

BLIND MAXIMUM LIKELIHOOD CHANNEL ESTIMATION FOR SPACE-TIME CODING SYSTEMS

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ABSTRACT

Sophisticated signal processing techniques have to be developed for capacity enhancement of future wireless communication systems. In recent years, space-time coding is proposed to provide significant capacity gains over the traditional communication systems in fading wireless channels. Space-time codes are obtained by combining channel coding, modulation, transmit diversity and optional receive diversity in order to provide diversity at the receiver and coding gain without sacrificing the bandwidth. In this paper, we consider the problem of blind estimation of space-time coded signals along with the channel parameters. Conditional maximum likelihood approach is considered and iterative solution is proposed. The proposed conditional maximum likelihood algorithm is based on iterative least squares with projection technique. The performance analysis issue of the proposed method is also studied. Finally, some simulation results are presented.

1. INTRODUCTION

The rapid growth in demand for a wide range of wireless services is a major driving force to provide high-data rate and high quality wireless access over fading channels [1]. However, wireless transmission is limited by available radio spectrum and impaired by path loss, interference from other users and fading caused by destructive addition of multipath. Therefore several physical layer related techniques have to be developed for future wireless systems to use the frequency resources as efficiently as possible. One

approach that shows real promise for substantial capacity enhancement is the use of diversity techniques [2]. Diversity techniques basically reduce the impact of fading due to multipath transmission and improve interference tolerance which in turn can be traded for increase capacity of the system. In recent years, the use of antenna array at the base station for transmit diversity has become increasingly popular since it is difficult to deploy more than one or two antennas at the portable unit. Transmit diversity techniques make several replicas of the signal available to the receiver with the hope that at least some of them are not severely attenuated. Moreover, the methods of transmitter diversity combined with channel coding have been employed at the transmitter, which is referred to as space-time coding, to introduce temporal and spatial correlation into signals transmitted from different antennas [2], [3]. The basic idea is to reuse the same frequency band simultaneously for parallel transmission channels to increase channel capacity [2], [3].

Unfortunately, employing antenna diversity at the transmitter is particularly challenging since the signals are combined in space prior to reception. Moreover, estimation of fading channels in space-time systems is further complicated, since the receiver estimates the path gain from each transmit antenna to each receive antenna. There has been considerable work reported in the literature on the estimation of channel information to improve performance of space-time coded systems operating on fading channels [4], [5], [6], [7]. In this paper we consider the problem of blind estimation of space-time coded signals along with the matrix of path gains. We propose conditional maximum likelihood (ML) approach which results in joint estimation of the channel matrix and the input sequences and is based on the iterative least

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squares with projection (ILSP) [8].

The performance of the proposed ML approach is explored based on the evaluation of Cramer-Rao bound (CRB). The CRB is a well known statistical tool that provides benchmarks for evaluating the performance of actual estimators. For the conditional estimator, the CRB derived in [9], is adapted to the present scenario.

2. SYSTEM MODEL

In the sequel, we consider a mobile communication system equipped with n transmit antennas and optional m receive antennas $m \leq n$. A general block diagram for the systems of interest is depicted in Figure 1. In this system, the source generates bit sequence $s(k)$, which are encoded by an error control code to produce codewords. The encoded data are parsed among n transmit antennas and then mapped by the modulator into discrete complex valued constellation points for transmission across channel. The modulated streams for all antennas are transmitted simultaneously. At the receiver, there are m receive antennas to collect the transmissions. Spatial channel link between each transmit and receive antenna is assumed to experience statistically independent fading.

The signals at each receive antenna is a noisy superposition of the faded versions of the n transmitted signals. The constellation points are scaled by a factor of E_s so that the average energy of constellation points is 1. Then we have the following complex base-band equivalent received signal at receive antenna j :

$$r_j(k) = \sum_{i=1}^n \alpha_{i,j}(k) c_i(k) + n_j(k) \quad (1)$$

where $\alpha_{i,j}(k)$ is the complex path gain from transmit antenna i to receive antenna j , $c_i(k)$ is the coded symbol transmitted from antenna i at time k , $n_j(k)$ is the additive white Gaussian noise sample for receive antenna j at time k . (1) can written in a matrix form

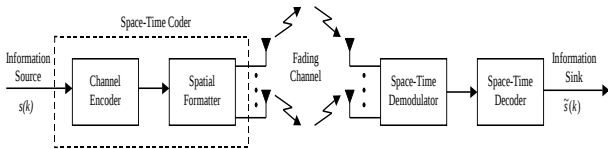


Figure 1: Space-time coding and decoding system

as

$$\mathbf{r}(k) = \mathbf{\Omega}(k) \mathbf{c}(k) + \mathbf{n}(k) \quad (2)$$

where $\mathbf{r}(k) = [r_1(k), \dots, r_m(k)]^T \in \mathbb{C}^{m \times 1}$ is the received signal vector, $\mathbf{c}(k) = [c_1(k), \dots, c_n(k)]^T \in \mathbb{C}^{n \times 1}$ is the code vector transmitted from the n transmit antennas at time k , $\mathbf{n}(k) = [n_1(k), \dots, n_m(k)]^T \in \mathbb{C}^{m \times 1}$

is the noise vector at the receive antennas and $\mathbf{\Omega}(k) \in \mathbb{C}^{m \times n}$ is the fading channel gain matrix given as

$$\mathbf{\Omega}(k) = \begin{bmatrix} \alpha_{1,1}(k) & \cdots & \alpha_{n,1}(k) \\ \vdots & \cdots & \vdots \\ \alpha_{1,m}(k) & \cdots & \alpha_{n,m}(k) \end{bmatrix}.$$

We impose following assumptions on model (2) for the rest of the paper:

AS1: Information sequence $s(k)$ is adopting finite complex values.

AS2: The noise vector $\mathbf{n}(k) = [n_1(k), \dots, n_m(k)]^T$ is Gaussian distributed with zero-mean and

$$\begin{aligned} \mathbb{E}[\mathbf{n}(k) \mathbf{n}^H(l)] &= \sigma^2 \mathbf{I} \delta_{k,l} \\ \mathbb{E}[\mathbf{n}(k) \mathbf{n}^T(l)] &= \mathbf{0} \end{aligned} \quad (3)$$

where $\mathbb{E}$ denotes expectation operator and $\delta_{k,l}$ is the Kronecker delta ($\delta_{k,l} = 1$ if $k = l$ and 0 otherwise).

Thus $\mathbf{n}(k)$ is assumed to be uncorrelated both temporally and spatially.

AS3: The fading channel is assumed to be quasi-static flat fading, so that during the transmission of L codeword symbols across any one of the links, the complex path gains do not change with time k , but are independent from one codeword transmission to the next, i.e.,

$$\alpha_{i,j}(k) = \alpha_{i,j}, \quad k = 1, 2, \dots, L. \quad (4)$$

The problem of estimating matrix of path gains along with the space-time coded signals from noisy observations $\mathbf{r}(L) = [\mathbf{r}^T(1), \dots, \mathbf{r}^T(L)]^T$ is the main concern of the paper. The traditional solution to this problem is to first estimate $\boldsymbol{\theta} = [\mathbf{\Omega}]$ from training sequence embedded in the input signal and then use these estimates as if they were the true parameters to obtain estimates of input sequence. As an alternative, we propose blind ML approach based on ILSP.

3. CONDITIONAL ML

An ML approach is developed in this section under **AS1**, **AS2**, **AS3** and the conditional signal model assumption (deterministic but unknown signal). The log-likelihood function is then given by,

$$\mathcal{L} = -\text{const} - mL \log \sigma^2 - \frac{1}{\sigma^2} \sum_{k=1}^L \|\mathbf{r}(k) - \mathbf{\Omega} \mathbf{c}(k)\|^2. \quad (5)$$

The conditional ML estimation can be obtained by jointly maximizing $\mathcal{L}$ over the unknown parameters $\mathbf{\Omega}$ and $\mathbf{c}(L) = [\mathbf{c}^T(1), \dots, \mathbf{c}^T(L)]^T$. After neglecting unnecessary terms, conditional ML yields the following minimization problem

$$\min_{\mathbf{\Omega}, \mathbf{c}(L)} \|\mathbf{r}(L) - \mathbf{\Omega} \mathbf{c}(L)\|^2. \quad (6)$$

Since the elements of $\mathbf{c}(L)$ are restricted to be finite alphabet, (6) results in a nonlinear separable optimization problem with mixed integer and continuous variables. Typically, the minimization problem in (6) is solved by alternatively minimizing with respect to $\mathbf{\Omega}$ and $\mathbf{c}(L)$ while keeping other parameters fixed. First we minimize (6) with respect to $\mathbf{\Omega}$. Then substitute $\hat{\mathbf{\Omega}}$ back into (6) and solve it for $\mathbf{c}(L)$. The ML estimate of $\mathbf{c}(L)$ in the second step can be obtained by enumeration. However, this search is computationally intensive since the number of possible $\mathbf{c}(L)$ matrices that need to be checked grows exponentially both with L and n . We now adopt a block conditional ML algorithm that has a lower computational complexity [8]. The proposed algorithm is based on ILSP [8]. It takes advantage of the ML estimator being separable in its continuous and integer variables.

Given an initial estimate $\hat{\mathbf{\Omega}}$ of $\mathbf{\Omega}$, the minimization of (6) with respect to $\mathbf{c}(L)$ is a least squares problem that can be solved in closed form. Each element of the solution is rounded-off to its closest discrete values (coded M-PSK signals). Then a better estimate of $\mathbf{\Omega}$ is obtained by minimizing (6) with respect to $\mathbf{\Omega}$, keeping $\hat{\mathbf{c}}(L)$ fixed. This minimization also results in least squares. This process continues until $\mathbf{\Omega}$ converges.

The following steps summarize the conditional ML algorithm:

Start with initial estimate $\mathbf{\Omega}_{(0)}$, $i = 0$

1. $i=i+1$

- $\mathbf{c}_i(L) = (\mathbf{\Omega}_{i-1}^* \mathbf{\Omega}_{i-1})^{-1} \mathbf{\Omega}_{i-1}^* \mathbf{r}(L)$.
- Project each element of $\mathbf{c}_i(L)$ to closest discrete values.
- $\mathbf{\Omega}_i = \mathbf{r} \mathbf{c}_i^*(L) (\mathbf{c}_i(L) \mathbf{c}_i^*(L))^{-1}$

2. Continue until $(\mathbf{\Omega}_i - \mathbf{\Omega}_{i-1}) = \mathbf{0}$.

The proposed conditional ML (ILSP) algorithm converges to a local minimum of (6). However, sufficiently good initialization provided from suboptimal techniques improve the possibility of global convergence and also reduce the number of iterations required.

4. CONDITIONAL CRB

The performance of the conditional ML method is assessed here by deriving their CRBs for the unbiased estimates of the nonrandom parameters. The CRB depends on the information on vector parameter $\boldsymbol{\theta}$ quantified by the Fisher information matrix (FIM) and provides a lower bound on the variance of the unbiased estimate (i.e., $\mathbb{E}\{\hat{\boldsymbol{\theta}}\} = \boldsymbol{\theta}$). Then the CRB for an unbiased estimator $\hat{\boldsymbol{\theta}}$ is bounded by the inverse of the FIM $\mathbf{J}(\boldsymbol{\theta})$:

$$\mathbb{E}\{(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})^T\} \geq \mathbf{J}^{-1}(\boldsymbol{\theta}). \quad (7)$$

The derivation of $\mathbf{J}(\boldsymbol{\theta})$ in (7) follows along the lines of [9]. We start constructing FIM by calculating the derivative of (5) with respect to $\boldsymbol{\tau} = [\mathbf{c}_r^T(1) \mathbf{c}_c^T(1) \cdots \mathbf{c}_r^T(L) \mathbf{c}_c^T(L) \quad \boldsymbol{\alpha}_r^T \boldsymbol{\alpha}_c^T]^T$ where

$$\begin{aligned} \mathbf{c}_r(k) &= \text{Re}\{[c_1(k), \dots, c_n(k)]^T\} \\ \mathbf{c}_c(k) &= \text{Im}\{[c_1(k), \dots, c_n(k)]^T\} \\ \boldsymbol{\alpha}_r^i &= \text{Re}\{[\alpha_{1,i}, \dots, \alpha_{m,i}]^T\} \\ \boldsymbol{\alpha}_r &= \text{Re}\{[\boldsymbol{\alpha}_1^T, \dots, \boldsymbol{\alpha}_n^T]^T\} \\ \boldsymbol{\alpha}_c^i &= \text{Im}\{[\alpha_{1,i}, \dots, \alpha_{m,i}]^T\} \\ \boldsymbol{\alpha}_c &= \text{Im}\{[\boldsymbol{\alpha}_1^T, \dots, \boldsymbol{\alpha}_n^T]^T\}. \end{aligned} \quad (8)$$

Taking the partial derivatives of (5), we then have

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \mathbf{c}_r(k)} &= \frac{2}{\sigma^2} \text{Re}\{\mathbf{\Omega}^H \mathbf{n}(k)\} & k = 1, \dots, L \\ \frac{\partial \mathcal{L}}{\partial \mathbf{c}_c(k)} &= \frac{2}{\sigma^2} \text{Im}\{\mathbf{\Omega}^H \mathbf{n}(k)\} & k = 1, \dots, L \\ \frac{\partial \mathcal{L}}{\partial \boldsymbol{\alpha}_r^i} &= \frac{2}{\sigma^2} \sum_{k=1}^L \text{Re}\{c_i^*(k) \mathbf{n}(k)\} & i = 1, \dots, n \\ \frac{\partial \mathcal{L}}{\partial \boldsymbol{\alpha}_r} &= \frac{2}{\sigma^2} \sum_{k=1}^L \text{Re}\{\mathbf{c}^*(k) \otimes \mathbf{n}(k)\} \\ \frac{\partial \mathcal{L}}{\partial \boldsymbol{\alpha}_c^i} &= \frac{2}{\sigma^2} \sum_{k=1}^L \text{Im}\{c_i(k) \mathbf{n}(k)\} & i = 1, \dots, n \\ \frac{\partial \mathcal{L}}{\partial \boldsymbol{\alpha}_c} &= \frac{2}{\sigma^2} \sum_{k=1}^L \text{Im}\{\mathbf{c}^*(k) \otimes \mathbf{n}(k)\}. \end{aligned} \quad (9)$$

We need the following assumption and results to obtain FIM, (see [9]):

$$\begin{aligned} E[\mathbf{n}(n) \mathbf{n}^H(m)] &= \sigma^2 \mathbf{I} \\ E[\mathbf{n}(n) \mathbf{n}^T(m)] &= 0 \\ E[\mathbf{n}^H(n) \mathbf{n}(n) \mathbf{n}^T(m)] &= 0. \end{aligned} \quad (10)$$

Using (9), (10) and taking expectations, we then obtain the entries of the Fisher information matrix for the conditional ML estimator, which are given by

$$\begin{aligned} E\left\{\frac{\partial \mathcal{L}}{\partial \mathbf{c}_r(n)} \cdot \frac{\partial \mathcal{L}}{\partial \mathbf{c}_r(m)}^T\right\} &= \frac{2}{\sigma^2} \text{Re}\{\mathbf{\Omega}^H \mathbf{\Omega}\} \delta_{n,m} = \mathbf{A} \\ E\left\{\frac{\partial \mathcal{L}}{\partial \mathbf{c}_r(n)} \cdot \frac{\partial \mathcal{L}}{\partial \mathbf{c}_c(m)}^T\right\} &= -\frac{2}{\sigma^2} \text{Im}\{\mathbf{\Omega}^H \mathbf{\Omega}\} \delta_{n,m} = \mathbf{B} \\ E\left\{\frac{\partial \mathcal{L}}{\partial \mathbf{c}_c(n)} \cdot \frac{\partial \mathcal{L}}{\partial \mathbf{c}_c(m)}^T\right\} &= \frac{2}{\sigma^2} \text{Re}\{\mathbf{\Omega}^H \mathbf{\Omega}\} \delta_{n,m} \\ E\left\{\frac{\partial \mathcal{L}}{\partial \mathbf{c}_r(k)} \cdot \frac{\partial \mathcal{L}}{\partial \boldsymbol{\alpha}_r}^T\right\} &= \frac{2}{\sigma^2} \text{Re}\{\mathbf{\Omega}^H \otimes \mathbf{c}^H(k)\} = \mathbf{C}_k \\ E\left\{\frac{\partial \mathcal{L}}{\partial \mathbf{c}_c(k)} \cdot \frac{\partial \mathcal{L}}{\partial \boldsymbol{\alpha}_r}^T\right\} &= \frac{2}{\sigma^2} \text{Im}\{\mathbf{\Omega}^H \otimes \mathbf{c}^H(k)\} = \mathbf{D}_k \end{aligned}$$

$$\begin{aligned}
E \left\{ \frac{\partial \mathcal{L}}{\partial \mathbf{c}_r(k)} \cdot \frac{\partial \mathcal{L}^T}{\partial \boldsymbol{\alpha}_c} \right\} &= -\frac{2}{\sigma^2} \text{Im} \left\{ \boldsymbol{\Omega}^H \otimes \mathbf{c}^H(k) \right\} \\
E \left\{ \frac{\partial \mathcal{L}}{\partial \mathbf{c}_c(k)} \cdot \frac{\partial \mathcal{L}^T}{\partial \boldsymbol{\alpha}_c} \right\} &= \frac{2}{\sigma^2} \text{Re} \left\{ \boldsymbol{\Omega}^H \otimes \mathbf{c}^H(k) \right\} \\
E \left\{ \frac{\partial \mathcal{L}}{\partial \boldsymbol{\alpha}_r} \cdot \frac{\partial \mathcal{L}^T}{\partial \boldsymbol{\alpha}_r} \right\} &= \frac{2}{\sigma^2} \sum_{k=1}^L \text{Re}[\mathbf{c}^*(k) \\
&\quad \otimes \mathbf{I}_m \otimes \mathbf{c}^H(k)] = \mathbf{E} \\
E \left\{ \frac{\partial \mathcal{L}}{\partial \boldsymbol{\alpha}_c} \cdot \frac{\partial \mathcal{L}^T}{\partial \boldsymbol{\alpha}_c} \right\} &= \frac{2}{\sigma^2} \sum_{k=1}^L \text{Re}[\mathbf{c}^*(k) \\
&\quad \otimes \mathbf{I}_m \otimes \mathbf{c}^H(k)] \\
E \left\{ \frac{\partial \mathcal{L}}{\partial \boldsymbol{\alpha}_r} \cdot \frac{\partial \mathcal{L}^T}{\partial \boldsymbol{\alpha}_c} \right\} &= -\frac{2}{\sigma^2} \sum_{k=1}^L \text{Im}[\mathbf{c}^*(k) \\
&\quad \otimes \mathbf{I}_m \otimes \mathbf{c}^H(k)] = -\mathbf{F} . \quad (11)
\end{aligned}$$

Then the FIM can be written in partitioned form as

$$\mathbf{J} = \left[\begin{array}{cc|c} \mathcal{H} & \mathbf{0} & \mathcal{C}_1 \\ & \ddots & \vdots \\ \mathbf{0} & \mathcal{H} & \mathcal{C}_L \\ \hline \mathcal{C}_1^T & \dots & \mathcal{C}_L^T \\ \hline & & \mathcal{E} \end{array} \right] \quad (12)$$

where

$$\begin{aligned}
\mathcal{H} &= \begin{bmatrix} \mathbf{A} & -\mathbf{B} \\ \mathbf{B} & \mathbf{A} \end{bmatrix}, \quad \mathcal{C}_k = \begin{bmatrix} \mathbf{C}_k & -\mathbf{D}_k \\ \mathbf{D}_k & \mathbf{C}_k \end{bmatrix}, \\
\mathcal{E} &= \begin{bmatrix} \mathbf{E} & -\mathbf{F} \\ \mathbf{F} & \mathbf{E} \end{bmatrix}. \quad (13)
\end{aligned}$$

The FIM can now be directly constructed. We can numerically compute the variance of individual parameter estimate by inverting the FIM $\text{CRB}(\boldsymbol{\tau}) = \text{diag} \{ \mathbf{J}^{-1}(\boldsymbol{\tau}) \}$.

5. SIMULATIONS

In this section, we illustrate some simulation results to evaluate the effectiveness and applicability of the proposed ML approaches. We consider the generator matrix form representation of the space-time coding system [10]. In this representation the stream of coded complex M -PSK symbols are obtained by applying mapping function $\mathcal{M}$ to the following matrix multiplication

$$\mathbf{c}(k) = \mathcal{M}(\mathbf{u}(k) \cdot \mathbf{G} \pmod{M}) \quad (14)$$

where $\mathbf{u}(k) = [s(lk+t-1), \dots, s(lk-t)]^T$ and $\mathbf{G}$ is the generator matrix with n columns and $l+s$ rows and $\mathcal{M}$ is a mapping function that maps integer values to the M -PSK symbols, $\mathcal{M}(x) = \exp(2\pi jx/M)$.

The performance of the proposed method was evaluated as a function of SNR (signal to noise ratio)

based on the Monte Carlo simulations. Conditional ML method was tested for 200 Monte Carlo trials per SNR point across range of SNR 's. The results were compared with hidden Markov model based unconditional ML and CRBs. In each trial, the estimation error of each parameter estimate from conditional and unconditional ML for the channel parameters were recorded. We consider following test case:

4-PSK space-time code example shown in Fig. 2 is considered with $n = 2$, $t = 2$ and generator matrix,

$$\mathbf{G} = \begin{bmatrix} 2 & 0 \\ 1 & 0 \\ 0 & 2 \\ 0 & 1 \end{bmatrix}$$

In this case, the coded 4-PSK symbols obtained from two current information bits are transmitted over the first antenna, whereas the coded 4-PSK symbols obtained from two preceding bits are transmitted over the second antenna simultaneously. The coded symbols are then transmitted through quasi-static fading channel matrix.

In Fig. 3, we have plotted the estimation error obtained from conditional and unconditional ML for the channel parameters as well as the conditional CRB.

6. CONCLUSIONS

In this paper, we presented the conditional ML approach to the problem of blind estimation of channel parameters along with the space-time coded sequence. We derived iterative ML algorithm based on the conditional signal model. Furthermore, the performance of the proposed algorithms is explored based on the derivation of the associated CRB. We also presented Monte Carlo simulations to verify the theoretically predicted estimator's performance. The examples demonstrated that proposed ML approach achieve the conditional CRB for high SNR values.

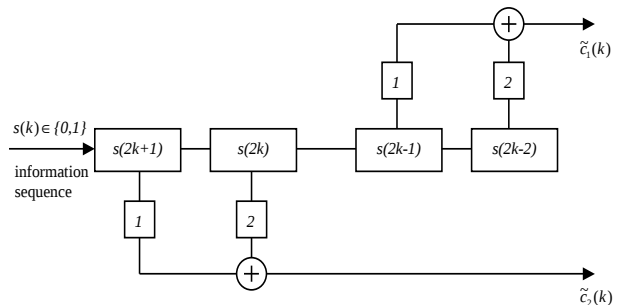


Figure 2: 4-state space-time coding system model

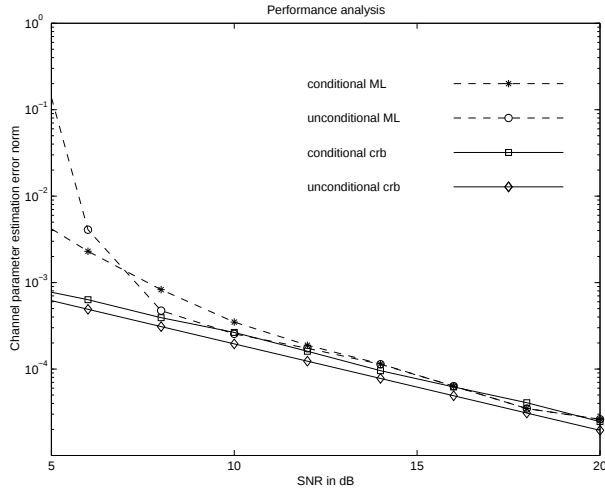


Figure 3: Channel Matrix Estimation Error Norm

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