LYAPUNOV-BASED STATCOM CONTROL

Huayuan Chen, Rujing Zhou and Youyi Wang

School of Electrical & Electronic Engineering
Nanyang Technological University, Singapore 639798

Abstract In this paper, we discuss the nonlinear controller design of STATic COMPensator (STATCOM) for voltage stability study. A state-space averaged model of the STATCOM is derived and is used for controller design. A Lyapunov-based nonlinear controller is proposed. The controller regulates the firing angle of the STATCOM to track a given reactive current reference. The controller achieves global asymptotic stability and is not affected by the operating point. Simulation results show that the performance of the nonlinear controller is much better than that of the conventional PI controller.

1. Introduction

Flexible AC Transmission System (FACTS) devices have received much attention in recent years. One important aspect of these devices is that they provide fast and reliable control over the three main transmission system parameters, namely, voltage magnitude, voltage phase angle and impedance [1]. This attractive feature enables these solid state controllers to be widely used in many fields of power systems, such as real and reactive power regulation, stability control and system damping [2].

The STATic COMPensator (STATCOM), also referred to as a STATic CONdenser (STATCON) or Advanced Static VAR Compensator (ASVC) is a second generation FACTS device used for shunt reactive power compensation. It is based on solid state synchronous voltage source that is analogous to an ideal synchronous machine. The STATCOM can be controlled either to provide or absorb reactive power. The performance of reactive compensation of a STATCOM is better than the conventional compensators such as the Static Var Compensator (SVC) [3]. Consequently, the STATCOM offers potential benefits in applications such as increasing steady-state loadability, improving the security of systems with heavy motor loads, improving transient stability and adding damping to power swings.

One important application of STATCOM is to improve power system voltage stability [4]. Voltage stability is concerned with the ability of power systems to maintain acceptable voltages at all nodes in the systems under normal and contingent conditions. Voltage instability will occur when a disturbance causes a progressive and uncontrollable decline in voltage magnitude. It is always associated with inadequate local reactive power support. There is an increasing interest with this problem [5]. However most of the research activities is still concentrated on static stability analysis. Relatively less attention has been paid to study voltage control and stabilization. In [6], the authors proposed a nonlinear controller for SVC and showed its effectiveness in voltage stability enhancement. Similarly, advanced controller can be designed for the STATCOM to improve voltage stability.

Because voltage instability evolves in a time-scale of minutes while the dynamics of the STATCOM is in a time-scale of milliseconds, the STATCOM can therefore be represented by its static characteristics in voltage stability study. The basic assumption is that the internal dynamics of the STATCOM is stable. It is not easy to guarantee this point. It is well-known that the STATCOM model is a nonlinear system. Because of the difficulties in controlling nonlinear systems, most of the previous works dealing with STATCOM control are confined to the application of the linear control theory to small-signal linearized models.

In this paper, we propose a new control law for the STATCOM based on Lyapunov stability theory without linearization. In our approach, Lyapunov's direct method which is an important tool for nonlinear system control design is adopted. The basic idea of the approach is to construct a positive definite scalar Lyapunov function for the system and to examine the function's time derivative. The proposed control law guarantees global asymptotic stability of the closed-loop system.

The paper is organized as follows. In Section 2, the mathematical model of the STATCOM is studied. A state-space averaged model is derived. In Section 3, Lyapunov-based nonlinear controller design is discussed. Simulation results are presented in Section 4 to test the performance of the controller. We conclude the paper in Section 5.

2. System Model

A. Mathematical Model

A simplified STATCOM model is shown in Fig. 1. The voltage source converter is connected to the sys-
tem bus through an inductance $L$ and a resistor $R$. $L$ represents the leakage of the coupling transformer, $R$ represents the converter and the transformer conduction losses. On the DC side, a capacitor $C$ is in parallel with a resistor $R$. The resistor $R$ represents the converter switching losses.

In Fig. 1, $E_a$, $E_b$, and $E_c$ are the line voltages, $V_a$, $V_b$, and $V_c$ are the converter output phase voltages, $V_{dc}$ and $I_{dc}$ are the DC voltage and current, $I_a$, $I_b$, and $I_c$ are the AC line currents.

![Fig. 1 STATCOM structure](image)

The dynamical equations for the STATCOM are expressed by

$$
\frac{dl_{abc}}{dt} = -\frac{R}{L} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} l_{abc} + \frac{1}{L} (E_{abc} - V_{abc})
$$

(1)

$$
\frac{dV_{dc}}{dt} = \frac{I_{dc}}{C} - \frac{V_{dc}}{R}
$$

(2)

where $l_{abc} = [I_a \ I_b \ I_c]^T$, $V_{abc} = [V_a \ V_b \ V_c]^T$, $E_{abc} = [E_a \ E_b \ E_c]^T$. The line voltages are given by:

$$
E_{abc} = \begin{bmatrix} \cos(\omega t + \theta) \\ \cos(\omega t + \theta - \frac{2\pi}{3}) \\ \cos(\omega t + \theta - \frac{4\pi}{3}) \end{bmatrix}, \text{where } E \text{ is the line}
$$

Note that the transformation $P_k$ is power invariant.

If the converter is lossless, we will get the following equations

$$
V_{abc} = V_{dc} S_{abc}
$$

(3)

$$
I_{dc} = I_{abc} S_{abc}
$$

(4)

For modeling and control design, it is very convenient to transform three-phase variables into a rotating $d$-$q$ frame using Park’s transformation. Define the following transformation

$$
I_{abc} = P_k I_{dq0}, \ E_{abc} = P_k E_{dq0} \text{ and } V_{abc} = P_k V_{dq0}
$$

where $P_k$ is defined as

$$
P_k = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) & \frac{1}{\sqrt{2}} \\ \cos(\omega t - \frac{2\pi}{3}) & \sin(\omega t - \frac{2\pi}{3}) & \frac{1}{\sqrt{2}} \\ \cos(\omega t - \frac{4\pi}{3}) & \sin(\omega t - \frac{4\pi}{3}) & \frac{1}{\sqrt{2}} \end{bmatrix}
$$

Applying the Park’s transformation to equation (1) and (2) and take into account the relations (3) and (4), we obtain the following time-invariant model

$$
\frac{dl_{dq}}{dt} = -\frac{R}{L} l_{dq} - \omega l_{dq} - \frac{K \cos(\alpha + \theta)}{L} V_{dc} + \frac{E_d}{L}
$$

(5)

$$
\frac{dl_{q}}{dt} = -\frac{R}{L} l_{q} + \omega l_{q} + \frac{K \sin(\alpha + \theta)}{L} V_{dc} + \frac{E_q}{L}
$$

(6)

$$
\frac{dV_{dc}}{dt} = \frac{1}{RC} V_{dc} + \frac{K \cos(\alpha + \theta)}{C} l_d
$$

(7)

where $K = \frac{\sqrt{3}}{\pi}$ and $E_d = E \cos(\theta)$, $E_q = -E \sin(\theta)$.

The real and reactive power of the STATCOM is in fact not affected by the line phase angle $\theta$ [7]. Without loss of generality, we can set $\theta = 0$ to simplify the model. Also the following per unit system on the STATCOM base is adopted:

$$
X_L = \frac{L_0}{Z_{base}}, \ X_C = \frac{1}{\cos(\theta) Z_{base}}, \ R' = \frac{R}{Z_{base}}, \ V_{base} = \frac{V}{Z_{base}}
$$

The simplified STATCOM model is given by

$$
\frac{dl_{dq}}{dt} = -\frac{R'}{X_L} \omega l_{dq} - \omega l_{dq} - \frac{K \cos(\alpha)}{X_L} V_{dc} + \frac{\omega E}{X_L}
$$

(10)

$$
\frac{dl_{q}}{dt} = -\frac{R'}{X_L} \omega l_{q} + \omega l_{q} + \frac{K \sin(\alpha)}{X_L} V_{dc}
$$

(11)

$$
\frac{dV_{dc}}{dt} = -\frac{X_C \omega}{R'} V_{dc} + X_C \cos(\alpha) l_d - X_C \sin(\alpha) l_q
$$

(12)

The firing angle $\alpha$ is the control input. The power equations (8) and (9) become
\[ P = EI_d \]  
\[ Q = EI_q \]

As can be seen, the reactive power is proportional to \( EI_q \) while the real power is proportional to \( EI_d \). To control the reactive power \( Q \), it is sufficient to control the reactive current \( I_q \).

### B. Steady-State Characteristics

Setting the equations (10) to (12) equal to zero, we can get the steady-state values \( I_{dq} \), \( I_{q0} \) and \( V_{dc0} \) corresponding to a given firing angle \( \alpha_0 \). Figs. 2-4 show the steady-state characteristics of the STATCOM. The EPRI 80 MVAR STATCOM [11] is used to generate the characteristics. The values are given as \( X_L = 0.15, R_S = 0.01, X_C = 0.88 \) and \( R' = 78 \), all in per unit. The bus voltage magnitude \( E \) is set to 1.0 per unit.

![Fig. 2 Steady-state characteristics \( I_q - \alpha \) ](image)

![Fig. 3 Steady-state characteristics \( I_d - \alpha \) ](image)

As can be seen, when firing angle is zero, the real and reactive current are not zero because of the power losses on \( R_S \) and reactive power consumed by the leakage \( L \). The real current of the STATCOM is almost constant and is very small compared to the reactive current. In steady state analysis, the real power losses can be neglected. The STATCOM becomes a pure reactive current source. Positive \( \alpha \) means the STATCOM works in the inductive region while negative value implies that it is in the capacitive region. The reactive current \( I_{q0} \) is almost proportional to the firing angle \( \alpha \). Also note that the capacitor voltage is not constant during firing angle variation.

![Fig. 4 Steady-state characteristics \( V_{dc0} - \alpha \) ](image)

In voltage stability study, we can use a variable reactive current source to represent the STATCOM. This requires the internal dynamics of the STATCOM is fast enough and is stable. Although the open loop system is asymptotically stable, its dynamic behavior is not in general satisfactory. In fact, as \( R_S \) approaches zero, the system becomes increasingly oscillatory. As stated previously, the conventional PI controllers are tuned around a particular operating point. They are not robust against changing operation conditions. In the following Section, we will discuss the nonlinear controller design based on Lyapunov stability theory.

### 3. Lyapunov-Based Reactive Current Control

The control of switched converters is divided into two groups. In the first case, the control law commands the switch position directly, such as the sliding mode control. The switching frequency is usually high. The second approach relies on the state-space averaged model. The control variable is the firing angle of the converter. In this paper, the controller design is based on the averaged model (10) to (12). Since we are interested with controlling the internal dynamics of the STATCOM, we assume that the line voltage \( E \) does not change with time. The control objective is to regulate the firing angle \( \alpha \), so that the reactive current \( I_q \) tracks the given reference current \( I_{q_{ref}} \) quickly and asymptotically. At the same time, all other states are bounded. As can be seen from the steady-state characteristics, once the reference reactive current \( I_{q_{ref}} \) is given, \( I_{dq0}, V_{dc0} \) and \( \alpha_0 \) are uniquely determined.

Define the following variables:
\[ x_i = I_d - I_{d0}, \quad x_2 = I_q - I_{q0}, \quad x_3 = V_{dc} - V_{dc0}, \]
\[ P_d = \cos \alpha, \quad P_{d0} = \cos \alpha_0, \quad P_q = \sin \alpha, \quad P_{q0} = \sin \alpha_0, \]
the incremental STATCOM model are thus obtained:
\[ \frac{dx_1}{dt} = \frac{1}{L} - R x_1 - \frac{1}{L} x_2 + KL_{d} V_{dc} + KL_{q} V_{q0} \]
\[ \frac{dx_2}{dt} = \frac{1}{L} - R x_2 + \frac{1}{L} x_1 + KL_{d} V_{dc} - KL_{q} V_{q0} \]
\[ \frac{1}{X_{C0}} \frac{dx_3}{dt} = -\frac{1}{R} x_3 + KL_{d} I_d - KL_{q} I_{q0} \]
\[ -KL_{q} I_{q} + KL_{q0} I_{q0} \]

Consider the positive definite Lyapunov function candidate:
\[ V(x) = \frac{1}{2} x_i^2 + \frac{1}{2} x_2^2 + \frac{1}{2} x_3^2 \]
Taking the derivative of (18) along any system trajectory gives
\[ \dot{V}(x) = -\lambda_1 x_1 - \lambda_2 x_2 - \lambda_3 x_3 \]
\[ + KL_{d} (V_{dc} - V_{dc0}) + KL_{q} (V_{q0} - V_{q00}) \]
\[ + KL_{q0} (V_{q0} - V_{q00}) + KL_{q0} (V_{q0} - V_{q00}) \]

Note that \( V(x) \) along any system trajectory is negative definite if
\[ P_d \lambda_1 + P_q \lambda_2 + \lambda_3 = 0 \]
where
\[ \lambda_1 = K (V_{dc} - V_{dc0}), \]
\[ \lambda_2 = K (V_{q0} - V_{q00}) \]
\[ \lambda_3 = (E - R x_0 - X_{L} I_{d0}) I_d \]
\[ - (R x_0 - X_{L} I_{d0}) I_q - \frac{V_{dc} V_{dc0}}{R} \]
Note that
\[ P_d^2 + P_q^2 = 1 \]
Solving (21) and (22) we get the following control law
\[ P_d = \frac{-\lambda_1 \lambda_2 + \lambda_2 \lambda_3 - \lambda_3 P_q}{\lambda_2^2 + \lambda_3^2} \]
\[ P_q = \frac{-\lambda_1 \lambda_3 - \lambda_3 \lambda_2 + \lambda_2 P_q}{(\lambda_2^2 + \lambda_3^2) \lambda_q} \]

4. Simulation

The performance of the proposed controller is tested through simulation study. For comparison, a PI controller which is given by
\[ \Delta \alpha = -K_p (I_q - I_{q0}) - K_i I_{q0} \]
where \( K_p = 500 \) is also studied. The transient responses of the real current \( I_d \), reactive current \( I_q \), and the capacitor voltage \( V_{dc} \) to positive and negative step references \( I_{q0} \) are studied. The results are shown in Fig. 5 through Fig. 10. In these figures, the dynamic behaviors under the proposed controller are identified by solid curves while those under the conventional PI controllers are shown by dashed curves. In Figs. 5 to 7 the reference \( I_{q0} \) jumps from 1 per unit to -1 per unit at 0.2 s. In Figs. 8 to 10 \( I_{q0} \) jumps from -1 per unit to 1 per unit at 0.2 s.
It is clearly shown that the transient responses follow the step change of references quickly under the proposed nonlinear controller. All transients reach steady state within 0.5 second. On the other hand, the responses are rather slow under the PI controller, although they approach steady-state finally. From Figs. 6 and 9, we see that the real current exhibits high frequency oscillation before it reaches steady-state. Also note that the dynamic behavior of positive transition is different from negative transition. The system is less damped during positive transition and the real power oscillation is severer. In both cases, the reactive current and capacitor voltage are asymptotically stable.

5. Conclusion

In this paper, a reactive current model of the STATCOM is studied. A Lyapunov-based reactive current controller is designed. Global asymptotic stability is guaranteed by the controller. Simulation results show that the reactive current tracks the reference value quickly and asymptotically. The performance of the proposed nonlinear controller is better than the conventional PI controller.

References