base manipulators [4], the dynamics of the space manipulator system is derived as:

$$M1(q)\ddot{q} + C1(q,\dot{q})\dot{q} = \begin{bmatrix} 0 \\ \tau_1' \\ \tau_2' \end{bmatrix} \quad (11)$$

where M1 and C1 are the inertia, and Coriolis and centrifugal matrices of the space manipulator system when the dynamics has derived using the DEM approach. The entries of these matrices are given in

Appendix B. τ_1' and τ_2' are the torques applied to the joints of the DEM.

Liang et. al [3] have shown that dynamics for a general space manipulator system derived using the VM and the DEM methods are equivalent. In this paper, we show the equivalence of the dynamics of a two-link planar space manipulator through simulations. We performed some control experiments. Following section presents simulation results.

III. Simulation Results

For simulation a two-link space manipulator system is used. The parameters of the space manipulator are given in Table 1.

Table. 1 Parameters of the space manipulator

Link	L_i (m)	R_i (m)	m_i (kg)	I_i (kg.m^2)
Base	-	0.5	4	0.4
1	0.5	0.5	1	0.1
2	0.5	0.5	1	0.1

The desired trajectories for the manipulator joints are

$\theta_1(t) = 0.5\sin(\pi t) rad$

$\theta_2(t) = 0.5\cos(\pi t) rad$

We factored out $\ddot{\alpha}$ in the first line of the equations (9) and (11) and substituted the results in the second and third lines of the corresponding equations. Thus, we obtained the open-loop relationship between the driving torques and the controlled joint angles. For the space manipulator

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = H\begin{pmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{pmatrix} + \gamma \qquad (12)$$

where

$$H = \begin{bmatrix} M(2,2) - k_1M(1,2) & M(2,3) - k_1M(1,3) \\ M(3,2) - k_2M(1,2) & M(3,3) - k_2M(1,3)\end{bmatrix} \qquad (13)$$

$$\gamma = \begin{pmatrix} C(1,1)\dot{\alpha} + C(1,2)\dot{\theta}_1 + C(1,3)\dot{\theta}_2 \\ C(2,1)\dot{\alpha} + C(2,2)\dot{\theta}_1 + C(2,3)\dot{\theta}_2 \\ C(3,1)\dot{\alpha} + C(3,2)\dot{\theta}_1 + C(3,3)\dot{\theta}_2 \end{pmatrix} \qquad (14)$$

and $k_1 = M(2,1)/M(1,1)$, $k_2 = M(3,1)/M(1,1)$.

For the DEM, relation between the driving torques and the joint angles are

$$\begin{bmatrix}\tau_1'\\ \tau_2'\end{bmatrix} = H'\begin{pmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{pmatrix} + \gamma' \qquad (15)$$

In H' and γ' entries of M1 and C1 are used. We selected a PD computed-torque controller [5] to control the manipulator's actuators. The form of the controller is

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = H(\ddot{\theta}_d + K_v\dot{e} + K_pe) + \gamma \qquad (16)$$

where $\ddot{\theta}_d$ is the vector of the desired trajectories, e is the trajectory error vector. Controller gains are selected as

$K_v = diag(10,10)$ and $K_p = diag(80,80)$.

Simulation results are given in Figures 3-8. Trajectories of the base angle and the joint angles for the space manipulator and the DEM are given in Fig. 3 and Fig. 4, respectively.

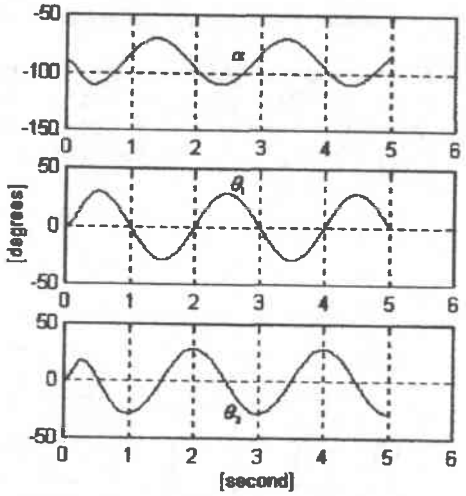

Fig.3 Base and joint angles of the space manipulator

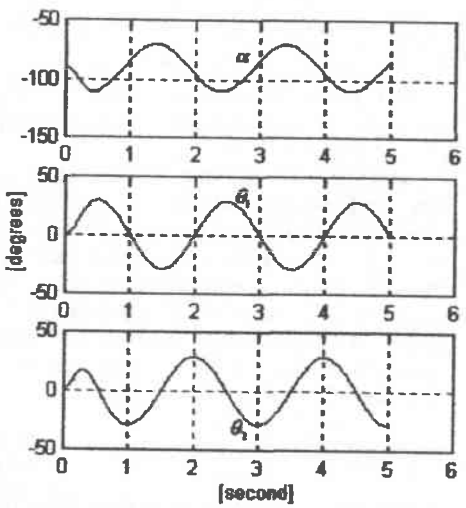

Fig.4 Base and joint angles of the DEM

As we can see, the space manipulator and the DEM behave identically when under the action of the same controller. The tracking errors of the joint angles of the DEM are shown in Fig. 5 and they go to zero in a short time.

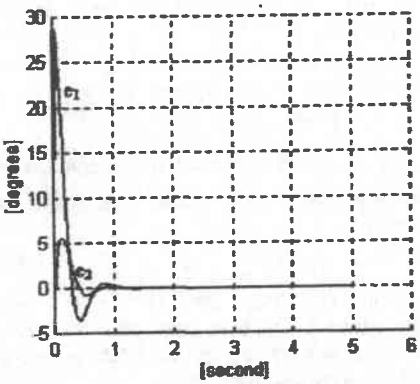

Fig.5 Tracking errors of the joint angles of the DEM

Differences between the base angles and the joint angles of the space manipulator and the DEM are given in Fig. 6. The maximum deviations are less than 0.3 and 0.02 degrees for the base angle and joint angles, respectively.

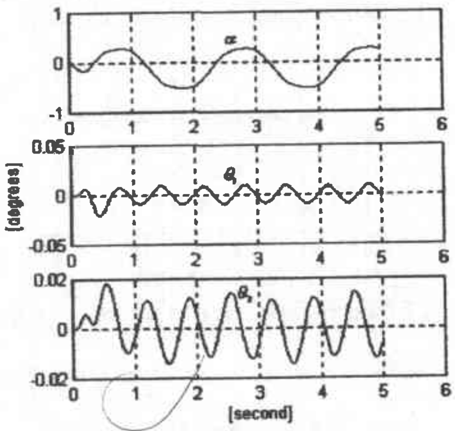

Fig.6 Differences between the base and the joint angles of the space manipulator and the DEM

Torques applied to the joints of the space manipulator and the DEM are given in Fig. 7 and Fig. 8. They behave identically, but the torques applied to the joints of the space manipulator are greater than that of the DEM. This experiment demonstrates that it is possible to simulate the behavior of a complex free-floating space manipulator system through the control of a simple fixed-base manipulator with a passive joint at the base.

IV. Conclusion

In this paper, dynamics of a two-link free-floating planar space manipulator is derived using the VM and the DEM approaches. The same PD computed-torque controller is applied to both dynamics and it is shown that the VM and the DEM behave identically.

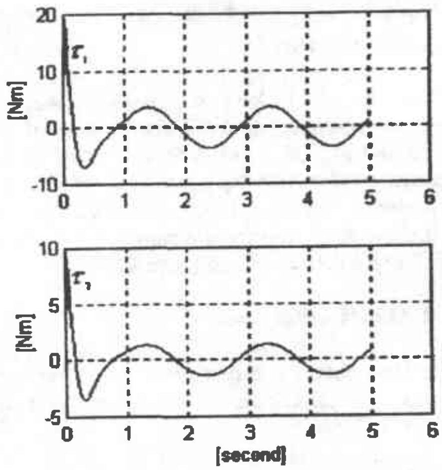

Fig. 7 Torques applied to the joints of the space manipulator

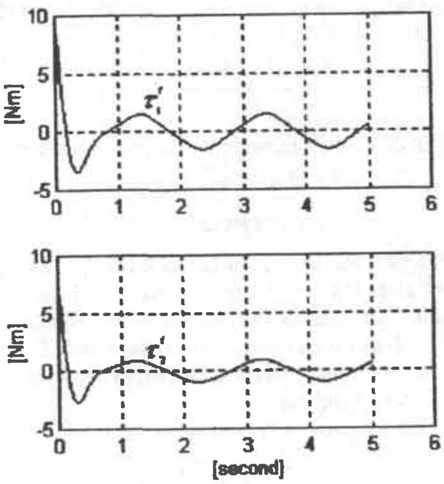

Fig.8 Torques applied to the joints of the DEM

Appendix A

The entries of $\mathbf{M}(q)$ and $\mathbf{C}(q,\dot{q})$ are given as follows:

$$M_{11} = a_1 + a_2 + a_3 + 2a_4\cos\theta_1 + 2a_5\cos(\theta_1+\theta_2) + 2a_6\cos\theta_2 + I_1 + I_2 + I_3$$

$$M_{12} = a_2 + a_3 + a_4\cos\theta_1 + a_5\cos(\theta_1+\theta_2) + 2a_6\cos\theta_2 + I_2 + I_3$$

$$M_{13} = a_3 + a_5\cos(\theta_1+\theta_2) + a_6\cos\theta_2 + I_3$$

$$M_{21} = M_{12}$$

$$M_{22} = a_2 + a_3 + 2a_6 \cos\theta_2 + I_2 + I_3$$
$$M_{23} = a_3 + a_6 \cos\theta_2 + I_3$$
$$M_{31} = M_{13}$$
$$M_{32} = M_{23}$$
$$M_{33} = a_3 + I_3$$

$$C_{11} = -a_4 \sin\theta_1\dot{\theta}_1 - a_5 \sin(\theta_1 + \theta_2)(\dot{\theta}_1 + \dot{\theta}_2) - a_6 \sin\theta_2\dot{\theta}_2$$
$$C_{12} = -a_4\dot{\alpha}_1 \sin\theta_1 - a_5\dot{\alpha}_{12} \sin(\theta_1 + \theta_2) - a_6 \sin\theta_2\dot{\theta}_2$$
$$C_{13} = -a_5\dot{\alpha}_{12} \sin(\theta_1 + \theta_2) - a_6\dot{\alpha}_{12} \sin\theta_2$$
$$C_{21} = a_4\dot{\alpha} \sin\theta_1 + a_5\dot{\alpha} \sin(\theta_1 + \theta_2) - a_6\dot{\theta}_2 \sin\theta_2$$
$$C_{22} = -a_6\dot{\theta}_2 \sin\theta_2$$
$$C_{23} = -a_6\dot{\alpha}_{12} \sin\theta_2$$
$$C_{31} = a_5\dot{\alpha} \sin(\theta_1 + \theta_2) + a_6\dot{\alpha}_1 \sin\theta_2$$
$$C_{32} = a_6\dot{\alpha}_1 \sin\theta_2$$
$$C_{33} = 0$$

where

$$a_1 = r_1^2(m_1 + m_2 + m_3)$$
$$a_2 = m_1A^2 + m_2C^2 + m_3D^2$$
$$a_3 = m_1B^2 + m_2B^2 + m_3E^2$$
$$a_4 = (m_1A + m_2C + m_3D)r_1$$
$$a_5 = (m_1B + m_2B + m_3E)r_1$$
$$a_6 = m_1AB + m_2BC + m_3DE$$

Appendix B

The entries of **M1**(q) and **C1**(q, $\dot{q}$)are in the same form of **M**(q) and **C**(q, $\dot{q}$), but a_i $i = 1,\ldots,6$. should be substituted by the following b_i $i = 1,.,6$. values.

$$b_1 = r_1^2(m_2' + m_3')$$
$$b_2 = m_2'l_2^2 + m_3'(r_2 + l_2)^2$$
$$b_3 = m_3'l_3^2$$
$$b_4 = r_1(m_2'l_2 + m_3'(r_2 + l_2))$$
$$b_5 = m_3'r_1l_3$$
$$b_6 = m_3'(r_2 + l_2)l_3$$

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