EXPERIMENTAL IDENTIFICATION OF AN ELECTROMECHANICAL SYSTEM RUNNING IN OPEN-LOOP CONDITIONS

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Keywords: DC motor, identification, modelling, least-squares estimation.

ABSTRACT

The current trends in development and deployment of electromechanical systems have facilitated the unified activities in the analysis and design of state-of-the-art motion devices, electric motors and digital controllers. This paper presents discrete-time on-line identification of a permanent-magnet DC motor in open-loop conditions. Studies are carried out by formulating the mathematical model using differential equations, and digital identification using plant input-output data. Forgetting factor dependent recursive least squares method is used to estimate the parameters of the motor which match input-output behaviour of the motor. Discrete-time data for parameter identification are obtained experimentally carrying out on a permanent-magnet DC motor set-up in laboratory. Root-mean-square-error criterion is used to check matching of actual output and predicted output. Results are presented which show variations in machine parameters.

1. INTRODUCTION

The problem of controlling electromechanical systems is very important one in many industrial applications [1]. Input-output relation of these systems must be researched to improve the steady-state and dynamic characteristics such as electric drives and DC motors. The main advantages of the DC motors are easy speed or position control and wide adjustable range to follow a predetermined speed or position trajectory under load [2, 3, 4]. These have been extensively used in several industrial applications [1, 2, 4, 5, 7, 8, 9]. Controller parameters of a DC motor have been calculated using the linear fixed motor parameters at an operating point. However, the fixed parameter controllers may not give desired performance under different operating conditions. Recently, there has been considerable development in adaptive control schemes for the servo systems of the DC motors with their industrial applications [4]. This has attracted extensive researches in the field of control engineering, especially in the areas of plant identification and control [1, 8, 10].

The objective of this research is to document our recent experimental studies on identification of a DC motor. A motor model is first developed that has two inputs and single output using ordinary linear differential equations. The motor, then, is modelled by linear difference equations from digital input-output data obtained experimentally. Root-mean-square-error method is used to check model matching. The tests are performed in open-loop conditions.

II. MODEL OF THE PLANT

An important step in designing a control system is a proper modelling of the plant to be controlled [8]. An exact plant model should produce output responses similar to those of the actual plant. The complexity of most physical plants, however, makes the development of exact models infeasible. Therefore, in order to design a controller that is reliable and easy to understand in practice, simplified plant models should be obtained around operating points [8]. Many plants may be modelled as a multimass plant with the masses connected with flexible shafts or springs [11]. In many cases the modelling is further simplified by considering a two-mass plant where the first mass represents the motor, the second mass represents the load, and the shaft is connected mass or inertia free [12]. The schematic diagram of the electromechanical plant, DC motor connected to a load, is given in Figure 1. The DC motor can be viewed as a two inputs and one output, where the motor armature voltage (or current) and external torque are the inputs and the velocity (or angular displacement) is the output. The linear equations describing the electrical components of the motor can be represented as [1, 6, 10]:

\[ v_a = R_a i_a + L_a \dot{i}_a + e_a \]  

Figure 1. A schematic diagram of the linear two-mass system.
\[ e_a = K_m \omega_m \]  \hspace{1cm} (2) \\
\[ T_m = K_m i_a \]  \hspace{1cm} (3)

where \( e_a \) is the motor armature voltage, \( R_a \) and \( L_a \) are the armature coil resistance and inductance, \( i_a \) is the armature current, \( e_r \) is the back electromotive-force voltage, \( K_m \) is the motor constant, \( T_m \) is the generated motor torque. The linear equations representing the plant mechanical components are given as [1, 13]:

\[ J_m \dot{\omega}_m = -B_m \omega_m - T_s + T_m \]  \hspace{1cm} (4) \\
\[ J_L \dot{\omega}_L = -B_l \omega_L + T_s - T_d \]  \hspace{1cm} (5) \\
\[ T_s = k_s \omega_s + B_s \dot{\omega}_s \]  \hspace{1cm} (6) \\
\[ \ddot{\omega}_m = \omega_m, \quad \ddot{\omega}_L = \omega_L, \quad \dot{\omega}_s = \dot{\omega}_s, \quad \omega_s = \omega_m - \omega_L \]  \hspace{1cm} (7)

where \( J_m \) is the motor moment of inertia, \( B_m \) is the motor viscous friction, \( T_s \) is the transmitted shaft torque, \( J_L \) is the load moment of inertia, \( B_l \) is the viscous load friction, \( T_d \) is the load torque disturbance, \( k_s \) is the shaft elasticity, and \( B_s \) is the inner damping coefficient of the shaft. The angles \( \omega_m, \theta_i, \theta_s \), \( \omega_a \), \( \omega_s \), \( \omega_L \), \( \omega_m \) are the their respective time derivatives; the motor angular velocity, the load angular velocity, and the difference angular velocity. Block diagram of the plant is illustrated in Figure 2 that includes transfer functions representations of inherent components. The transfer functions are obtained from Eqs. (1-7) as:

\[ G_1(s) = \frac{1}{L_a s + R_a}, \quad G_2(s) = \frac{1}{J_m s + B_m}, \]
\[ G_3(s) = \frac{B_s s + k_s}{s}, \quad G_4(s) = \frac{1}{J_L s + B_L} \]  \hspace{1cm} (8)

where \( G_1, G_2, G_3, G_4 \in \mathbb{R}(s) \) (\( \mathbb{R}(s) \) denotes set of all real rational functions), the variable \( s \) denotes complex frequency, \( s = j \omega, \quad j = \sqrt{-1}, \quad \omega \) is the frequency. The relation between the output (\( \omega_L \)) and inputs (\( V_a \)) is of interest of the present paper. Fourth-order transfer function between \( \omega_L \) and \( V_a \) occurs.

**III. EXPERIMENTAL SET-UP**

Two signal generators are used to supply input voltage to the motor in experimental set-up. One of the signal generators is to produce constant input voltage and the other is used for variable input voltage to provide perturbation signal for identification. A computer (Pentium II MMX 300 MHz 128 MB RAM) is used to generate zero mean independent random noise signal to corrupt the output speed signal. The noise signal is generated in Matlab and the signal data file is converted into a wav file that is sent to the motor input using a sound card (Crystal PnP Audio soundcard) used as D/A converter. The measured input-output data are transferred to another computer (A computer (Pentium II MMX 300 MHz 128 MB RAM) by a Data Acquisition card (ADVANTECH PCL-1800, 130 kHz in speed, 12 bit high-speed A/D converter with a conversion time of 2.5 µs.). The output shaft speed is measured from an optical sensor (as rev/s) and a tachogenerator (as Volts) connected to the motor shaft. The DC motor operates at ±12 volts armature voltage input with a maximum output shaft speed of 2400 rev/min. The motor drives a shaft that carries disks which operate various transducers, and a tachogenerator. A low pass filter is used to filter the output speed signal from high frequency noise components. The motor speeds at different input armature voltages are measured to obtain the tachogenerator characteristics. The results are given in Table 1. It has almost a linear characteristics with a calculated gain of 2.15 Volt/rad/s.

**Table 1. DC motor responses for different input armature voltages.**

<table>
<thead>
<tr>
<th>Applied input (Volt) (( V_a ))</th>
<th>Speed (rev/s)</th>
<th>Speed (rad/s)</th>
<th>Tacho output (Volt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>7</td>
<td>0.7330</td>
<td>1.60</td>
</tr>
<tr>
<td>3.0</td>
<td>11</td>
<td>1.1519</td>
<td>2.52</td>
</tr>
<tr>
<td>4.0</td>
<td>15</td>
<td>1.5708</td>
<td>3.50</td>
</tr>
<tr>
<td>5.0</td>
<td>19</td>
<td>1.9896</td>
<td>4.46</td>
</tr>
<tr>
<td>6.0</td>
<td>24</td>
<td>2.5132</td>
<td>5.42</td>
</tr>
<tr>
<td>7.0</td>
<td>28</td>
<td>2.9320</td>
<td>6.37</td>
</tr>
<tr>
<td>8.0</td>
<td>32</td>
<td>3.3510</td>
<td>7.32</td>
</tr>
<tr>
<td>9.0</td>
<td>37</td>
<td>3.8746</td>
<td>8.32</td>
</tr>
<tr>
<td>10.0</td>
<td>41</td>
<td>4.2935</td>
<td>9.26</td>
</tr>
</tbody>
</table>

**IV. PROCESS REACTION METHOD**

Process reaction curve method is one of the widely used approaches to predetermine the dynamic behaviour of a plant under load or no-load conditions [15]. Some dynamical properties of the plant can be obtained using the process reaction curve method such as rise time, settling time, time constant, time delay. A 7.5 V in

![Figure 2. Block diagram of the linear two-mass system.](image-url)
magnitude step input armature voltage is applied to the motor and the output shaft speed response from the tachogenerator is obtained such that the steady-state output value is 6.65 V. The rise time is about 0.321 s while delay time at the beginning is about 0.008 s. The steady-state gain is 0.886 in magnitude, the time delay is \( T = 0.15 \) s.

**V. DIGITAL IDENTIFICATION**

The role of identification consists in describing the behaviour of a given plant by a model suitably selected within an appropriate class of systems [16,17]. The selection criterion exploits the information contained in the observation data available over finite time horizon. Sample time that is also an important parameter to estimate suitable parameter set should be chosen to be fast relative to the plant dynamics [18]. Otherwise, unsuitable choice of the sample time may cause small plant gain, non-minimum phase plant, aliasing and stability problems.

Recursive least squares (RLS) methods that have been widely used with several advantages such as easy numerical solution and fast parameter convergence gives a consistent modelling accuracy over a wide range of operating conditions and is the best linear unbiased estimate [19,20]. We consider the plant as a discrete-time transfer function, \( G(z^{-1}) \):

\[
G(z^{-1}) = z^{-d} \frac{B(z^{-1})}{A(z^{-1})} \in R(z^{-1})
\]

where \( d \) is the discrete dead time, \( z^{-1} \) is the time shift operator, \( A(z^{-1}) \) and \( B(z^{-1}) \) are the plant polynomials with real coefficients, \( A(z^{-1}), B(z^{-1}) \in R[z^{-1}] \):

\[
A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_{n_a} z^{-n_a} \quad (10a)
\]

\[
B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_{n_b} z^{-n_b} \quad (10b)
\]

where \( a_i \) (\( i = 1, 2, \ldots, n_a \)), \( b_j \) (\( j = 0, 1, 2, \ldots, n_b \)) \( \in \mathbb{R} \), \( n_a \) and \( n_b \) represent the order of the plant model polynomials, respectively, with \( n_a \geq n_b \), \( R \) denotes set of all real numbers, \( R[.] \) denotes set of finite polynomials with real coefficients. The relationship between the plant output \( y(k) \) and the plant input \( u(k) \) can be written as

\[
y(k) + \sum_{i=1}^{n_a} a_i y(k-i) = \sum_{j=1}^{n_b} b_j u(k-d-j) + e(k)
\]

where the term \( e(k) \) represents the effects of residual errors in modeling the plant, and disturbances that affect the plant. Eq. (11) can be rearranged in a compact form:

\[
y(k) = \phi^T(k) \hat{\theta} + e(k)
\]

where the data vector \( \phi(k) \) includes the past values of input and output, and \( \theta \) is the plant parameter vector:

\[
\phi(k) = [-y(k-1) -y(k-2) -\ldots -y(k-n_a)]^T
\]

\[
u(k-d-1) u(k-d-2) -\ldots u(k-d-n_b)]^T
\]

\[
\theta = [a_1 \ a_2 \ \ldots \ a_{n_a} \ b_0 \ b_1 \ \ldots \ b_{n_b}]^T
\]

Furthermore, the sequence of errors \( \{e(k)\} \) is independent uncorrelated noise sequence of variance \( \sigma_e^2 \) and \( E[e(t)] = 0 \). The plant parameters, \( a_i \) and \( b_j \) are assumed to be unknown and should be estimated. The estimation model of the plant is

\[
G_e(z^{-1}) = z^{-d} \frac{B_e(z^{-1})}{A_e(z^{-1})} \in R(z^{-1})
\]

where

\[
A_e(z^{-1}) = 1 + \hat{a}_1 z^{-1} + \hat{a}_2 z^{-2} + \ldots + \hat{a}_{n_a} z^{-n_a} \in R[z^{-1}]
\]

\[
B_e(z^{-1}) = \hat{b}_0 + \hat{b}_1 z^{-1} + \hat{b}_2 z^{-2} + \ldots + \hat{b}_{n_m} z^{-n_m} \in R[z^{-1}]
\]

where \( n_a \) and \( n_m \) represent the order of the estimated plant model polynomials \( A_e(z^{-1}) \) and \( B_e(z^{-1}) \), respectively, with \( n_a \geq n_m \), the symbol ‘\( \wedge \)’ denotes an estimated parameter. The parameter estimates vector is given

\[
\hat{\theta} = [\hat{a}_1 \ \hat{a}_2 \ \ldots \ \hat{a}_{n_a} \ \hat{b}_0 \ \hat{b}_1 \ \ldots \ \hat{b}_{n_b}]^T
\]

The estimated model output is

\[
\hat{y}(k) = \phi^T(k) \hat{\theta}(k-1)
\]

The model prediction error, \( e(k) \) that is the difference between the plant output and the estimated model output is a key variable in recursive least squares algorithm, is defined as

\[
e(k) = y(k)-\phi^T(k) \hat{\theta}(k-1)
\]

The error is used to update the parameter estimate as

\[
\hat{\theta}(k) = \hat{\theta}(k-1) + P(k) \phi(k) e(k)
\]

where the estimator covariance matrix \( P(k) \) is updated using

\[
P(k) = \frac{1}{\lambda} P(k-1) I_z - \frac{\phi(k) \phi^T(k) P(k-1)}{\lambda + \phi^T(k) P(k-1) \phi(k)}
\]

where the subscript ‘\( z \)’ is the dimension of the identity matrix, \( z = n_a + n_b + 1 \), \( \lambda \) is the forgetting factor,
0 < \lambda \leq 1$. In general, choosing $0.98 < \lambda < 0.995$ gives a good balance between convergence speed and noise susceptibility [19]. In the experiments, the sample frequency was chosen to be $f_s = 25 \text{Hz}$ that satisfies requirement of choice of the sample time [18]. The initial values of parameters are taken to be zero ($\hat{a}_i(0) = 0$, $\hat{b}_j(0) = 0$). The initial covariance matrix diagonal values are taken $P(0) = 10I$ with a forgetting factor of $\lambda = 0.985$. The robustness of the covariance matrix is achieved using a Biernan’s U-D factorization algorithm [18]. Square wave signal is one of the commonly used perturbation signals that can be used in open-loop identification [18] and adaptive control applications [19, 21]. The important point is that duration of the square wave should be sufficient to excite the slowest plant mode. A square wave periodic signal with a frequency of $f = 0.8 \text{Hz}$ that is superimposed on a 5.0 volts dc signal was applied to the plant input. This perturbation signal is sufficient to excite the slowest plant mode since the plant has a rise of 0.321 s. The input signal (armature voltage) is not correlated with noise signals to obtain a bias free parameter estimates [14]. The estimated parameters for a third-order model are shown in Figure 3 and Figure 4 that converge after a certain samples. The speed convergence depends on the forgetting factor used, here $\lambda = 0.985$. More faster parameter convergence can be obtained if value of the forgetting factor is reduced.

**VI. MODEL VALIDATION**

After any plant identification process some form of model validation should be performed. The method is to visually compare the model response to that of the actual plant response [22]. Major deficiencies in the model structure and parameter estimates would give rise to obvious errors in the model output sequence. Root-mean-square (RMS) error method is one of the commonly used approaches for model validation [8]. The RMS errors between the actual plant output and predicted model output should be compared to find a proper model structure. The estimation is performed using different model orders for comparison. The RMS errors calculated for different estimation model orders are 0.02902 for the first order, 0.02342 for the second order, 0.01706 for the third order, 0.01680 for the fourth order, 0.01646 for the sixth order and 0.01652 for the seventh order. A third order model and fourth order model appears to be suitable. Further increase in the model order brought no significant improvement in the performance of predicted models. As compared to first-order and second-order model, it was found that the third order model contributed to the improvement of the predicted output.

**VII. CONCLUSIONS**

In this paper, we deal with the problem of modelling and identifying of an armature controlled DC motor from input-output data taking into account the prior information on the dynamical nature of the plant. The aim of this research is also to highlight some of the more practical implications of plant identification and to describe the well-established algorithm (RLS) used to perform plant analysis. Process reaction curve method is used to ensure the steady-state and transient behaviour of the permanent-magnet DC motor coupled to a shaft and load.

A real-time implementation of the RLS estimator is presented on the DC motor. The open-loop experimental tests conducted successfully demonstrate the ease of the computer based parameter identification method. The measured data obtained experimentally from real-time set-up are used instantly by a software program that runs in Matlab environment to identify unknown plant parameters. A third-order discrete-time linear model is shown to be flexible enough to fit the observations well. Thus, this model and motor parameter identification procedure allows adaptive control designers to develop control algorithms explicitly. It also became apparent
that the order of the suitable linear model was lower than
the theoretical one.

**RECOMMENDATIONS FOR FUTURE WORK**

The visual comparison of the actual output and predicted
output responses, as shown in Figure 5, does not show
some obvious errors due to the non-linearities within the
plant. The non-linearities appear to manifest themselves
in the inconsistent peaking of the plant responses. To
improve the model further would require an approach
outside the conventional linear techniques capabilities.

![Figure 5. Actual (solid line) and predicted (dotted line) outputs.](image)

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