# PATTERN SYNTHESIS WITH UNIFORM CIRCULAR ARRAYS FOR INTERCELL INTERFERENCE REDUCTION


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## ABSTRACT

**This paper investigates the use of uniform circular arrays (UCAs) having specially synthesized patterns at the base stations of WCDMA cellular systems, and the realized reduction in intercell interference. UCAs are very practical for this type of application as they can provide 360 degrees of coverage. UCA patterns that are steering-invariant and where the sidelobe levels are controllable are synthesized. The decrease in the ratio of intercell interference to intracell power resulting from the use of these arrays in a beam-steering scheme will be assessed, and the advantages and disadvantages of each pattern type will be discussed.**


## I. INTRODUCTION

The number of served users in 3G WCDMA cellular networks is downlink limited in many scenarios due to increased interference [1]. Advanced antennas are used to reduce interference [2], and adaptive beamforming employed to increase cell coverage and user capacity through antenna gain and interference rejection [3]. Two beamforming methods are normally considered: the fixed beam, and the steered beam. The first makes use of a specified number of fixed beams to cover a cell sector, whereas the second allows pointing the beam towards a specific user.

With beam steering, the antenna pattern needs to remain unaltered irrespective of the look direction. This was an assumption in [4], but this property was obtained in [5] where uniform circular arrays with Chebyshev patterns were designed, using a technique first proposed in [6]. UCAs, where *uniform* means *equi-spaced*, are very practical for deployment at base stations since they provide all-azimuth, i.e. 360 degrees, coverage.

Interesting UCA patterns are obtained by first transforming the UCA into a virtual uniform linear array (ULA) using the technique of [6], and then applying a special excitation to this virtual ULA, which is transformed back into a UCA. The transformation guarantees the steering-invariance property of the resulting UCA pattern, whereas the choice of the ULA excitation is responsible for the sidelobe level control and the directivity of the pattern. Three excitation types that are good candidates are the famous Dolph-Chebyshev, the modified-Chebyshev [7], and the discretized Taylor One-Parameter [8] excitations. The first two result in *equi*-ripple sidelobes, and the third in decaying sidelobes. A Dolph-Chebyshev distribution gives the smallest beamwidth for a prescribed sidelobe level, or the lowest sidelobe level for a desired beamwidth. The modified-Chebyshev distribution solves the directivity saturation problem of Chebyshev distribution for high numbers of elements at the cost of a slightly larger beamwidth. The number of sidelobes is smaller. A Taylor One-Parameter distribution is the most directive with a beamwidth smaller than that of modified-Chebyshev but still larger than conventional Chebyshev. Because of the decaying behavior of the sidelobes, this distribution is most practical for use because it leads to the least noise and interference from the far-out sidelobes, as compared to the two other distributions. A note is that applying a uniform excitation to the ULA is not suitable when sidelobe level control is a priority. A uniform distribution is a special case of the Taylor One-Parameter distribution.

The obtained UCAs, which will be respectively called the Chebyshev, modified-Chebyshev and Taylor UCAs, have steering-invariant patterns and controllable sidelobe levels. As expected, the use of these arrays at the base station of WCDMA cellular systems in a beam-steering scenario leads to substantial decrease in the ratio of intercell interference to intracell power, as compared to the omnidirectional or the 3-sector cases. In the omnidirectional case, there is one sector per cell and the base station (BS) is installed at the center of the cell and equipped with an omnidirectional antenna. The 3-sector case is when three antennas are used at the BS, each covering 120 azimuthal degrees [9]. Among the three UCA types, the Taylor UCAs result in the smallest ratio of intercell interference to intracell power, followed in order by Chebyshev and modified-Chebyshev UCAs.

## II. PROBLEM FORMULATION

The ratio of intercell interference to received intracell power is given as [9]

$$F_{k,l} = \frac{\sum_{j=1, j \neq l}^{J} P_{T,j} g_{k,j}}{P_{T,l} g_{k,l}} \tag{1}$$

where $l$ denotes the mobile station (MS) of interest, $k$ the serving BS, $P_T$ the total BS transmit power, and $J$ the number of cell sectors in the network. In (1), $g_{k,j}$ is expressed as

$$g_{k,j} = K\left(d_{kj}\right)^{-n} \xi_{kj} G_{kj} \tag{2}$$

where $K$ is the path gain, $n$ the pathloss exponent, $d_{kj}$ the distance from BS $k$ to MS $j$, $G_{kj}$ the antenna gain from BS $k$ in the direction of MS $j$, and $\xi_{kj}$ is the lognormal shadowing from BS $k$ to MS $j$. $\xi_{kj}$ is a zero-mean Gaussian random variable with variance $\sigma^2$.

For a UCA with $N$ isotropic elements and radius $r$, let $\mathbf{a}_c\left(\theta\right)$ denote its array response vector. Let $\mathbf{a}_v\left(\theta\right)$ be the array response vector of the corresponding virtual ULA. $\mathbf{a}_v\left(\theta\right)$ is given by [6]

$$\begin{aligned} \mathbf{a}_v\left(\theta\right) &= \mathbf{J}\left(N,\lambda,r\right)\, \mathbf{F}\left(N,h\right)\, \mathbf{a}_c\left(\theta\right) \\ &\approx \left[e^{-jh\theta},\ldots,1,\ldots,e^{jh\theta}\right]^T . \end{aligned} \tag{3}$$

In (3), $\theta$ is the azimuth angle,

$$\begin{aligned} \mathbf{J}\left(N,\lambda,r\right) &= \operatorname{diag}\left\{\left(j^m \sqrt{N} J_m\left(2\pi r/\lambda\right)\right)^{-1}\right\}, \\ m &= -h,\ldots,0,\ldots,h \end{aligned} \tag{4}$$

and

$$\mathbf{F}\left(N,h\right) = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & \omega^{-h} & \omega^{-2h} & \cdots & \omega^{-(N-1)h} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & \omega^{-1} & \omega^{-2} & \cdots & \omega^{-(N-1)} \\ 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega^{1} & \omega^{2} & \cdots & \omega^{(N-1)} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & \omega^{h} & \omega^{2h} & \cdots & \omega^{(N-1)h} \end{bmatrix}. \tag{5}$$

In (4), $J_m(\cdot)$ is the Bessel function of the first kind and order $m$, $\lambda$ is the wavelength, and in (5), $\omega = e^{j2\pi/N}$. The size of the virtual ULA is $N_v = 2h+1$, and $h$ is given by

$$h = \max\left\{h \,\middle|\, h \leq \frac{N-1}{2} \text{ and } \frac{J_{h-N}(2\pi r/\lambda)}{J_h(2\pi r/\lambda)} < \varepsilon\right\} \tag{6}$$

for some predetermined $\varepsilon$. The approximation in (3) requires that $N \,\square\, 2\pi r/\lambda$. From (3), it follows that:

$$\begin{aligned} &\mathbf{C}^T\left(\theta_s,h\right)\, \mathbf{a}_v\left(\theta\right) = \\ &\qquad \mathbf{C}^T\left(\theta_s,h\right) \mathbf{J}\left(N,\lambda,r\right)\, \mathbf{F}\left(N,h\right)\, \mathbf{a}_c\left(\theta\right) \end{aligned} \tag{7}$$

where $\theta_s$ is the steering angle and $\mathbf{C}$ is a ($2h$+1)-element vector given by

$$\begin{aligned} \mathbf{C}\left(\theta_s,h\right) = \Big[ & I_{-h}e^{jh\theta_s}, \quad \cdots \quad I_{-1}e^{j\theta_s}, \quad I_0, \\ & I_1 e^{-j\theta_s}, \quad \cdots \quad I_h e^{-jh\theta_s} \Big]^T, \end{aligned} \tag{8}$$

and $\mathbf{I}_v = \left[I_{-h}, \ \cdots \ I_{-1}, \ I_0, \ I_1, \ \cdots \ I_h\right]$ is the coefficients vector of the virtual ULA. From (7), it is deduced that the coefficients vector of the UCA is:

$$\mathbf{D} = \mathbf{C}\ \mathbf{J}\ \mathbf{F}\,. \tag{9}$$

In the above derivations, the array elements were considered to be isotropic and mutual coupling was not incorporated. In fact, (9) will remain unchanged for the case of non-isotropic elements, and a method described in [10] makes it easy to account for mutual coupling in the formulation.

For a modified Chebyshev linear array with $N_v = 2h+1$ elements and a sidelobe level ratio equal to $R$, the coefficients have mirror symmetry and are given by

$$I_m = \frac{1}{N_v} + \frac{2}{N_v\ R} \sum_{t=1}^{h} \cos\frac{2mt\pi}{N_v}\left[T_{2h/q}\left(\gamma\cos\frac{t\pi}{N_v}\right)\right]^q \tag{10}$$

where $-h \leq m \leq h$, $T_p(\cdot)$ denotes a Chebyshev polynomial of order $p$, $q$ is an integer greater than unity, and $\gamma = \cosh\left[q\cosh^{-1}\left(R^{1/q}\right)/2h\right]$. When $q=1$, the linear array is a conventional Dolph-Chebyshev array.

For the Taylor UCA case, the elements of the virtual ULA should have the following coefficients [8, 11]

$$I_m = \mathrm{I}_0\left[\beta\left(1-[m/h)]^2\right)^{1/2}\right] \tag{11}$$

where $\mathrm{I}_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind, and $\beta$ is a parameter that controls the maximal sidelobe level (MSLL).

After obtaining the coefficients from (10) or (11), **C** and consequently **D** can be computed. The UCA pattern in the azimuth plane is given by

$$P = \sum_{n=1}^{N} d_n \exp\left\{ jkr\left[ \cos\left( \theta - \frac{2(n-1)\pi}{N} \right) - \cos\left( \theta_s - \frac{2(n-1)\pi}{N} \right) \right] \right\} \tag{12}$$

where $d_n$ denotes the n-th element of **D**, and $k = 2\pi / \lambda$. The pattern can also be deduced from (7).

## III. SIMULATION RESULTS

As a first example, we take a UCA with $N$ = 35, MSSL = –20 dB, and an inter-element spacing $d = 0.194\lambda$. The radius $r = d/\left(2\sin\left(\pi / N\right)\right) = 1.084\lambda$. For $\varepsilon = 0.05$, $h$ = 16 and the virtual linear array has 33 elements. The in-plane azimuth array pattern of the Taylor UCA is plotted in Fig. 1 for $\theta_s = -90^\circ$, $0^\circ$, and $60^\circ$. As can be seen, the three patterns are the same except for the angle shift. For this example, $\beta = 2.222$, which is independent of the look direction. The same remark holds for the other UCA types.

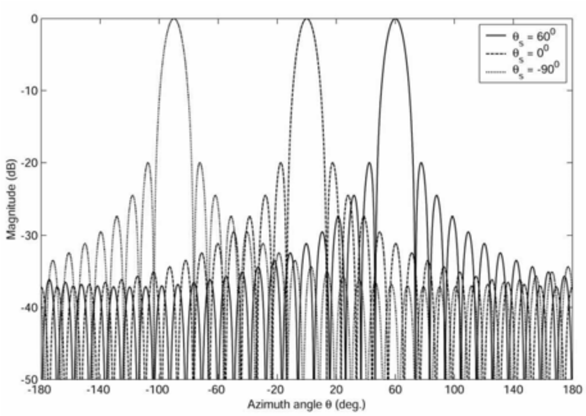


Figure 1. Normalized array patterns in the azimuth plane for the Taylor UCA for $N$ = 35, MSSL = –20 dB, and $d = 0.194\lambda$

In the second example, $N$ is 39, MSLL = –20 dB, $r = \lambda / 2$, and $\theta_s = 0^\circ$. Fig. 2 depicts the in-plane azimuth array patterns of Chebyshev, modified-Chebyshev ($q$ = 2) and Taylor UCAs. A note is that the Taylor UCA always has a narrower main lobe than the modified-Chebyshev design, but still a wider one compared to the Dolph-Chebyshev design. The modified-Chebyshev UCA has the smallest number of sidelobes. The maximum-to-minimum absolute coefficient ratio (dynamic range of the taper weights) is smallest for the Taylor UCA, followed respectively by that of the modified-Chebyshev and Chebyshev cases. The same comparison holds for the maximum phase difference in the coefficients.

To assess the decrease in the intercell interference to intracell power ratio, a simulation model based on a symmetric network of equivalently equipped BSs is adopted. The simulated network consists of 7 hexagonal cells. The MSs are assumed uniformly distributed over the network and present only in the azimuth plane, and the BSs are assumed to transmit at their maximum power.

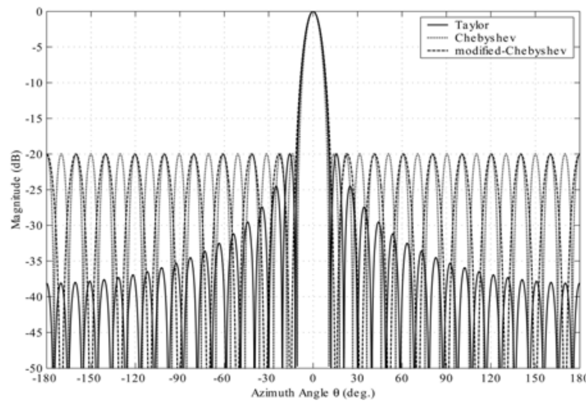


Figure 2. Patterns for $N$ = 39, MSSL = –20 dB, $r = \lambda / 2$, and $\theta_s = 0^\circ$

Fast fading is considered averaged out by perfect power control, diversity, and channel coding. In the simulation, $K = -50$ dB, $\sigma = 8$ dB, and the pathloss exponent $n$ is varied from 2 to 5, which is the interval of practical values appearing in empirical measurements [12]. The average of 10000 independent iterations is taken. In each iteration, a user is created at a random location and (1) is computed for this user. Finally the average value of $F$ is calculated. The UCA assumed at each BS has 33 elements and a MSLL of -20 dB. The elements are considered isotropic and mutual coupling disregarded. Suitable antenna elements can be used though, and the Matrix Pencil method can be employed to compensate for the mutual coupling effects. The resulting average value of $F$ for the Dolph-Chebyshev, modified-Chebyshev and Taylor designs compared to each others and to the omnidirectional case is plotted in Fig. 3 versus $n$.

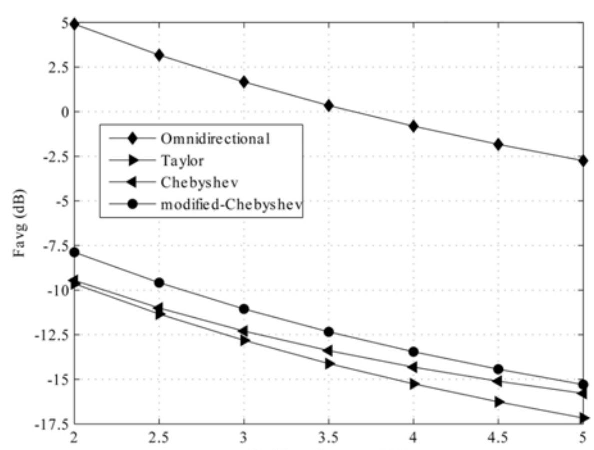


Figure 3. Average $F$ versus pathloss exponent $n$

Evidently, the use of beam steering with the three UCA types results in much lower *F* in comparison with the omnidirectional case, and this is due to the narrow beam and low sidelobes in the patterns of the UCAs. As expected, *F* decreases with increasing *n*. A larger *n* means more power attenuation, and consequently less interference with other users. The Taylor UCA results in the smallest *F*, followed by Chebyshev and then modified-Chebyshev. The decay in the sidelobes of the Taylor UCA leads to less interference from the direction of the sidelobes, which explains the superiority of the Taylor UCA in terms of the ratio of intercell interference to received intracell power *F*. On the other hand, the Chebyshev and modified-Chebyshev UCAs both have a constant sidelobe level, but the former has a narrower beamwidth, which also explains why it resulted in smaller *F*.

## IV. CONCLUSION

With the foreseen growth in the number of 3G WCDMA subscribers, the need to reduce the value of the ratio of intercell interference to received intracell power is more important than ever. One key technique used in this respect is the employment of advanced antenna arrays at the base stations, combined with the use of adaptive beamforming. Uniform circular arrays are a good candidate for playing the key role in this scenario, as they provide 360 degrees of coverage in the azimuthal plane. This paper presented the Chebyshev, modified-Chebyshev and Taylor UCAs whose patterns are independent of the steering angle and their sidelobe level is controllable. Deploying these arrays at the base stations of WCDMA cellular systems significantly reduces the ratio of intercell interference to intracell power, compared to the case of the cell being served by an omnidirectional antenna. The three types compared, it was shown that the Taylor UCAs lead to the smallest ratio of intercell interference to intracell power. They are also the most directive for large number of array elements. The Chebyshev UCAs always have the narrowest beam, whereas the modified-Chebyshev UCAs generate the least number of sidelobes.

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