

# THE OBSERVATION OF CHAOTIC OSCILLATIONS IN POWER SYSTEMS WITH STATIC VAR COMPENSATOR

Kadir ABACI      M.Ali YALÇIN      Yılmaz UYAROĞLU      Murat YILDIZ

*Sakarya University Dept. of Electrical and Electronics Engineering Esentepe Campus, Sakarya TURKEY*

kabaci@engineer.com, yalcin@sakarya.edu.tr, uyaroglu@sakarya.edu.tr, myildiz@sakarya.edu.tr

## Abstract:

An electric power system shows a chaotic behavior on an interval of loading state. Another type of system instability is voltage collapse when the system is too loaded. Voltage-collapse can be simply described as “not able to supply enough reactive power to the load bus.” By changing its reactive power output, a Static Var Compensator (SVC) regulates the voltage at the point to which it is connected. Voltage collapse dynamics were modeled by the movement of the system state along a particular trajectory at a saddle node bifurcation. In voltage collapse analysis, bifurcation points needed in order to the sample power system are attained. Chaotic behavior has been observed in computer simulations for a sample system to the 5 bus Hale network including SVC.

**Keywords:** Voltage collapse, Saddle-node bifurcation, SVC, Chaotic behavior

## INTRODUCTION

In voltage collapse analysis, bifurcation points needed in order to the sample power system are attained by using computational method. During the past decade utilities have reported serious complications in maintaining network stability in their power systems, particularly voltage stability, as some events occur and parameters change in the systems [1]. Chaotic phenomena are one type of deterministic oscillation existing in deterministic systems. Study on chaotic phenomena is one important part of power system stability studies. Early studies such as Ajarapu and Lee, Chiang [7], Wang and Tan, mainly focused on interpreting the behavior of chaotic oscillations in power systems. Later studies such as Wu, Rajesh, Yu and Jia, Srivastava and Abed began to concern the interaction of chaotic motion and system dynamic components, the relationship of power system stability region and chaos, and the methods of preventing and eliminating power system chaotic oscillations [2].

Hopf and saddle-node bifurcations(SNB) have been recognized as some of the reasons, albeit not the only ones, for voltage stability problems in a variety of power system models [4,6]. In the choice of bifurcation parameters, they indicated that the reactive power demand at the load bus as a bifurcation parameter is unrealistic and cannot characterize a wide range of operating conditions [3]. In practice, the most severe system and load conditions are generally known and allow the required rating of the SVC to be determined [5]. It is a fast acting static reactive power compensator, which can

damp out the power oscillations and cope with the voltage problem due to reactive power deficit. We consider a general nonlinear system described by a set of differential equations in n-dimensional Euclidian space,

$$\dot{x} = f(x) \quad (1)$$

$$\dot{x} = f(x, \lambda) \quad (2)$$

where generally the state vector  $x$  may consist of generator angle, generator angular velocity, load voltage magnitude, etc. the parameter  $\lambda$  may be real, reactive power or input power to the generator, etc. At a fixed point (a equilibrium point or a steady-state solution)  $x(\lambda_0)$ , since the right-hand term of Eq.(2) becomes zero, its stability is dominated by the eigenvalues of the Jacobian  $J = \partial f / \partial x$  evaluated at  $x(\lambda_0)$  [3].

Next, we discuss four steady-state behaviors associated with the nonlinear system (1): equilibrium points, saddle nodes and chaos. Chaos, also called strange attractor, has no generally accepted precise mathematical definition. Usually, from a practical point of view, it can be defined as bounded steady-state behavior which does not fall into the categories of the other three steady-state behaviors, i.e., equilibrium points, periodic solutions, and quasi-periodic solutions [7]. At a SNB point, two equilibrium points, generally one stable and one unstable, coalesce and become a saddle-node point, and then disappear as the parameter passes through the bifurcation value. At a bifurcation the Jacobian has a zero eigenvalue and hence the determinant of the Jacobian is zero. Therefore, the necessary conditions for SNB are given following,

$$f(x_0, \lambda_0) = 0, \quad \det J(f(x_0, \lambda_0)) = 0$$

SNB is considered as a main reason for dynamic instability of the system (2) and is associated with voltage collapse problems in power systems [8,9,10].

In this paper, chaotic oscillations are determined in power systems controlled static var compensator. Firstly, chaotic phase portraits are observed in a simple power system. The final section of this paper concentrates on applying the results obtained for the sample system to the 5 bus Hale network. The concepts of Power System Model are firstly defined in section 2. In section 3, bifurcation analysis and chaos are described in sample power system. Finally, simulation results are shown to demonstrate the effectiveness of our proposed method.

## 2. POWER SYSTEM MODEL

The simple two bus system is shown in figure 1. The p.u dynamic equations that represent this system, using a basic dynamic generator model, a frequency and voltage dependent dynamic model for the load and dynamic SVC model, are given by

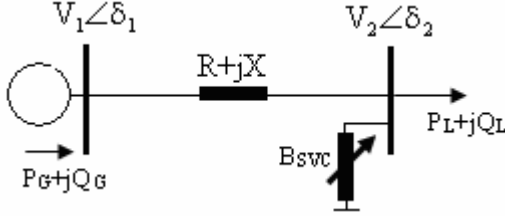


Figure 1. A basic power system having SVC at the end of transmission line.

### 2.1 Generator Dynamic Model.

The dynamic equations for machine are given by:

$$\dot{\omega} = \frac{1}{M}(P_M - P_G - D_G \omega) \quad (3)$$

$$\dot{\delta} = \omega - \frac{1}{D_L}(P_L - P_D) \quad (4)$$

### 2.2 Load Model

The dynamic equations for load at a bus are given by:

$$\dot{V}_2 = \frac{1}{\tau}(Q_L - Q_D) \quad (5)$$

### 2.3 Dynamic Model of SVC

In this paper, SVC has been represented by Basic Model-1 [12].

$$\dot{B}_{ref} = \frac{1}{T_R}[-B_{ref} + K_R(V_{ref} - V_2)] \quad (6)$$

$$\dot{B}_{SVC} = \frac{1}{T_B}(-B_{SVC} + B_{ref}) \quad (7)$$

where  $T_R$  and  $K_R$  are time and gain constants of voltage regulator, and also  $T_B$  ve  $B_{ref}$  are time constant and reference susceptance values of SVC.

$$\delta = \delta_1 - \delta_2$$

$$P_G = V_1^2 G - V_1 V_2 (G \cos \delta - B \sin \delta) \quad (8)$$

$$P_L = -V_2^2 G + V_1 V_2 (G \cos \delta + B \sin \delta) \quad (9)$$

$$Q_G = V_1^2 B - V_1 V_2 (G \sin \delta + B \cos \delta) \quad (10)$$

$$Q_L = -V_2^2 (B - B_{SVC}) - V_1 V_2 (G \cos \delta - B \sin \delta) \quad (11)$$

$$G = \frac{R}{R^2 + X^2}, \quad B = \frac{X}{R^2 + X^2}$$

$Q_G$  is used to represent generator reactive limits. If  $Q_{G_{min}} \leq Q_G \leq Q_{G_{max}}$ , the generator voltage  $V_1$  is

assumed to be controlled to represent somewhat the control actions of a voltage regulator or AVR; thus, neglecting droop and the control system dynamics, the voltage regulator is modeled here by keeping the generator terminal voltage at the fixed value  $V_1 = V_{10} = 1$ . where the generator inertia and damping constants are represented by  $M$  and  $D_G$ , and  $D_L$  and  $\tau$  stand for the dynamic load frequency and voltage time constants respectively [11]

## 3. BIFURCATION ANALYSIS AND CHAOS IN A SIMPLE POWER SYSTEM

The steady state load demand is modeled through the parameter  $P_d$ , under the assumption that reactive power demand is directly proportional to the active power demand, i.e.  $Q_d = k.P_d$ ; this parameter is used here to carry out the voltage collapse studies. SVC operated capacitive mode figures out compensation effect for power system stability. To simplify the stability analysis, the resistance is neglected ( $R=0$ ),  $P_m = P_d$ . The initial loading condition, as considered or not, as discussed below. The p.u time constants are assumed to be  $M = 0.9$ ,  $D_G = 0.001$ ,  $D_L = 0.01$ ,  $\tau = 40$ ; the load power factor is assumed to be 0.97 lagging, i.e.,  $k = 0.25$ , and reactance of transmission line  $X = 0.5$  pu. SVC have injected reactive power at 0.518 pu value in order to  $V_2 = 1.0$  pu value of load bus, meanwhile, voltage of load bus is  $1.0 \angle -30.0^0$  pu value, afterwards load flow analysis. Values belonging to limit points of system,  $X^* = [\omega; \delta; V; B_{svc}; P_d]$  state variables vector is attained as  $[0.0; 0.6629; 0.8561; 0.518; 1.0537]$ .

At the end of simulation, from graphics at the  $P_d = 1.06$  pu value achieved figure 2.a), system can be noticed to lead to be unstable at the  $P_d = 1.06$  pu value. Fig. 2.b depicts the  $P_d$ -V curve for the simple system. As expected the "nose" is the SNB point and the load active power demand at the SNB point is  $P_d^{SNB} = 1.0537$  p.u.

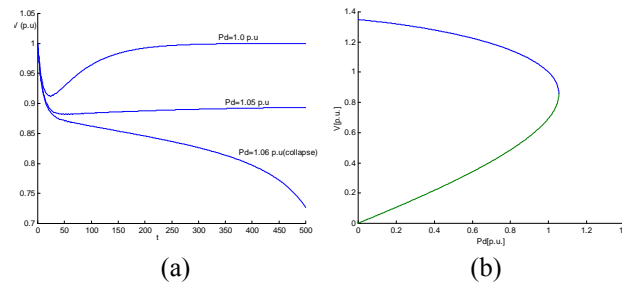


Figure 2. Graphic simulations, a) Variations of voltage  $V_2$  for different  $P_d$  values, b)  $P_d$ -V curve for  $B_{SVC} = 0.518$  p.u.

Phase portraits of state space for three different operating points are shown in fig. 3. Since  $P_d$  is less than  $P_d^{SNB}$  value of 1 p.u and 1.05 p.u, the stability of the oscillations and convergence of the equilibrium point are shown in fig.3.a and fig.3.b respectively; whereas in fig.3.c, the

system goes into voltage collapse induced by the saddle node bifurcation, since  $P_d=1.06 > P_d^{SNB}$ .

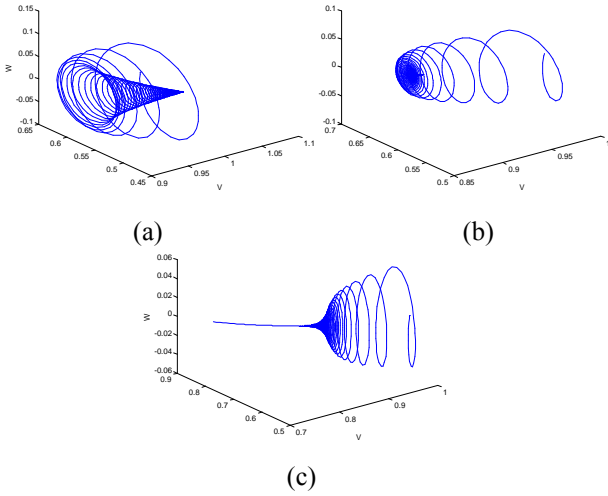


Fig.3. Phase portraits, at the different load values, ( $V_2$ - $\delta$ - $w$ ) a)  $P_d=1.0$ , b)  $P_d=1.05$ , c)  $P_d=1.06$

The voltage collapse is caused by the load dynamic. However, observe that, as a consequence of the voltage instability, also the generator rotor angle presents an unstable trajectory in figure 4b.

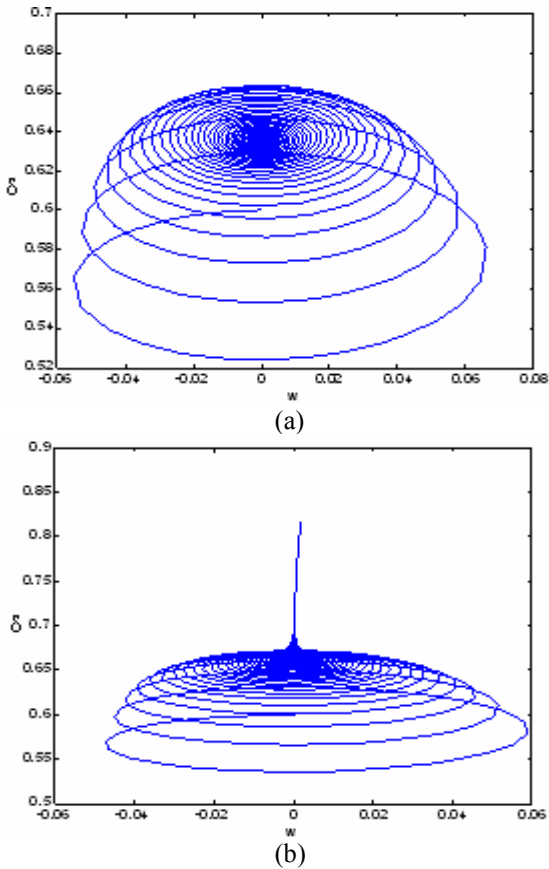


Figure 4. Phase portraits between generator angle  $\delta$ , and gen. angular velocity  $w$ , a)  $P_d=1.05$ , b)  $P_d=1.06$  (instability)

#### 4. APPLIED TO N-BUS POWER SYSTEMS

The our method explained at appendix is tested for well-known the 5 bus Hale network as shown in figure 5. The generators are set to control voltage magnitudes at the Slack bus North and the PV bus South at 1.06 p.u and 1 p.u., respectively. One SVC is placed at the Lake to keep voltage magnitude at that bus at 1 p.u. Nodal voltages are given in Table 1. The SVC injects 20.47 MVAR into Lake and keeps the nodal voltage magnitude at 1 p.u.

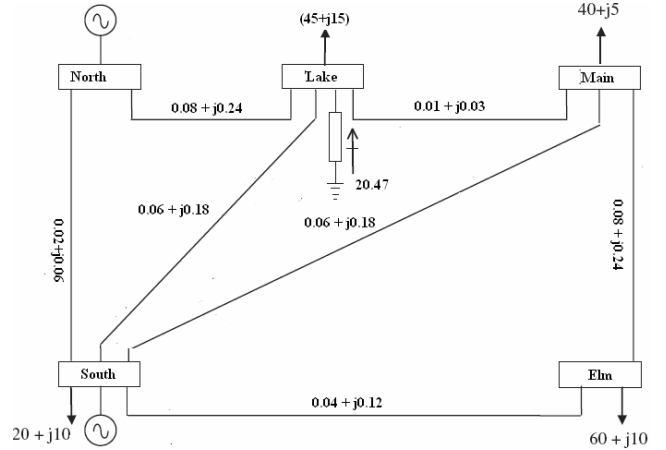


Figure 5. Five bus Hale network [13] ( $S_{base}=100$  MVA)

According to results obtained from load flow conclusions, the sample network shown in figure 5, is changed into basic two-bus power system using “The Reducing Bus Method”. Bus admittance matrix  $Y_{bus_{red}}$

$$Y_{bus_{red}} = \begin{bmatrix} 3.4135 - j10.3112 & -3.2273 + j9.8877 \\ -3.2273 + j9.8877 & 3.8427 - j10.2099 \end{bmatrix}$$

Where, the parameters of pi equivalent circuit having the 2-bus are  $\dot{A}=1.0478+j0.0466$ ,  $\dot{B}=0.0298+j0.0914$ ,  $\dot{C}=0.8302 +j0.7572$ ,  $\dot{D}=1.0443+j0.0044$ , respectively. It is shown that voltage of Lake bus, susceptance of SVC, generator angular velocity, and generator angle for stable operation points of system in figure 6.

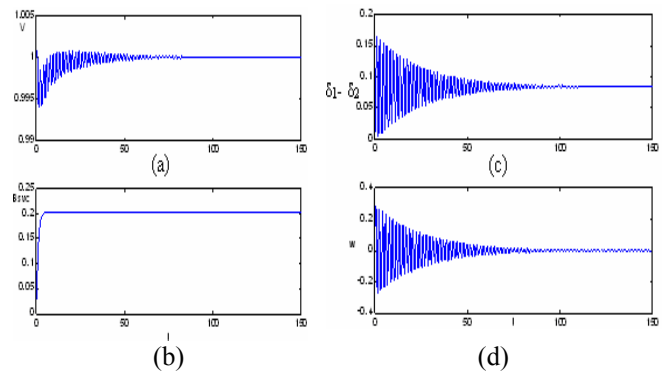


Figure 6. For  $P_d=0.45$  at stable operating points, a) Load Voltage b) SVC susceptance c) Difference with load and Generator angle ( $\delta = \delta_1 - \delta_2$ ) d) generator angular velocity

Our obtained conclusions that result from load flow simulations shows similarity with given values at Table 1.

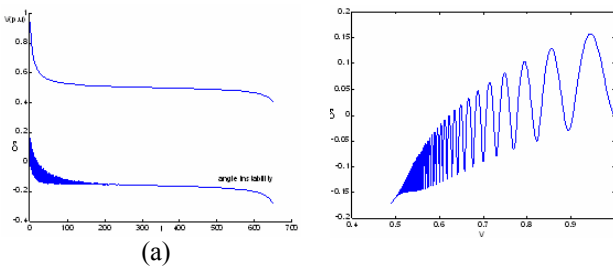
Table 1. Nodal voltages of modified network

Nodal voltage	Network bus				
	North	South	Lake	Main	Elm
Magnitude(p.u.)	1.00	1.000	1.000	0.994	0.975
Phase angle (radian)	0.0	-0.036	-0.084	-0.089	-0.101

#### 4.1. Hopf and saddle-node bifurcations

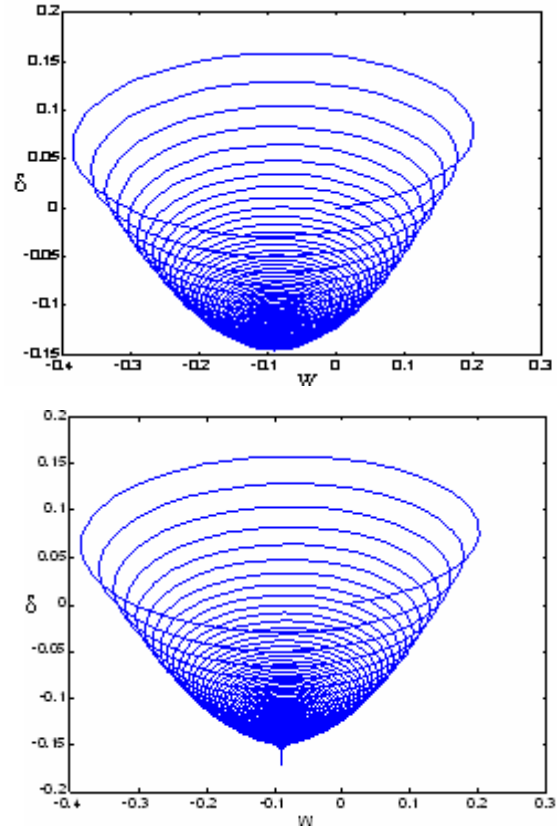
In this section, we analyze the stability and bifurcation points Eq.(3,4,5,7) where  $P_d$  is taken as a bifurcation parameter. These bifurcations are the saddle-node bifurcation and the Hopf bifurcation. All the results are obtained by detailed numerical simulations, and preliminary analyses are based on the mathematical theories and methods are given as appendix. The Jacobian matrix at the equilibrium has two eigenvalues of the system passed imaginary axis. So there is a Hopf bifurcation HB.  $\{-0.0391 \pm j3.4140, -0.2762, -0.100\}$ . It can be said that other eigenvalues have not imaginary part. The values concerning with limit points of system  $f(x^*) = [w^*; \delta^*; V^*; B_{svc}^*; P_d^*]^T$  state variables vector are achieved as  $[-0.0894; -0.1596; 0.5004; 0.2047; 8.8347]^T$ . At a  $P_d^{SNB} = 8.8347$  p.u., a saddle-node bifurcation SNB occurs which the Jacobian has a simple zero eigenvalue. The V- $\delta$  graphics between voltage magnitude and angle are very significant with respect to show state space system having the different loading condition. of the system is chosen as  $P_d^1 = 8.84$  p.u as shown in the figure 7.

Then, the obtained conclusions due to  $P_d^1 > P_d^{SNB}$  have shown to diverge from stable operation point of system. After the system is bifurcated, the power system converging to the voltage collapse has diverged from equilibrium point.



(b) Figure 7. For  $P_d^1 = 8.84$  p.u. a) Time series between generator voltage and generator angle b) Phase portrait between generator voltage and generator angle

It is chosen  $P_d^2 = 8.829$  p.u aiming to show being at the stable operating point prior to reach the SNB point of system. It is obviously seen being  $P_d^2 < P_d^{SNB}$ . The chaotic behaviour at the  $P_d^1$  and  $P_d^2$  points of the system can be observed in the figure 8.



(b) Figure 8. Phase portraits of generator angular velocity and generator angle for different values of  $P_d$  a)  $P_d^1 = 8.829$  (Stable), b)  $P_d^2 = 8.84$  (Unstable).

Phase portraits are achieved plotting three dimension graphics for  $P_d^1$  value (Fig.9), It can be shown that the power system undergoes a subcritical Hopf bifurcation at  $P_d^2 = 8.84$ , which is followed closely by the saddle-node bifurcation,  $P_d^{SNB}$ . For the power system example in equilibrium with  $P_d^1$  near  $P_d^{SNB}$ , if any perturbations take the operating point beyond the saddle-node, a fast monotonous divergence, i.e. a voltage collapse, can be expected.

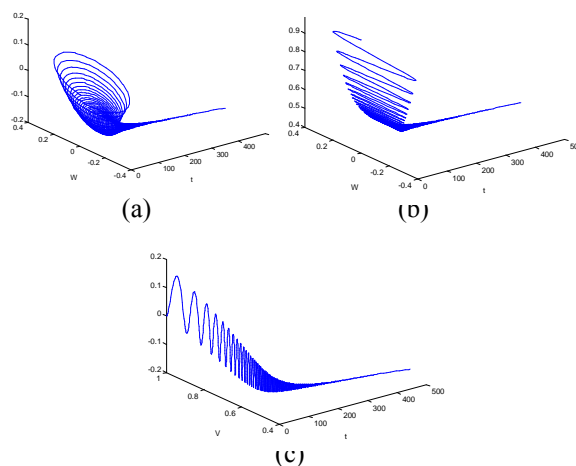


Figure 9. For  $P_d^1=8.84$  three dimension phase portraits ( $t=500$  sec.) a)t-w- $\delta$  b) t-w-V c)t- $\delta$ -V

## 5.CONCLUSIONS

In this paper, the relationships between chaos and power system instability are deeply studied.

A simple power system and 5 bus 2 generators Hale network have been observed in Computer simulations to become a chaotic system over a range of loading conditions. We illustrate that chaos can induce voltage collapse, angle divergence or voltage collapse with angle instability simultaneously when its stable condition is broken. In general, static bifurcations (saddle-node bifurcations) can be related to system collapse, and the oscillations of the quasi-periodic orbits or weak chaos affect the quality of electric power by distorting the voltage and current wave forms and can also cause the power system collapse in certain circumstances. However they are considered as one of the clues for the collapse of complicated power systems or stability margin narrowing in parameter space. All the studies are helpful to understand the various instability modes and to find effective anti-chaos strategies in power systems. Finally, simulation results are shown to demonstrate the effectiveness of the proposed method. Our proposed power system modeling significantly improves the power system stability and provides Hopf bifurcation and chaos control. Future work will concentrate on studying different FACTS devices and their effects on the stability of the power systems and the synthesis of a general "bifurcation control" technique for FACTS or TCSC controllers.

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