SOLUTION OF ELECTROSTATIC FIELD PROBLEM WITH PARABOLIC BOUNDARY ELEMENTS

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ABSTRACT

In this study, electrostatic field problems were investigated using parabolic boundary elements in the Boundary Element Method (BEM). Moreover, constant and linear boundary elements were examined and three different programs were developed for every three elements in MATLAB. Coaxial cable specimens having homogenous and non-homogenous regions were considered. In these practices, potential values at any internal points were obtained with the programs developed in MATLAB after unknown values were calculated. These results were compared with analytical results.

I. INTRODUCTION

Many problems in engineering analyses are in the form of region problems. The last developed method BEM is one of the numerical solution methods to solve these problems. Values obtained with solution of the electrostatic field problems were calculated as rapidly and accurately. [1]

BEM has been applied to both two-dimensional and three-dimensional regions. In engineering analyses, there are some types of equation that express the problem regions mathematical. These equations are in general Laplace, Helmholtz, Wave, Diffusion and Navier's equations. In this study, two-dimensional electrostatic fields were examined with the solution of Laplace equation.

II. BOUNDARY ELEMENT METHOD

In the problems that are analyzed by BEM, the boundary of the region is divided arbitrarily. Each piece is called "a boundary element" at boundaries. On these elements, there are some values that are known or to be calculated. These values are the potentials (u) on the definition points of the element (nodes) or the fluxes (q) that are the derivates of the potentials with respect to the normal.

In BEM, general equation used for solution of the problem is a Boundary Integral Equation. The Boundary Integral Equation, which is obtained using Green Theorem applied to 2D region integral, is 1D integral in the boundary of problem domain.

$$c_i\, u_i + \int_S u\, q^*\, dS = \int_S q\, u^*\, dS \tag{1}$$

In this equation, the potential to be calculated for the node i on the boundary is denoted as "u_i". u^* is the fundamental solution of the 2D Laplace equation:

$$u^* = \frac{1}{2\pi} \ln\left(\frac{1}{r}\right) \tag{2}$$

First, unknown values on the boundaries have to be calculated for analysis with BEM. Latter, the values of internal points are calculated using these values of boundary. [1]

Different boundary elements had been developed for analyses of problem regions. These element that are commonly constant, linear and parabolic. According to this, in the constant element there is a node at the center of the element. Also in the linear element, there are two node and are at the end of the element. In the parabolic element, there are three nodes. One of these nodes is at the center and others are at the end of the element. These boundary elements have been defined with different interpolation functions. [2]

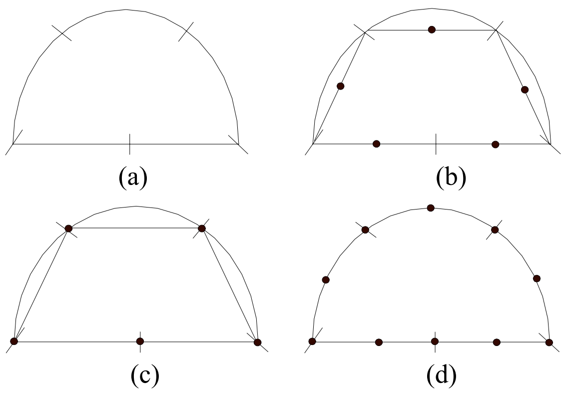

Figure 1. (a) Divided problem region, (b) Constant element, (c) Linear element, (d) Parabolic element

CONSTANT ELEMENT

In elements of this kind it is assumed that the variables of the boundary have a constant value along the element and these values defined with one node are at the center of the element. After the boundary of the problem domain is analyzed with N elements, the Boundary Integral Equation can be written for node i on the element as with follows:

$$c_i\, u_i + \sum_{j=1}^{N} \int_{S_j} u\, q^*\, dS = \sum_{j=1}^{N} \int_{S_j} q\, u^*\, dS \qquad (3)$$

A linear equation system is obtained after integrals belonging to elements are solved by numerical integration method:

$$Hu = Gq \qquad (4)$$

If this equation system is solved, unknown values (u and/or q) on the boundary will be obtained. Afterwards, the formulation of the internal point is used to calculate the potential of any point i through the boundary values that are known and obtained from eqn.3. [3]

$$u_i = \sum_{j=1}^{N} \int_{S_j} q\, u^*\, dS - \sum_{j=1}^{N} \int_{S_j} u\, q^*\, dS \qquad (5)$$

LINEAR ELEMENT

In element of this kind, it is assumed that the boundary values, u and q, have a linear evolution between two nodes, which are both the ends of the element.

The functions of the linear interpolation defining the physical values on the elements are as follows: [4]

$$\phi_1 = \frac{1}{2}(1-\xi) \qquad \phi_2 = \frac{1}{2}(1+\xi) \qquad (6)$$

In these equations, the values of ξ are local coordinate that varies between the end points (1 to –1). The general equation on boundary points can be written, by substituting the linear interpolation functions into eqn.3, as:

$$c_i\, u_i + \sum_{j=1}^{N} \int_{S_j} \left[\phi_1 \ \ \phi_2\right] \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} q^*\, dS = \sum_{i=1}^{N} \int_{S_j} \left[\phi_1 \ \ \phi_2\right] \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} u^*\, dS \qquad (7)$$

Values u and q on the boundary are obtained when this equation is solved. Moreover, coefficient c_i must be calculated depending on the angle that is between the former and the following elements from the node.

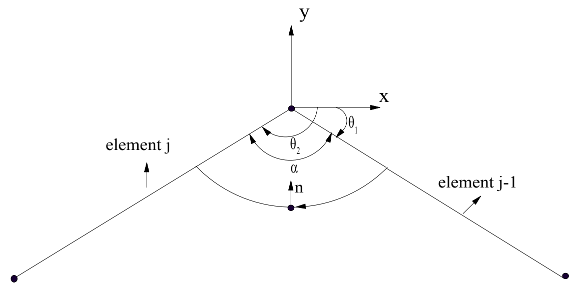

Figure 2. The angle between two elements

$$c_i = \frac{\theta_2 - \theta_1}{2\pi} = \frac{\alpha}{2\pi} \qquad (8)$$

According to the values (u and q) on the boundary, the potential on any internal point i is obtained from eqn.5.

PARABOLIC ELEMENT

In element of this kind it is assumed that the node values (u and q) on the element have a parabolic evolution along the element.

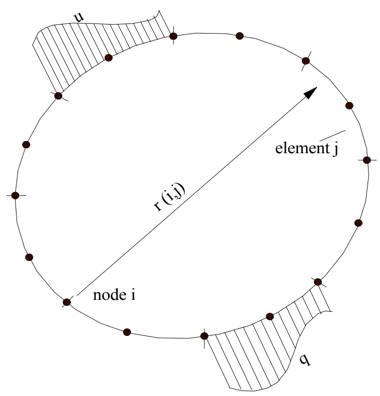

Figure 3. General illustration of the parabolic element on the problem domain

In this method, one of the nodes is at the center of the element and others are at the ends of the element. In fig.3, the raked region refers to variation of the values on the element j.

In the parabolic element, the boundary of the element has changeability as equal to the boundary of the problem. Above feature, provides very important advantage and obtained results are quite accurate.

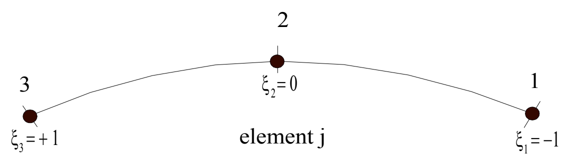

Figure 4. The parabolic element with local coordinate

In the parabolic element, the interpolation functions of the nodes are defined as the following: [4]

$$\phi_1 = \frac{1}{2}\xi(\xi-1),\ \phi_2 = 1-\xi^2,\ \phi_3 = \frac{1}{2}\xi(\xi+1) \qquad (9)$$

By substituting the values u and q on nodes and the interpolation functions into eqn.3 we obtain:

$$c_i\, u_i + \sum_{j=1}^{N} \int_{S_j} \begin{bmatrix}\phi_1 & \phi_2 & \phi_3\end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} q^*\, dS = \sum_{j=1}^{N} \int_{S_j} \begin{bmatrix}\phi_1 & \phi_2 & \phi_3\end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} u^*\, dS \qquad (10)$$

When eqn.10 is solved using the numerical integration method, unknown values (u and/or q) on boundary are obtained. Coefficient c_i in eqn.10 is also calculated by eqn.8.

As in the linear element, the potential on any internal point i is obtained from eqn.5 that was added the functions of the parabolic interpolation.

SUBREGIONS

If the analyzed problem has non-homogenous structure, the numerical procedures can apply after the problem region is separated into homogenous subregions having the same properties. In the between joining these regions, the boundary conditions are continuity of the potential and discontinuity of the flux. When the equation sets belonging to each subregion are added, a general equation is obtained for all regions. [3]

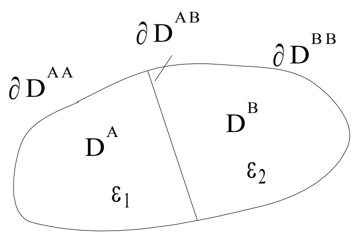

Figure 5. Geometrical definitions of subregions

In fig.5, D region was divided to A and B subregion having ε_1 and ε_2 dielectric constants.

The boundary values at the interface are the same.

$$u^{AB} = u^{BA} \qquad q^{AB} = q^{BA} \qquad (11)$$

III. APPLICATIONS

The programs concerning to three different elements were written to solve 2D electrostatic field problems. The programs of the constant, linear and parabolic elements are called SABEL, LINEL and PAREL, respectively. As an example of application, a quarter section of the coaxial cable sample was examined. In the first application, after these three programs were applied to quarter section of the coaxial cable sample that was divided into 28 boundary elements, solutions at the definition points were obtained. These solutions were compared with the analytical solution values at the definition points. Latter, the program PAREL was applied after the coaxial cable sample was divided into two subregions that have two different dielectric constants. Thus, the solution values were illustrated with the equipotential contours.

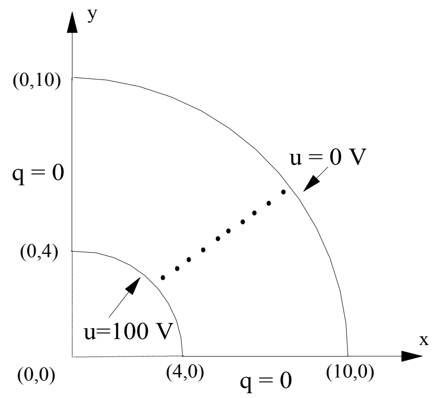

Figure 6. A quarter section of the coaxial cable

Table 1. The potential distribution of a quarter section of the coaxial cable divided into 28 boundary elements in radial direction

Cal. Pnt.	Coordinates x	y	SABEL Program	LINEL Program	PAREL Program	Analytical Solves
1	3.25	3.25	88.22	82.63	82.47	84.74
2	3.67	3.67	73.48	66.77	69.88	71.36
3	4.10	4.10	60.52	55.07	58.34	59.44
4	4.52	4.52	44.42	44.50	47.83	48.70
5	4.94	4.94	32.95	34.84	38.22	38.92
6	5.37	5.37	24.12	25.96	29.39	29.95
7	5.79	5.79	15.89	17.72	21.21	21.65
8	6.22	6.22	8.18	10.06	13.61	13.95
9	6.64	6.64	1.10	2.88	6.51	6.75

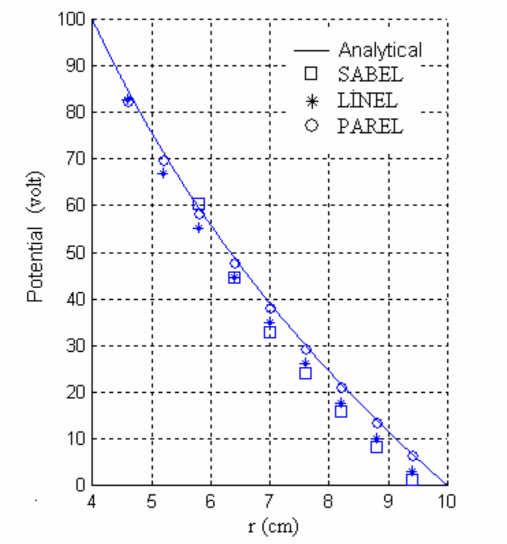

Figure 7. The potential distribution in the radial direction

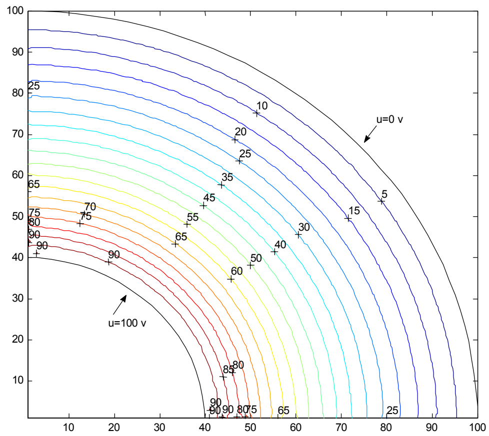

Figure 8. The Equipotential contours

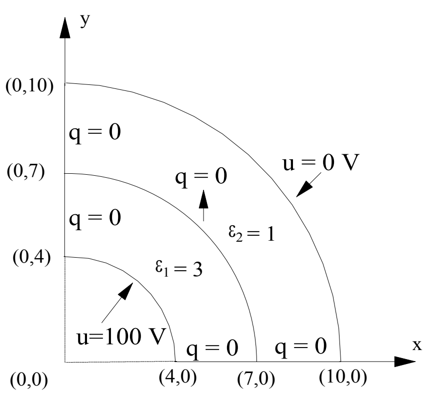

Figure 9. A quarter section of the coaxial cable that was divided in two subregions

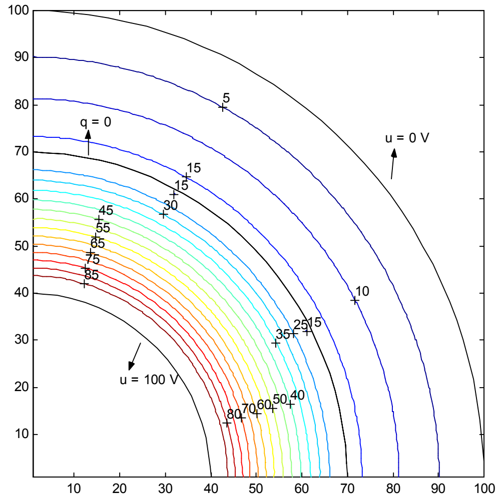

Figure 10. The Equipotential contours

IV. CONCLUSION

In this study, after the mathematical theory of BEM was investigated, 2D electrostatic field problems were solved with parabolic element used in this method. For this purpose, a computer program named PAREL was written in MATLAB by using parabolic element. Also, constant and linear elements that were other functions of the method were described and their numerical formulations were given. For these elements the programs SABEL and LINEL were written. First, in the coaxial cable system chosen as an example of application, the boundary of the quarter section coaxial cable was divided into 28 elements and results were obtained separately with these three programs. Since the geometry of the parabolic element was coincident with the boundary, it is observed that the potential values at the internal points are very precise. In the second application, the quarter section cable is divided in to two subregions having dielectric constants 1 and 3 respectively. When the PAREL program was applied, the precise values of the solution were obtained.

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