AN ANT COLONY OPTIMIZATION METHOD WITH LOCAL SEARCH FOR STABILITY ANALYSIS OF LINEAR TIME-INVARIANT TIME-DELAY SYSTEMS

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ABSTRACT

A hybrid search method based on Ant Colony Optimization for Continuous Domains ($ACO_{\mathbb{R}}$) and the Nelder-Mead Simplex algorithm is proposed to calculate the stability margin of linear time-invariant time-delay systems. The effectiveness of the Nelder-Mead Simplex procedure is combined with the global search power of the $ACO_{\mathbb{R}}$ method to obtain the stability margin of linear time-invariant time-delay systems. Also, the problem is formulated in a way that results in a constant-dimension search space (the complex plane) regardless of the order of the system. Simulation results indicate successful assessment of stability for the systems under examination.

I. INTRODUCTION

Time-delays are present in many real-world systems including telecommunication and mechanical systems. Time-delays can have an adverse effect on the stability of a system [2]. This necessitates having proper tools for the stability analysis of time-delay systems. Many researchers have investigated stability of time-delay systems [1, 3, 4, 5, 6, 7, 8], and it is still an active research area.

In this paper the stability analysis of linear time-invariant time-delay (LTITD) systems in state-space representation is investigated. It is assumed that the dynamics of the system is represented by the state-space equation:

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{x}(t-\tau) \tag{1}$$

where A and B are constant matrices and τ is the constant time lag.

Lee and Dianat [6] used Lyapunov's direct method for the stability analysis of an LTITD system. They proved that the system is asymptotically stable if and only if for all matrices $Q(t)$ that satisfy

$$\dot{Q}(t) = (A+Q(0))Q(t);\; 0 \le t \le \tau \tag{2}$$

with the boundary condition $Q(\tau) = B$ and $Q(t) = 0$ for $t \notin [0,\tau]$, the eigenvalues of $(A+Q(0))$ have negative real parts. It can be seen easily that if $Q(t)$ is a solution of equation (2), it must satisfy the nonlinear matrix equation:

$$Q(\tau) = e^{(A+Q(0))\tau} Q(0) = B \tag{3}$$

Based on Lee and Dianat's work, Kim and Bae [5] formulated the stability analysis of system (1) as a constrained optimization problem,

$$\begin{aligned} &\Gamma(A,B,\tau) = -\max_{Q(0),i} real(\lambda_i(A+Q(0))) \\ &s.t.:\; e^{(A+Q(0))\tau} Q(0) = B \end{aligned} \tag{4}$$

and to solve this problem, they used constrained simulated annealing and Newton's method.

In this paper, ant colony optimization for continuous domains ($ACO_{\mathbb{R}}$) [12] is combined with Nelder-Mead simplex local search method (NM) to obtain solutions to this constrained optimization problem.

The characteristic equation of system (1) can be written as,

$$C(s) = \det(sI - (A + Be^{-\tau s})) = 0 \tag{5}$$

The linear time-invariant system (1) is asymptotically stable if and only if all its characteristic roots lie in the left half of the complex s-plane. Thus, the stability margin of the system (1) can be reformulated as:

$$\begin{aligned} &\Gamma(A,B,\tau) = -\max_{s} real(s) \\ &s.t.:\; \det(sI - (A+Be^{-\tau s})) = 0 \end{aligned} \tag{6}$$

The paper is organized as follows: In section II, the proposed method is introduced. Section III presents the simulation results. Section IV concludes the paper and gives directions for further research.

II. STABILITY ANALYSIS USING $ACO_{\mathbb{R}}$ AND NM METHODS

Informally, constrained optimization problem (6) is the problem of finding the *rightmost* zero of the characteristic equation, $C(s)=0$. One way to find the solution is using an iterated local search (ILS) procedure which produces different zeros of $C(s)$ until a root with maximal real part is found.

The problem of finding a function's root has been extensively studied in the literature and various methods have been suggested [10]. One method is formulating the problem as an optimization problem; if a function $F(s)$ has a zero at s_0, $|F(s)|$ reaches its minimum value at $s=s_0$. The value $|F(s)|$ can be used as the cost function for any heuristic search method. In this paper the NM method is used to find the zeros of the characteristic equation (5). The interested reader is referred to [9, 10] for details on the Nelder-Mead simplex method.

Since in a local search method the final answer is dependent on the starting point, the choice of the starting point should be diverse enough to ensure that all zeros of the characteristic equation are obtained.

In this paper we use $ACO_{\mathbb{R}}$ to find the starting points of the NM algorithm. Let $M(s)$ be a function that assigns to each point on the complex plane, the result of the NM algorithm. That is, if s_0 is the starting point and s_1 is the result of the local search, then we have $s_1 = M(s_0)$. The constrained optimization problem (6) can be converted to the following equivalent constrained optimization problem:

$$\begin{aligned} &\Gamma(A,B,\tau) = -\max_{s} real(M(s)) \\ &s.t.:\ C(M(s)) = 0 \end{aligned} \tag{7}$$

It is noted that the local search algorithm may end up in a local minimum of $|C(s)|$, thus violating the constraint.

In order to solve the above constrained optimization problem, a method based on $ACO_{\mathbb{R}}$ and stochastic ranking [11] is used. A summary of this algorithm to find the stability margin of the system (1) is presented in the following:

1. Create an archive of random solutions and apply the NM algorithm to obtain the cost $real(M(s))$ and the constraint violation $|C(M(s))|$.
2. Sort the solution archive according to stochastic ranking as described in [11].
3. While termination conditions are not met:
   - Create a number of new solutions based on the archive and the $ACO_{\mathbb{R}}$ procedure as described in [12] and apply the NM algorithm to obtain the cost $real(M(s))$ and constraint violation $|C(M(s))|$; add the solutions to the end of the solution archive
   - Sort the solution archive according to stochastic ranking as described in [11]; this will cause the worst solutions to move to the end of the archive and finally to get eliminated.

It is noted that in the present work a somewhat modified bi-objective version of the NM algorithm is used. Instead of solely comparing the cost values at different points on the complex plane, the solutions are compared based on the following criteria:

1. $|C(s)|$, the cost values
2. In the case of equal costs, absolute value of the real parts

In this way a bias towards the right half-plane is introduced into the local search which aids the global search to find the rightmost zero faster.

III. SIMULATION RESULTS

Two examples are taken from [5] to show the ability of the proposed method to calculate the stability margin of LTITD systems. The solution archive for the $ACO_{\mathbb{R}}$ global search has a size of 10 and 2 new solutions are generated at each iteration. The search process is terminated if after 10 iterations no new zero of the characteristic equation is found. The other settings of the algorithm are typical settings as mentioned in [12] with the stochastic ranking probability parameter P_f equal to 0.475.

Example 1. Consider the linear time-delay system of the form:

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}-2 & 0\\ 0 & -0.9\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix}\begin{bmatrix}x_1(t-\tau)\\ x_2(t-\tau)\end{bmatrix} \tag{8}$$

In [7] this system has been examined as an uncertain time-delay system, and the stability of this system is delay dependent. Based on analytical results presented in [3, 4, 8], it is known that for time-delay values greater than 6.1726 the system in unstable.

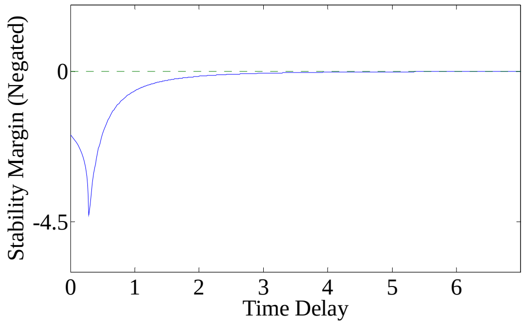

Figure 1. Stability margin vs. time delay for example 1

In [5] the authors have obtained the stability margin of the system for time delays in the range $[0,7]$ using constrained simulated annealing. They have also used Newton's method [10] as the local search method.

In this work, the stability margins for the system (8) have been computed for a range of time delays similar to [5]. The results are graphically shown in Figure 1.

As seen in Figure 1, the stability margin characterized by $-\Gamma(A,B,\tau)$ (see (7)) increases for delay values up to 0.29 where the stability margin is approximately 4.3. For time delay values larger than 0.29, the stability margin of the system decreases until the system becomes unstable at 6.17. This is in agreement with the results reported in [3, 4, 5, 8].

The results and the graph produced by the proposed method are similar to those given in [5]. In other words, at each time delay value, both methods have obtained similar values for the stability margin of the system.

Example 2. Consider the linear time-delay system of the form:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \alpha \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(t-\tau) \\ x_2(t-\tau) \end{bmatrix} \quad (9)$$

where $\alpha = 1.5$ is a constant. The characteristic equation of this system is $C(s) = s^2 + s + 1 + \alpha s e^{-\tau s} = 0$. It can be seen that this system is stable for $\tau = 0$.

In [5] the stability margin of the system has been obtained for time delays in the range $[0,2]$ using constrained simulated annealing, followed by Newton's method [10] as the local search algorithm.

In this work, the stability margins for the system (9) have been computed for a range of time delays similar to [5]. The results are graphically shown in Figure 2.

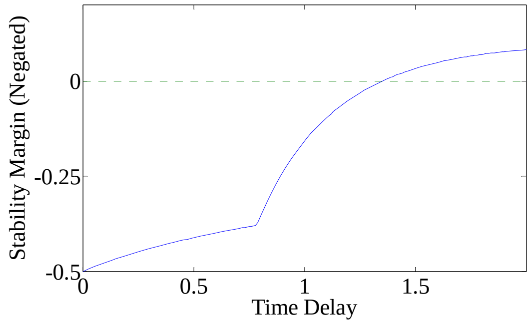

Figure 2. Stability margin vs. time delay for example 2

It is seen in Figure 2 that for $\tau = 0$, the stability margin of the system characterized by $-\Gamma(A,B,\tau)$ (see (7)) decreases uniformly by increasing time delay values. The rate of decrease increases at $\tau \simeq 0.79$. The system becomes unstable at $\tau \simeq 1.34$.

It is noted that on this second example, also, our results match with those reported by Kim and Bae [5]. Both methods have found identical values for the stability margin at any value of time delay in the range $[0,2]$.

IV. CONCLUSIONS AND FUTURE WORK

A new method for the stability assessment of linear time-invariant time-delay systems using the characteristic equation of the system has been proposed. Simulation results agree with those reported in the literature.

In comparison with Kim and Bae's work [5], the method has the advantage of a reduced dimensionality of the search space. While in their method the dimension of the search space grows quadratically with the order of the system, our method has a search space with a constant dimension of 2 (the complex plane). For example, if the order of the time-delay system is 6, the method in [5] needs to search a 36-dimensional space while our method still searches the complex plane for the roots of the characteristic equation.

Also, as it is known, in using the Newton's method, the need for calculating the derivatives of the function can cause difficulties. It requires more computational effort to obtain the roots of the characteristic equation. Also, numerical errors can be large if the characteristic function is too steep at the point where the derivative is calculated.

In our approach, Newton's method is substituted by the simple and effective method of Nelder-Mead. The effectiveness of the Nelder-Mead simplex algorithm is combined with the global search power of the $ACO_{\mathbb{R}}$ method to obtain the stability margin of LTITD systems.

The proposed method can also be used for the robust stabilization of LTITD systems. As seen in this paper, the proposed method can effectively give the stability margin of an LTITD system at different time-delay values. A heuristic search procedure can be used to find the parameters of a controller for the system and then the proposed method can verify if the resulted system is robustly stable for a certain range of time-delays.

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