A New Tuning Method for $PI^{\lambda}D^{\mu}$ Controller

Celaleddin Yeroglu[1] , Cem Onat[2] and Nusret Tan[1]

[1] Inonu University, Electrical and Electronics Engineering Department, 44280, Malatya, Turkey
[2] Inonu University, Mechanical Engineering Department, 44280, Malatya, Turkey

cyeroglu@inonu.edu.tr, conat@inonu.edu.tr, ntan@inonu.edu.tr

Abstract

The paper presents development of a new tuning method for fractional order *PID* controller for the systems which have integer order transfer functions. All the parameters of the controller, namely proportional gain k_p, integral gain k_i, derivative gain k_d, fractional order of integrator λ and fractional order of differentiator μ can be obtained by using this method. It is clearly shown that the fractional order controller, which the parameters obtained by the proposed method, gives better response than the integer order one for the same system.

1. Introduction

There is no doubt that the most common controllers used in industry have been *PID* controllers for many years. Widespread usage of these algorithms has motivated many researchers to look for better design methods or alternative controllers [1, 2]. For example, the fractional order algorithm for the control of dynamic systems has been introduced and performance of CRONE (French abbreviation for Commande Robuste d'Ordre Non Entier), over the *PID* controller, has been demonstrated by Oustaloup [3, 4]. Podlubny has proposed a generalization of the *PID* controller as $PI^{\lambda}D^{\mu}$ controller which is known as fractional order *PID* controller, where λ is the non-integer order of integrator and μ is the non-integer order of the differentiator term. He also demonstrated that the $PI^{\lambda}D^{\mu}$ controller has better response than classical *PID* controller [5, 6]. Frequency domain approaches of $PI^{\lambda}D^{\mu}$ controller are studied in [7]. Also many valuable studies have been done for fractional order controllers and their implementations in [8-13].

Crucial importance of tuning of the controllers cannot be underestimated. Thus, many tuning techniques for obtaining the parameters of the controllers were introduced during last few decades. The most well known tuning rules for classical controllers are given by Ziegler-Nichols [1] and Åström-Hägglund [2] which have been the milestones for developments of many other methods. Tuning methods of $PI^{\lambda}D^{\mu}$ controllers are a new research subject. Some results related with this subject are given in [14-16]. Reference [17] proposes a method based on optimization strategies. Tuning of H_{∞} controllers for fractional SISO system suggested in [18]. A new design method for PI^{α} controller is given in [19]. Some tuning rules for robustness to plant uncertainty for PI^{λ} controller are given in [20]. However in order to achieve better results, there are still needs for new methods to obtain the parameters of $PI^{\lambda}D^{\mu}$ controllers.

Controller tuning is the process of obtaining the controller parameters to meet given performance specifications. Especially, Zigler-Nichols rules are useful when mathematical model of the plant are not known [1].

A point on a Nyquist curve of the plant $G(j\omega)$ can be moved to another position on $G(j\omega)C(j\omega)$ by choosing appropriate *PI* or *PID* parameters of $C(j\omega)$. Thus, Åström-Hägglund used this property of Nyquist curve for their tuning method, which can provide transferring one point on a Nyquist curve to a desired position, to achieve specified phase and gain margins [21, 22].

The aim of this paper is to introduce a new tuning method for a $PI^{\lambda}D^{\mu}$ controller, which is inspired from classical Zigler-Nichols and Åström-Hägglund tuning methods. The proposed method uses classical Zigler-Nichols tuning rules to obtain the values of k_p and k_i. The initial value of k_d is obtained using Åström-Hägglund method. In order to achieve specified phase margin, two nonlinear equations have been obtained using critical point information, namely critical frequency ω_c and critical gain k_c using the idea of Åström-Hägglund tuning method. Fine tuning has been done for k_d to achieve the best numerical solutions of these two equations. The values of λ and μ are obtained from these equations using an optimization toolbox of MATLAB. Tuning of the controller parameters may be required to achieve better step response of the system. In that case, an optimization model, which has been developed using Simulink MATLAB, is used. This optimization model uses the controller parameters obtained by proposed method, as initial values. Then it produces new values for the controller parameters.

The paper is organized as follows: A brief mathematical background is given in Section 2. Computation of controller parameters is given in Section 3. Tuning method for fractional order controller is given in Section 4. An application of the proposed method is given in Section 5. Section 6 includes concluding remarks.

2. Brief Mathematical Background

Orders of fractional calculus are real numbers [23]. Many different definitions for general fractional integro-differential operation can be found in the literature. Among them the most commonly used for general fractional integro-differential expressions are given by Cauchy, Riemann-Liouville, Grünwald-Letnikov and Caputo [7]. Caputo expression for fractional order differentiation is given as;

$$ {}_0D_t^{\alpha} y(t) = \frac{1}{\Gamma(1-\gamma)} \int_0^t \frac{y^{(m+1)}(\tau)}{(t-\tau)^{\gamma}} d\tau \qquad (1) $$

where $\alpha = m+\gamma$, m is an integer and $0<\gamma<1$. Caputo expression for fractional order integration is defined as [24];

$$ {}_0D_t^{\gamma} = \frac{1}{\Gamma(-\gamma)} \int_0^t \frac{y(\tau)}{(t-\tau)^{1+\gamma}} d\tau \quad , \quad \gamma < 0 \qquad (2) $$

Due to easy calculations, Laplace domain is commonly used to express the fractional integro-differential operations. Thus, Laplace transform of fractional order differentiation can be given as [24];

$$ L\left[{}_0D_t^{\alpha} f(t)\right] = s^{\alpha} L\left[f(t)\right] - \sum_{k=1}^{n-1} s^k \left[{}_0D_t^{\alpha-k-1} f(t)\right]_{t=0} \qquad (3) $$

If the derivatives of the function $f(t)$ are all equal to zero, the following equation can be written [24].

$$ L\left[{}_0D_t^{\alpha} f(t)\right] = s^{\alpha} L\left[f(t)\right] \qquad (4) $$

A fractional differential equation for a fractional order control system can be written as:

$$ \begin{aligned} & a_n \frac{d^{\alpha_n} y(t)}{dt^{\alpha_n}} + a_{n-1} \frac{d^{\alpha_{n-1}} y(t)}{dt^{\alpha_{n-1}}} + \cdots\cdots + a_0 \frac{d^{\alpha_0} y(t)}{dt^{\alpha_0}} \\ & = b_m \frac{d^{\beta_m} x(t)}{dt^{\beta_m}} + b_{m-1} \frac{d^{\beta_{m-1}} x(t)}{dt^{\beta_{m-1}}} + \cdots\cdots + b_0 \frac{d^{\beta_0} x(t)}{dt^{\beta_0}} \end{aligned} \qquad (5) $$

where $y(t)$ is output and $x(t)$ is the input of the system. The Laplace transform of Eq. (5) can be obtained as [11].

$$ G(s) = \frac{Y(s)}{X(s)} = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \ldots. + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \ldots.. + a_0 s^{\alpha_0}} \qquad (6) $$

where $\alpha_n > \alpha_{n-1} > \ldots. > \alpha_0 \geq 0$ and $\beta_m > \beta_{m-1} > \ldots. > \beta_0 \geq 0$ are satisfied, a_k $(k = 0,1,2,\ldots,n)$ and b_k $(k = 0,1,2,\ldots,n)$ are constants.

The analysis of the Laplace transform and inverse Laplace transform of fractional integro-differential operation at time domain are quite complicated. But frequency domain analysis of the fractional order control system is same as the integer order one. Since the power $j\omega$ is a real number, one can write the fractional power of $j\omega$ as follows,

$$ (j\omega)^{\alpha} = \omega^{\alpha} (\cos\frac{\pi}{2}\alpha + j\sin\frac{\pi}{2}\alpha) \qquad (7) $$

where α is a real number. Thus, the frequency domain expression of the fractional order control system can easily be obtained by substituting s with $j\omega$ in the Laplace transform of the transfer function of the fractional order control system.

3. Computation of $PI^{\lambda}D^{\mu}$ Controller Parameters

Consider the negative unity feedback control system shown in Fig. 1.

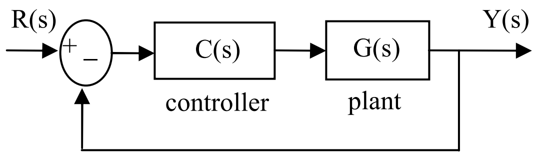

Fig. 1. Negative unity feedback system

Transfer function of the plant is an integer order. However the controller of the system is a fractional order $PI^{\lambda}D^{\mu}$ controller of the form,

$$ C(s) = k_p + \frac{k_i}{s^{\lambda}} + k_d s^{\mu} \qquad (8) $$

One can obtain a classical *PID* controller by taking $\lambda=1$ and $\mu=1$ as,

$$ C(s) = k_p + \frac{k_i}{s} + k_d s \qquad (9) $$

In this study, a method has been proposed to obtain the proportional gain constant k_p, the constant of integral term k_i, the constant of derivative term k_d , the fractional order of differentiator λ and the fractional order of integrator μ . The method uses classical Zigler-Nichols tuning rules to obtain k_p and k_i. Initial value of k_d is obtained from Åström-Hägglund method, then some fine tunings has bees done for better numerical solution. The fractional orders λ and μ are obtained to achieve specified phase margin using the idea of Åström-Hägglund tuning method.

Let ϕ_{pm} be the required phase margin and ω_{cp} be the frequency of the critical point on the Nyquist curve of $G(s)$ at which $\arg(G(j\omega_{cp})) = -180^{o}$ and define gain margin as,

$$ g_m = \frac{1}{\left|G(j\omega_{cp})\right|} = k_c \qquad (10) $$

Then, in order to make the phase margin of the system equal to ϕ_{pm} and $\left|C(j\omega_{cp})\,G(j\omega_{cp})\right| = 1$, the following equation must be satisfied.

$$ C(j\omega_{cp}) = \frac{1}{\left|G(j\omega_{cp})\right|} e^{j\phi_{pm}} = k_c \cos\phi_{pm} + jk_c \sin\phi_{pm} \qquad (11) $$

Then, one can write $C(j\omega_{cp})$ using Eqs. (8) and (11) as;

$$C(j\omega_{cp}) = k_p + k_i \omega_{cp}^{-\lambda} \cos(\frac{pi}{2}\lambda) + k_d \omega_{cp}^{\mu} \cos(\frac{pi}{2}\mu)$$
$$+ j[-k_i \omega_{cp}^{-\lambda} \sin(\frac{pi}{2}\lambda) + k_d \omega_{cp}^{\mu} \sin(\frac{pi}{2}\mu)] \quad (12)$$

Considering Eqs. (11) and (12), one can obtain,

$$f_1(\lambda,\mu) = k_p + k_i \omega_{cp}^{-\lambda} \cos(\frac{pi}{2}\lambda) +$$
$$k_d \omega_{cp}^{\mu} \cos(\frac{pi}{2}\mu) - k_c(\cos\phi_{pm}) = 0 \quad (13)$$

and

$$f_2(\lambda,\mu) = -k_i \omega_{cp}^{-\lambda} \sin(\frac{pi}{2}\lambda) +$$
$$k_d \omega_{cp}^{\mu} \sin(\frac{pi}{2}\mu) - k_c(\sin\phi_{pm}) = 0 \quad (14)$$

The numerical solutions for λ and μ can be obtained from Eqs. (13) and (14).

4. Tuning Method for $PI^{\lambda}D^{\mu}$ Controller

All the parameters of the $PI^{\lambda}D^{\mu}$ controller, which are given in Eq. (8), can be obtained by using the following procedure.

1- Specify the value of required phase margin ϕ_{pm}.
2- Obtain k_p and k_i from classical Zigler-Nichols rules.
3- Obtain Eqs. (13) and (14).
4- Specify the initial value for k_d using the Åström-Hägglund method.
5- Simulation results show that especially variation on k_d effect the numerical solution of the equations seriously. Therefore, fine tuning can be required for k_d to achieve the best numerical solution for the Eqs. (13) and (14).
6- Find the numerical solutions for λ and μ from Eqs. (13) and (14) by considering the new value of k_d.
7- If the step response of the system is not satisfactory enough, an optimization can be done by using optimization model to get better values for the controller parameters.

5. Application of the Proposed Method

Consider the negative unity feedback system given in Fig. 1. The transfer function of the system is given as,

$$G(s) = \frac{1}{s(s+1)(s+2)} \quad (15)$$

Phase crossover frequency of the system can be obtained as $\omega_{cp} = \sqrt{2}$ and gain margin of the system can be obtained as $k_c = 6$. Constants of proportional, integral and derivative terms of the controller has been obtained by using the classical Zigler-Nicholes rules as $k_p = 3.6$, $k_i = 1.63$ and $k_d = 1.98$. Let *PID* controller obtained from Zigler-Nichols method be *C₁(s)* as,

$$C_1(s) = 3.6 + \frac{1.63}{s} + 1.98s \quad (16)$$

Using the classical Åström-Hägglund method, the values of *PID* controller parameters have been calculated for specified phase margins which are shown in Table 1. Let $C_2(s)$ shows *PID* controller obtained from Åström-Hägglund method. For example, from Table 1, the *PID* controller for $\phi_{pm} = 40^o$ is given as,

$$C_2(s) = 4.59 + \frac{1.51}{s} + 3.48s \quad (17)$$

The proposed method takes the values of k_p and k_i from Zigler-Nicholes method. The initial values for derivative term k_d have been obtained by using the Åström-Hägglund method for the specified phase margins. Fine tuning has been done for the term k_d to achieve the best numerical solution of the Eqs. (13) and (14) for each specified phase margin. These two equations have been solved by using optimization toolbox "fsolve" of the MATLAB to obtain numerical values of λ and μ by considering the new value of k_d for each specified phase margin. Table 1 shows all the values of k_p, k_i, k_d, λ and μ for each of the specified phase margin. Let $C_3(s)$ shows the $PI^{\lambda}D^{\mu}$ controller which can be written from Table 1 for $\phi_{pm} = 40^o$ as,

$$C_3(s) = 3.6 + \frac{1.63}{s^{1.39}} + 3.75s^{0.79} \quad (18)$$

In order to obtain better step response, an optimization model has been developed using Simulink Library of MATLAB by considering Least Square Method for optimization. This optimization model has been used to get new optimized values for the parameters k_p, k_i, k_d, λ and μ. Let $C_4(s)$ shows the $PI^{\lambda}D^{\mu}$ controller with optimized values, which can be written from Table 2 as,

$$C_4(s) = 1.44 + \frac{2.64}{s^{0.51}} + 5.48s^{0.86} \quad (19)$$

Table 1. The values of *PID* controller parameters calculated by Åström-Hägglund and the $PI^{\lambda}D^{\mu}$ controller parameters obtained by proposed method for $\phi_{pm} = 40^o;45^o;50^o;55^o;60^o$

	Åström-Hägglund			Proposed method				
ϕ_{pm}	k_p	k_i	k_d	k_p	k_i	k_d	λ	μ
40°	4.59	1.51	3.48	3.6	1.63	3.75	1.39	0.79
45°	4.24	1.24	3.62	3.6	1.63	3.90	1.35	0.86
50°	3.86	0.99	3.75	3.6	1.63	4.05	1.31	0.91
55°	3.44	0.77	3.86	3.6	1.63	4.20	1.27	0.97
60°	3.00	0.57	3.96	3.6	1.63	4.30	1.32	1.01

Consequently, four type of controller namely $C_1(s)$, $C_2(s)$, $C_3(s)$ and $C_4(s)$, have been obtained for the given plant as follows,

- Parameters of $C_1(s)$ are calculated using Zigler-Nichols method, such as $k_p = 3.6$, $k_i = 1.63$ and $k_d = 1.98$ for all specified phase margin
- Parameters of $C_2(s)$ are calculated using Åström-Hägglund method, for specified phase margins as shown in Table 1.
- Parameters of $C_3(s)$ are obtained using the proposed method for the specified phase margins as shown in Table 1.
- Optimization model has been used to get better step response for the controller $C_3(s)$. Values of the parameters of $C_3(s)$ are taken as initial values for optimization. Then, new values for k_p, k_i, k_d, λ and μ are obtained for $C_4(s)$ as shown in Table 2.

Table 2 shows the optimization results of the controller parameters, which is obtained by taking the values of $C_3(s)$ for $\phi_{pm} = 40^o; 50^o; 60^o$ as initial value respectively. As seen from Table 2, the optimization results for the controller parameters for different phase margin are close to each other.

Table 2. The values of $PI^{\lambda}D^{\mu}$ controller parameters obtained by optimization model by taking the values of C3(s) for $\phi_{pm} = 40^o; 50^o; 60^o$, as initial values for optimization.

	Optimization Results				
Referance Values	k_p	k_i	k_d	λ	μ
For $\phi_{pm} = 40^o$	1.4379	2.6427	5.4828	0.5148	0.8630
For $\phi_{pm} = 50^o$	1.9299	2.3928	5.4086	0.5274	0.8756
For $\phi_{pm} = 60^o$	1.4571	2.5298	5.3283	0.4953	0.8622

Step responses of the system for $C_1(s)$, $C_2(s)$ and $C_4(s)$ for 40^o phase margin are given in Fig. 2 and the performance specifications for this value of phase margin are shown in Table 3, where one can clearly observe that the proposed method has better response than the others.

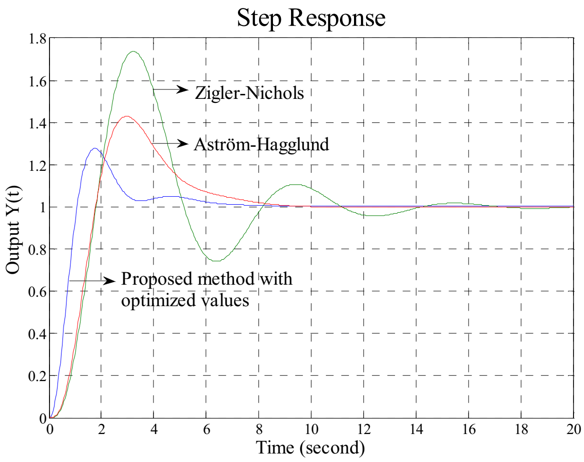

Fig. 2. Step responses of the system for $C_1(s)$, $C_2(s)$ and $C_4(s)$ for 40^o phase margin.

Table 3. Step response specifications of $C_1(s)$, $C_2(s)$ and $C_4(s)$

Step response specifications	Zigler-Nichols *PID*	Åström Hägglund *PID*	Proposed Fractional *PID*
Max. Overshoot (%)	73.5	43.0	27.9
Peak time (s)	3.25	2.95	1.74
Rise time (s)	1.67	1.66	0.96
Settling time (%5)	12.5	6.67	4.65
Settling time (%2)	13.6	8.00	6.20

As known, step response of the system gives valuable information such as, maximum overshoot, rise time, peak time and settling time. Thus, step responses of the system for $C_1(s)$, $C_2(s)$ and $C_4(s)$ are obtained by using the simulink model and "nintblocks" of MATLAB, which is developed by Duarte Valério [25]. Table 3 gives step response specifications of the system for $C_1(s)$, $C_2(s)$ and $C_4(s)$. One can conclude from Fig. 2 and Table 3 that the performance specifications of the proposed method are much better than the Zigler-Nichols and Åström-Hägglund tuning methods. Especially, maximum overshoot, rise time and settling time of the system are much better for the proposed method.

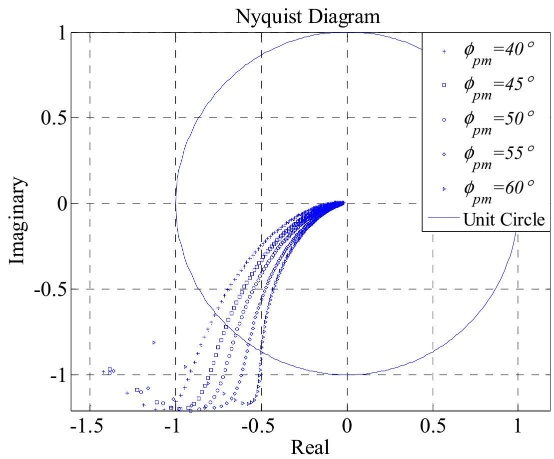

Fig. 3. Nyquist plot of the system for $C_3(s)$ for the phase margins $\phi_{pm} = 40^o; 45^o; 50^o; 55^o; 60^o$.

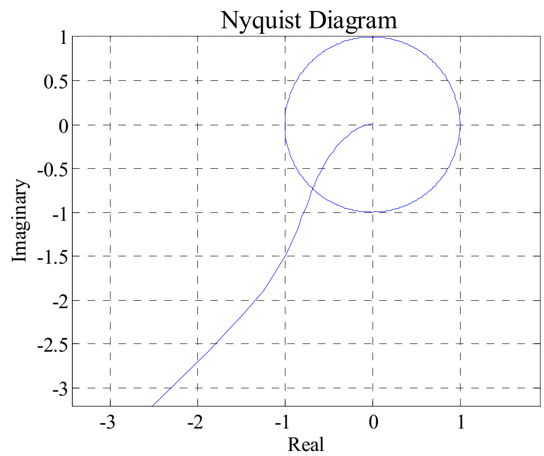

Fig. 4. Nyquist plot of the system for $C_4(s)$.

Nyquist plots of the system for the $C_3(s)$ for phase margins $\phi_{pm} = 40^o; 45^o; 50^o; 55^o; 60^o$ and Nyquist plot of the system for $C_4(s)$ are obtained by using the toolbox, which is developed in MATLAB by C. Yeroglu and N. Tan [26]. Nyquist plots are given for the frequency range of $0.5 < \omega < 100$ as shown in Fig. 3 and 4. From Fig. 3, it can be seen that the system satisfies each of the specified phase margin for C_3(s). As seen from Fig. 4, the values of the gain and phase margins of the system for $C_4(s)$ are suitable.

6. Conclusion

A method for tuning of $PI^{\lambda}D^{\mu}$ controller has been proposed. The presented method is based on the idea of using Zigler-Nichols and Åström-Hägglund method together. k_p and k_i parameters of $PI^{\lambda}D^{\mu}$ controller have been computed from Zigler-Nichols method and the remaining parameters k_d , λ and μ have been found from Åström-Hägglund method using critical point information. Values of the controller parameters are optimized to achieve better step response. The simulation results demonstrated that the $PI^{\lambda}D^{\mu}$ controller has better response than the classical *PID* controllers.

It is necessary to point out that there are many other tuning methods in the literature for PID controllers, which may give better results than Zigler-Nichols and Åström-Hägglund methods for some cases. Some tuning methods for $PI^{\lambda}D^{\mu}$ controller are also proposed in recent years. The comparison study of the proposed methods for tuning of $PI^{\lambda}D^{\mu}$ controllers certainly will be very important. Research in this direction can be done in the future work.

7. References

[1] K. Ogata, "Modern Control Engineering", Prentice Hall, New Jersey, 2002.

[2] K. Åström and T. Hägglund, "PID controllers: Theory, Design and Tuning", *Instrument society of America*, Nort Carolina, 1995.

[3] A. Oustaloup,"LaDerivation Non Entiere", Paris, 1995.

[4] A. Oustaloup, F. Levron, B. Mathieu and F. M. Nanot, "Frequency-Band Complex Noninteger Differentiator: Characterization and Synthesis", *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, Vol. 47, No. 1, 2000.

[5] I. Podlubny, "Fractional Differential Equations", Academic Press, San Diego, California, 1999.

[6] I. Podlubny, "Fractional-order systems and $PI^{\lambda}D^{\mu}$ controllers", *IEEE Transactions on Automatic Control*, vol. 44, no. 1, pp. 208–214, 1999.

[7] B. Vinagre, I. Podlubny, L Dorcak and V. Feliu, "On fractional PID controllers: A frequency domain approach", *IFAC workshop on digital control. Past, present and future of PID control*, pp. 53–58, Terrasa, Spain, 2000.

[8] I. Petras and M. Hypiusova, "Design of fractional-order controllers via H_∞ norm minimization", *Selected Topics in Modeling and Control*, vol. 3, pp. 50–54, 2002.

[9] Y. Q.Chen and K. L. Moore, "Discritization schemes for fractional order differentiators and integrators" *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, 2002, vol. 49(3), pp.363–367.

[10] Y. Q. Chen, B. M. Vinagre, and I. Podlubny, "Continued fraction expansion approaches to discretizing fractional order derivatives—An expository review", *Nonlinear Dynamics*, vol.38, pp.155–170, 2004.

[11] D. Xue, Y. Q. Chen, "A comparative Introduction of Four Fractional Order Controllers", *Proceeding the 4th World Congress, Intelligent Control and Auto, v*ol. 4 pp.3228-3235, 2002.

[12] S.E. Hamamci "An algorithm for stabilization of fractional-order time delay systems using fractional-order PID controllers", *IEEE Trans. On Automatic Control*, vol. 52, pp. 1964-1969, 2007.

[13] M.Ö.Efe, Kesirli dereceli kayan kipli denetim, pp:622-627, *TOK'08* ,İstanbul, 2008.

[14] R. S. Barbosa, J. A. Tenerio, Machado and Isabel. M. Ferreira, "Tuning of PID controllers based Bode's Ideal transfer function, *Nonlinear Daynamics*, vol. 38, pp.305-321, 2004.

[15] C. A. Monje, B. M. Vinagre, V. Feliu, Y.Q. Chen, "Tuning and auto-tuning of fractional order controllers for industry applications", *Control Engineering Practice*, vol. 16, pp. 798–812, 2008.

[16] K. Bettou, A. Charef, F. Mesquine, "A new design method for fractional $PI^{\lambda}D^{\mu}$ controller", *IJ-STA*, vol. 2, pp. 414-429, 2008.

[17] M. Zhuang and D. P. Atherton, "Automatic tuning of optimum PID controllers", *IEEE Proceedings-Part D:Control Theory and Applications,* vol. 140, issue 3, pp. 216-224, 1993

[18] R. Malti, M. Aoun, O. Cois, A. Oustaloup, and F. Levron, "H_2 norm of fractional differential systems", In *ASME 2003 design engineering technical conferences and computers and information in engineering conference*, Chicago, USA, 2003.

[19] G. Maione and P. Lino, "New Tuning Rules for Fractional PI^{α} controllers", *Nonlinear Dynamics*, 2006.

[20] C.A. Monje, A.J. Calderon, B.M.Vinagre, Y.Q.Chen and V. Feliu, "On Fractional PI^{λ} Controllers: Some Tuning Rules for Robustness to Plant Uncertainties", *Nonlinear Dynamics*, vol. 38, pp. 369-381, 2004.

[21] K. J. Åström and T. Hägglund, "Automatic Tuning of Simple Regulators", *IFAC 9th World Congress*, Budapest, 1984.

[22] K. J. Åström and T. Hägglund, "Automatic Tuning of Simple Regulators with specification on phase and amplitude margins", *Automatica*, Vol. 20, No. 5, pp. 645-651, 1984.

[23] W. Jifeng, L. Yuankai, "Frequncy domain Analysis and Applications for Fractional Order Control System", *IOP, Journal of Physics:Conf. 13*, pp.268-273, 2005.

[24] D. Xue, Y.Q. Chen, D. P. Atherton "Linear Feedback Control Analysis and Design with MATLAB", Advances in Design and Control, Siam, 2007.

[25] D. Valério, "Ninteger v. 2.3 Fractional control toolbox for MatLab", *http://web.ist.utl.pt/~duarte.valerio*, 2005.

[26] C. Yeroglu, N. Tan, "Development of a Toolbox for Frequency Response Analysis of Fractional Order Control Systems", *19th European Conference on Circuit Theory and Design*, Antalya, August 2009.