

A COMPUTER SIMULATION OF DILUTION OF PRECISION IN THE GLOBAL POSITIONING SYSTEM USING MATLAB

Fevzi Aytaç Kaya
e-mail: aytacfk@gmail.com

Gazi University, Faculty of Engineering & Architecture
Department of Electrical&Electronics Engineering
06570 - Maltepe, Ankara, Turkey

Müzeyyen Sarıtaş
e-mail: muzeyyen@gazi.edu.tr

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ABSTRACT

The aim of this study is to design a computer simulation of the Dilution of Precision (DOP) which affects the GPS accuracy. DOP simulator is a usefull interface which provides a fast and practical analysis of both the DOP parameters and satellite geometry. The simulator was designed as a graphical user interface in MATLAB.

I. INTRODUCTION

GPS devices are designed to send any positional data they calculate, regardless of its accuracy. Indeed, the latitude, longitude and altitude reported by GPS devices are frequently inaccurate by three hundred meters. This inaccuracy can lead to poor business intelligence, or, worse still, get an end-user into a precarious situation. Developers who wish to write location-based services must get smart about GPS precision. Fortunately, GPS devices provide information which can be used to make 300m inaccuracies a thing of the past. This information is known as "Dilution of Precision," and is the key to writing commercial GPS applications [1]. The Dilution of Precision (DOP) is a value of probability for the geometrical effect on GPS accuracy.

GPS devices calculate the position using a technique called "3-D multilateration," which is the process of figuring out where a number of spheres intersect. In the case of GPS, each sphere has a satellite at its center. The radius of the sphere is the calculated distance from the satellite to the GPS device. Ideally, these spheres would intersect at exactly one point, causing there to be only one possible solution to the current location, but in reality, the intersection forms more of an oddly shaped area. The device could be located within any point in the area, forcing devices to choose from many possibilities. Figure 1 shows such an area created from three satellites. The current location could be any point within the gray-colored area. Precision is said to be "diluted" when the

area grows larger, which leads to this article's focus: dilution of precision. The monitoring and control of dilution of precision is the key to writing high-precision applications [1].

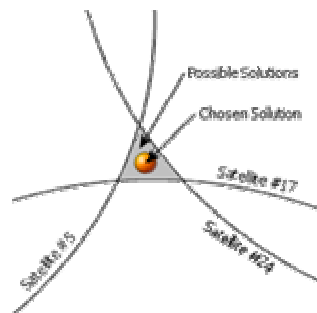


Figure 1: GPS devices must choose one of several possible solutions to the current location [1].

Several external sources introduce errors into a GPS position estimated by a GPS receiver. One important factor in determining positional accuracy is the constellation, or geometry, of the group of satellites from which signals are being received. DOP only depends on the position of the satellites: how many satellites you can see, how high they are in the sky, and the bearing towards them. This is often referred to as the geometry [2]. The computed position can vary depending on which satellites are used for the measurement. Different satellite geometries can magnify or lessen the position error. A greater angle between the satellites lowers the DOP, and provides a better measurement. A higher DOP indicates poor satellite geometry, and an inferior measurement configuration, or in other words: the lower the value the greater the confidence in the solution.

II. THEORETICAL BACKGROUND

Satellite navigation depends on accurate range measurements in order to determine the position of the user. Because the receiver clock is generally not synchronized with GPS system time, the range measurements are erroneous and are therefore called pseudoranges [3]. A pseudorange equation is thus:

$$PR_i = \sqrt{(X-X_i)^2 + (Y-Y_i)^2 + (Z-Z_i)^2} + ct_B \quad (1)$$

where: PR_i is the pseudorange to the i^{th} satellite (m)
 (X, Y, Z) is the unknown three dimensional user position,
 (X_i, Y_i, Z_i) is the known three dimensional satellite position (m),
 t_B is the unknown receiver clock offset (s),
 c is the speed of light (m/s).

Because there are four unknowns, at least four measurements are required to solve for X, Y, Z and t_B . Because the pseudorange equations are nonlinear, the system must be linearized in order to derive a linear relationship between pseudorange errors and position errors. If we linearize the GPS pseudo-ranges by using a first order Taylor Series expansion, we obtain a linear equation of the form:

$$\begin{pmatrix} \partial PR_1 \\ \partial PR_2 \\ \partial PR_3 \\ \partial PR_4 \end{pmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & 1 \\ h_{21} & h_{22} & h_{23} & 1 \\ h_{31} & h_{32} & h_{33} & 1 \\ h_{41} & h_{42} & h_{43} & 1 \end{bmatrix} \begin{pmatrix} \partial X \\ \partial Y \\ \partial Z \\ ct_B \end{pmatrix} \quad (2)$$

where the h_{mn} elements represent direction cosines to each of the satellites.

In general, this equation can be written as:

$$\partial \mathbf{Y} = \mathbf{H} \partial \boldsymbol{\beta} \quad (3)$$

Thus, a receiver modifies an initial estimate of $\boldsymbol{\beta}$ using $\partial \boldsymbol{\beta}$ and iterates until convergence is achieved.

The column of ones in \mathbf{H} shows that the receiver clock offset (ct_B) biases each pseudorange measurement by exactly the same amount [3]. In practice, there may be interchannel biases that affect each measurement differently. However, great care is taken by receiver manufacturers to calibrate these effects.

SATELLITE GEOMETRY

The GPS satellite geometry relates position errors to range measurement errors. Considering equation 1 with m pseudorange measurements. Then $\partial \mathbf{Y}$ is an $m \times 1$ vector, \mathbf{H} is an $m \times 4$ matrix, and as usual $\partial \boldsymbol{\beta}$ is a 4×1

vector. If we think of $\partial \boldsymbol{\beta}$ as a zero-mean vector containing the errors in the estimated user state, then we are interested in the statistics of $\partial \boldsymbol{\beta}$ because that will characterize the expected position errors. Using the generalized inverse of \mathbf{H} , we find the covariance of $\partial \boldsymbol{\beta}$ [4]:

$$\begin{aligned} \text{cov}(\partial \boldsymbol{\beta}) &= E[\partial \boldsymbol{\beta} \partial \boldsymbol{\beta}^T] = E[(\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \partial \mathbf{Y} \partial \mathbf{Y}^T \mathbf{H} (\mathbf{H}^T \mathbf{H})^{-T}] \\ &= (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T E[\partial \mathbf{Y} \partial \mathbf{Y}^T] \mathbf{H} (\mathbf{H}^T \mathbf{H})^{-T} \\ &= (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \text{cov}(\partial \mathbf{Y}) \mathbf{H} (\mathbf{H}^T \mathbf{H})^{-T} \end{aligned} \quad (4)$$

Now we have the covariance of $\partial \mathbf{Y}$, the pseudorange errors. These are assumed to be uncorrelated, Gaussian random variables. As such, they are statistically independent which results in a diagonal covariance matrix. Furthermore, the range measurement errors are assumed to have the same variance (σ_r) for each satellite. So, we have:

$$\text{cov}(\partial \mathbf{Y}) = \sigma_r^2 \mathbf{I} \quad (5)$$

which results in :

$$\begin{aligned} E[\partial \boldsymbol{\beta} \partial \boldsymbol{\beta}^T] &= \sigma_r^2 (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{H} (\mathbf{H}^T \mathbf{H})^{-T} \\ &= \sigma_r^2 (\mathbf{H}^T \mathbf{H})^{-T} \end{aligned} \quad (6)$$

It can easily be shown that $\mathbf{H}^T \mathbf{H}$ is symmetric, so the transpose is unnecessary:

$$\text{cov}(\partial \boldsymbol{\beta}) = \sigma_r^2 (\mathbf{H}^T \mathbf{H})^{-1} \quad (7)$$

Now the relationship between range measurement errors and position errors becomes clear. Let $\mathbf{G} = (\mathbf{H}^T \mathbf{H})^{-1}$ so that $\text{cov}(\partial \boldsymbol{\beta}) = \sigma_r^2 \mathbf{G}$. Using $\mathbf{B} = ct_B$ and expanding Eq. 7, we get:

$$\begin{bmatrix} \sigma_x^2 & \text{cov}(X,Y) & \text{cov}(X,Z) & \text{cov}(X,B) \\ \text{cov}(Y,X) & \sigma_y^2 & \text{cov}(Y,Z) & \text{cov}(Y,B) \\ \text{cov}(Z,X) & \text{cov}(Z,Y) & \sigma_z^2 & \text{cov}(Z,B) \\ \text{cov}(B,X) & \text{cov}(B,Y) & \text{cov}(B,Z) & \sigma_B^2 \end{bmatrix} = \begin{bmatrix} G_{xx} & G_{xy} & G_{xz} & G_{xB} \\ G_{yx} & G_{yy} & G_{yz} & G_{yB} \\ G_{zx} & G_{zy} & G_{zz} & G_{zB} \\ G_{Bx} & G_{By} & G_{Bz} & G_{BB} \end{bmatrix} \sigma_r^2 \quad (8)$$

The elements of \mathbf{G} give a measure of the satellite geometry called the dilution of precision (DOP) [5]. Various DOP values can be calculated from the diagonal elements of \mathbf{G} . For example:

$$\begin{aligned} \sigma_x^2 + \sigma_y^2 + \sigma_z^2 + \sigma_B^2 &= (G_{xx} + G_{yy} + G_{zz} + G_{BB}) \sigma_r^2 \\ \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 + \sigma_B^2} &= GDOP \sigma_r \end{aligned} \quad (9)$$

where GDOP is the Geometric Dilution of Precision defined as:

$$GDOP = \sqrt{G_{xx} + G_{yy} + G_{zz} + G_{BB}} \quad (10)$$

Other DOPs are as follows:

$$PDOP = \frac{\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}}{\sigma_r} = \sqrt{G_{xx} + G_{yy} + G_{zz}}$$

$$HDOP = \frac{\sqrt{\sigma_x^2 + \sigma_y^2}}{\sigma_r} = \sqrt{G_{xx} + G_{yy}}$$

$$VDOP = \frac{\sigma_z}{\sigma_r} = \sqrt{G_{zz}}$$

$$TDOP = \frac{\sigma_B}{\sigma_r} = \sqrt{G_{BB}} \quad (11)$$

where:

PDOP is the Position Dilution of Precision

HDOP is the Horizontal Dilution of Precision

VDOP is the Vertical Dilution of Precision

TDOP is the Time Dilution of Precision

While each of these GDOP terms can be individually computed, they are formed from covariances and so are not independent of each other. A high TDOP (time dilution of precision), for example, will cause receiver clock errors which will eventually result in increased position errors. Good GDOP, a small value representing a large unit-vector-volume, results when angles from receiver to satellites are different. Whereas poor GDOP, a large value representing a small unit vector-volume, results when angles from receiver to the set of satellites

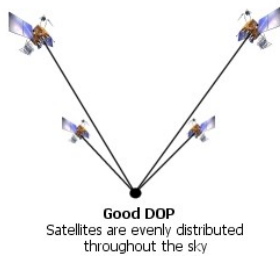


Figure 2 : Good satellite geometry [1].

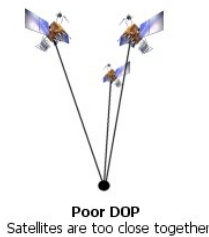


Figure 3 : Poor satellite geometry [1].

used are similar. Clustered satellites give poor GDOP values (Figures 2,3). The higher the DOP, the weaker the geometry.

ECEF COORDINATE REFERENCE FRAME

The Cartesian coordinate frame of reference used in GPS is called Earth-Centered, Earth-Fixed (ECEF). ECEF uses three-dimensional XYZ coordinates (in meters) to describe the location of a GPS user or satellite. The term "Earth-Centered" comes from the fact that the origin of the axis (0,0,0) is located at the mass center of gravity. The term "Earth-Fixed" implies that the axes are fixed with respect to the earth (that is, they rotate with the earth). The Z-axis pierces the North Pole, and the XY-axis defines the equatorial plane [6].

III. COMPUTER SIMULATION

The fundamental aim in designing the Dilution of Precision computer simulator, namely the DOP Simulator, is to obtain a useful interface which provides a fast and practical analysis of both the DOP parameters and the satellite geometry. The simulator was designed as a graphical user interface in MATLAB [7].

The DOP Simulator, when started, asks the user to enter the ECEF coordinates of the four satellites and the GPS receiver with respect to the ECEF coordinate reference frame in meters. The user enters the coordinates to be simulated and presses the **Analyze** button. The program gets the coordinates as inputs and calculates the DOP parameters according to the equations which were mentioned above (Equations 1 to 11). The results are displayed in the corresponding windows. All the parameters, namely GDOP, PDOP, TDOP, HDOP, and VDOP are calculated and displayed. Later the results are rated according to levels between 1 and 50 (Table 1).

Table 1: DOP Ratings

DOP	RATING
1	IDEAL
2-3	EXCELLENT
4-6	GOOD
7-8	MODERATE
9-20	FAIR
21-50	POOR

Finally the description of the rating associated with each parameter is stated in an **EXPLANATIONS** window. To see the explanation associated with each parameter, the user should click the **EXPLAIN** button which is located under the rating of each parameter (Figure 4). Finally, the program shows the satellite geometry (on the right top of the interface) associated with the coordinates that the user has entered. This provides the opportunity to see both the DOP values and the satellite geometry at the same time so that a better imagination is created in the users

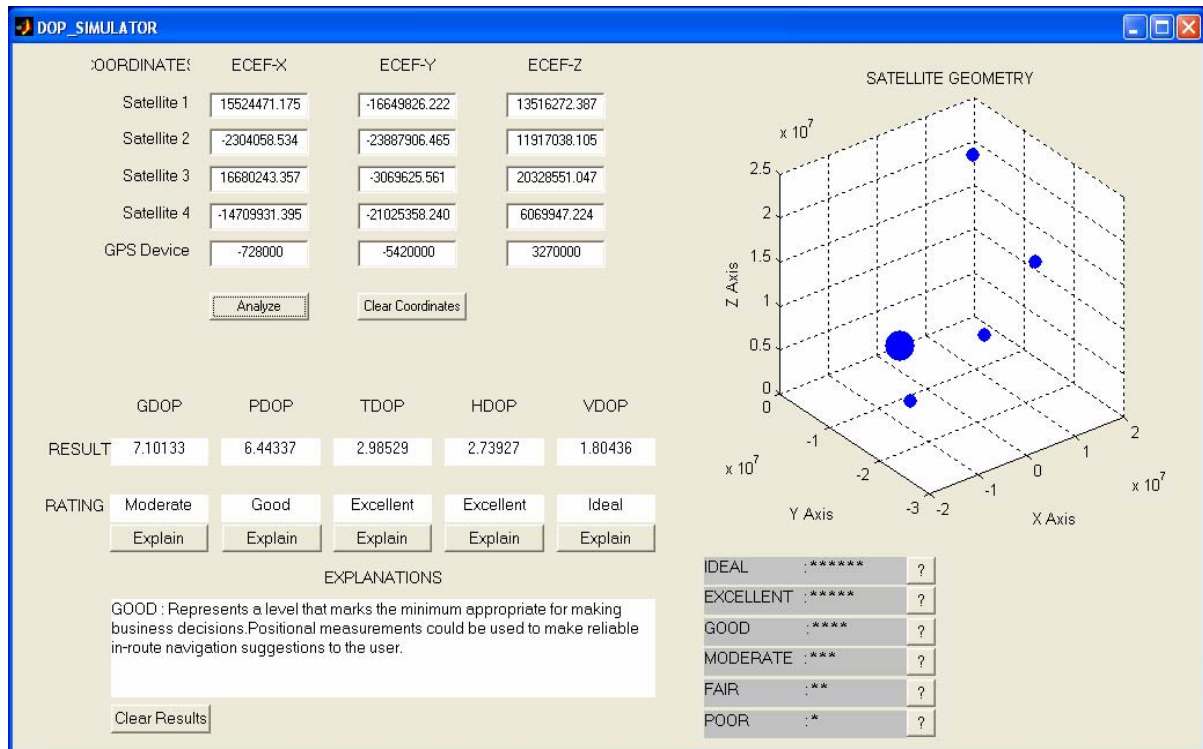


Figure 4: The Dilution of Precision computer simulation user interface.

mind. The satellites are simulated with four spheres that have equal volumes. The GPS device is simulated with a sphere having a bigger volume than the four satellites. From another point of view; the user can clear the coordinates using the **Clear Coordinates** button and enter different new coordinates to see how the DOP values change with the satellite geometry and also how the precision of the positioning is **diluted**. The user can also decide on which parameter mostly causes the dilution of precision by looking both at the **RESULTS** and the **RATINGS**. That is; the user can consider whether the dilution of precision occurred vertically, horizontally, positional or a timing error.

The DOP Simulator has some other properties. One of these is; the program detects invalid inputs and warns the user to correct the mistake. That is, when a user enters for example a letter between the numbers in one of the coordinates by mistake, the program detects the invalid input and warns the user with a message saying which of the coordinates he has entered wrong and asks to correct it. Another property is the additional explanation shortcuts to each of the ratings. If the user wants to compare the ratings, he can simply click the ? button near the rating names which are located at the right bottom of the interface. These shortcuts directly give the explanations of the DOP ratings.

IV. RESULTS AND CONCLUSION

The graphical user interface, which is named as the DOP Simulator was operated and various different coordinates were tried. The results proved that the DOP parameters change according to the satellite geometry. The DOP values were decreased, as the angles between the satellites were increased, as a result the accuracy was increased. However the simulator is considered as a sufficient interface, the user can plot the coordinates by writing the necessary code in the MATLAB command window and do detailed analysis using the camera view option. This option is not used in the simulation since it will be a time consuming process which is a contradiction to the design criteria of the simulation.

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