Trellis Coded Quantization/Modulation over Rician Fading Channel

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Abstract

In this letter, bit error performance of trellis coded quantization/modulation (TCQ/TCM) schemes over Rician fading channel is modified by taking into account the effects of quantization noise and fading channel parameter estimation error. The importance of the consideration of quantization noise error is stated in the evaluation of performance analysis and as an example 8-PSK Combined TCQ/TCM scheme is studied for different fading parameters to support the modification.

1 Introduction

Since trellis coded quantization (TCQ) is a computationally efficient source coding technique that was developed by exploiting signal set expansion and set partitioning ideas from trellis coded modulation (TCM), it naturally lends itself to construct joint source/channel coding systems together with TCM [1], [2], [3]. As a result, joint source/channel coding results in substantial performance improvement [1]. In the present paper, we consider TCQ/TCM signals propagating through a Rician fading environment. We develop an analytical approach to further elaborate the performance analysis of TCQ/TCM systems by taking into account the effects of quantization noise and channel parameters estimation error, as explained later.

2 Performance Analysis

A TCQ coder takes independent, identically distributed (i.i.d.) source outputs which are generated according to some continuous probability density function (p.d.f.) $f(\bullet)$ and assigns a R bit binary words to each source sample. The coded bits are then applied to TCM producing TCQ/TCM modulated symbols.

If we consider TCQ/TCM signals over Rician fading communication environment, at the i^{th} signalling interval the received signal has the following form,

$$r(i) = \rho(i)w(i) + v(i) \tag{1}$$

where $\rho(i)$ represents a normalized fading magnitude having the Rician (p.d.f.), $w(i)$ is a channel input generated by TCQ/TCM scheme, and $v(i)$ is a zero mean, i.i.d., additive Gaussian noise sequence with variance σ_v^2.

In the sequel we investigate the performance degradation on TCQ/TCM systems due to quantization noise, channel parameters estimation error and channel noise. Details of the analytical approach is given below.

Quantization Noise: Some specific assumptions on the statistical nature of the quantization noise allows to handle the quantizer as a classical signal plus noise system as in communication theory. Motivated by this, the output of the TCQ scheme can also be modelled as the input signal plus the quantization noise [4].

We now need to find an expression for the variance of the quantization noise. Based on the assumption on the input signal p.d.f., the closed form expressions for the quantizer error variance would be obtained. As an example, let the input be a uniformly distributed random variable with variance σ_x^2 then uniform quantizer's error variance is [4],

$$\sigma_q^2 = 0.25 \times 2^{-2R}\sigma_x^2 . \tag{2}$$

Channel Estimation Error: If we consider the blind estimation of the Rician channel parameters from set of observations via maximum likelihood (ML) estimation techniques, the ML metric [5], which is proportional to the conditional p.d.f. of the received data sequence of length N (given w) needs to be minimized and is of the form [2]

$$m_j(\rho) = E_{\mathbf{w}\in\mathcal{W}}\left\{m_j(\rho \mid \mathbf{w})\right\} = E_{\mathbf{w}\in\mathcal{W}}\left\{\frac{1}{(2\sigma_v^2)^N}\exp\left(-\frac{1}{2\sigma_v^2}\sum_{i=1}^{N}|r(i)-\rho_j w(i)|^2\right)\right\} . \tag{3}$$

where $\mathbf{w} = [w(1), w(2), \cdots, w(N)]$ and $\mathcal{W}$ is the set of all possible transmitted sequences.

We assume that ρ_j is estimated from maximum likelihood metric via some iterative technique [5], we are now interested in variance of the estimator. The variance of any unbiased estimator $\hat{\rho_j}$ must satisfy

$$\sigma_{\rho_j}^2 \geq \frac{1}{-E[\frac{\partial^2 \log(m_j(\rho))}{\partial^2 \rho}]} . \tag{4}$$

The evaluation of the variance requires $\log[m_j(\rho)] = \log[E_{\mathbf{w}}(m_j(\rho \mid \mathbf{w}))]$ which is analytically intractable due to nature of (3). However, a valid bound can still be obtained from the conditional likelihood function $m_j(\rho \mid \mathbf{w})$, [6]. Since the logarithmic function is concave, we can employ Jensen's inequality, and obtain $\log[m_j(\rho)] \leq E_{\mathbf{w}}\{\log[m_j(\rho \mid \mathbf{w})]\}$, i.e.,

$$\log(m_j(\rho)) \leq -\frac{N}{2}\log(\sigma_v^2) - \frac{N}{2}\log(2\pi) - \frac{1}{2\sigma_v^2}E_{\mathbf{w}}\left\{\sum_{n=0}^{N-1}|r(i)-\rho_j w(i)|^2\right\} . \tag{5}$$

It turns out from the approximate likelihood of (5) that the variance of the estimator is of the form

$$\sigma_{\rho_j}^2 \geq \frac{\sigma_w^2}{N} \tag{6}$$

[2] Slowly varying fading channels are assumed where $\rho(i)$ remains constant over the j^{th} observation interval.

where σ_w^2 is the variance of the TCQ encoded data sequence.

Therefore, Eq. (6) provides an approximate bound which may not be tight but is much easier to compute. Moreover, this term can be treated as an additional Gaussian noise [6].

Since we are interested in evaluating performance degradation of the TCQ/TCM system due to quantization noise, fading channel parameter estimation error and AWGN, we need to consider total effects of these noise sources on the overall system. To achieve this goal, the noise effect due to quantization must be carried to the output of the Rician channel. Then the modified quantization noise variance becomes $\sigma_{q_m}^2 = \rho_j^2\sigma_q^2$. Based on these results, the total variance of these noise sources can be written as $\sigma_t^2 = \sigma_{q_m}^2 + \sigma_{\rho_j}^2 + \sigma_v^2$. The modified signal to noise ratio SNR_m is then defined as,

$$SNR_m = \frac{\sigma_x^2}{\sigma_t^2} \tag{7}$$

However if we apply central limit theorem, all these noise sources could be treated as a single additive Gaussian noise source, which helps the modification of bit error upper bounds [4].

3 Examples: 8-PSK Combined TCQ/TCM System

In this section, we investigate the effects of the noise sources on the 8-PSK Combined TCQ/TCM system. Encoding rate in this scheme is chosen as $R = 2$ bits/sample, however extensions to the higher rates straightforward but are omitted due to lack of space. The system consists of a four state combined TCQ/TCM structure employing an 8-PSK constellation. On the branches of the proposed combined structure, there is one to one correspondence between the signal set and quantization levels. From every state emanates two adjacent branches each of which contains two paralel transitions.

For AWGN channels, an upper bound on the average bit error probability [7], for $n/n+1$ rate encoder, P_b assuming ideal interleaving/deinterleaving, can be modified as,

$$P_b \leq \frac{1}{n}\frac{\partial T(W,I)}{\partial I}\Big|_{I=1,W=\exp\left(\frac{-n\times SNR_m}{4}\right)} \tag{8}$$

where SNR_m is the modified signal to noise ratio and $T(W,I)$ is the transfer function of the pair-state transition diagram that takes into account an enumeration of all the distance and error

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properties associated with the trellis code, and with the number of states. In the presence of fading, evaluation of P_b depends on the proposed decoding metric, the presence or absence of the channel state information (CSI) and the type of detection used. In case of coherent detection and ideal CSI with an additive maximum likelihood metric, the transfer function $\overline{T(W,I)}$ of the modified transition diagram is obtained by merely replacing the factor W^{β} in each branch label gain, by $\overline{W^{\beta\rho^2}}$ where the overline denotes averaging over the fading amplitude ρ, and β represents the squared Euclidean distance between any two channel signals. The analytical upper bounds are derived using modified transfer function of this scheme as in [3] and results are presented in Fig. 1.

4 Conclusion

In this paper the effects of quantization noise, fading channel parameter estimation error and AWGN is taken into account to modify bit error performance of the TCQ/TCM system. The interesting observation made from this modification is that quantization noise effect shows increasing characteristic at high SNR values and as K increases. Therefore quantization noise effect should be considered in the performance analysis evaluation of TCQ/TCM systems especially at high SNR values.

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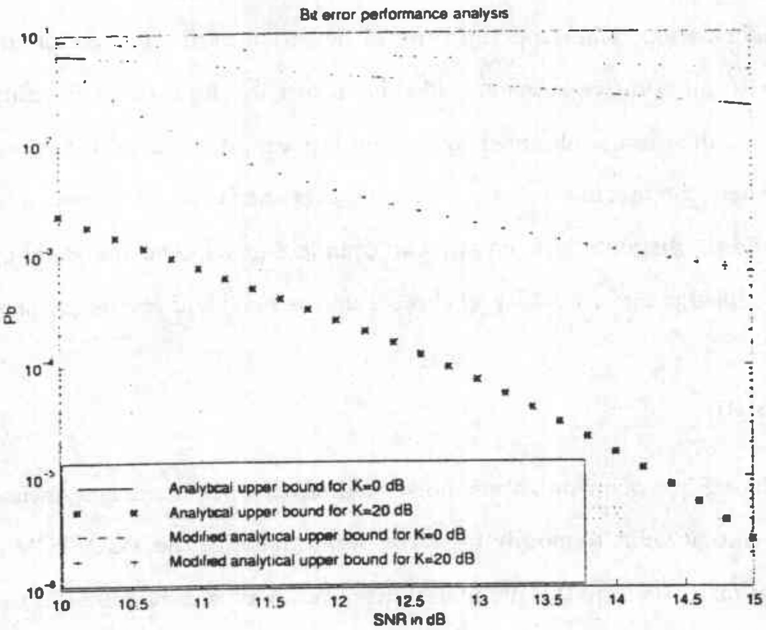

Figure 1: Modified Performance Analysis for uniform quantization

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