# Extended Symmetrical Aperture Direction Finding Using Correlative Interferometer Method

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# Abstract

Correlative Interferometry is the most commonly used practical Direction Finding method. It is based on comparing the measured phase differences with the recorded and saved phase differences. In order to achieve an unambiguous direction of arrival (DOA) estimate, classical methods require antenna aperture within half wavelength. In this paper, we compare the effect of the increased number of non-parallel antenna pairs. It is observed that, increasing the number of non-parallel antenna pairs (baselines), unambiguous DOA estimates can be made for a symmetrically placed antenna array with antenna apertures greater than the half wavelength.

#### 1. Introduction

Determining the direction of wireless transmission sources has been popular since World War I. It has widespread use in numerous application areas. Many different methods utilizing the amplitude and the phase of the incoming RF signal are used in direction finding. Watson-Watt [1], Pseudo-Doppler [1], [2], Interferometer [3] and MUSIC [4] are common classical DF techniques.

In this work, we investigate the correlative interferometer method in detail and try to improve the DF accuracy and bandwidth.

Interferometer methods depend on the phase differences of two or more antennas. Correlative interferometer method takes the measured phase differences between antennas and correlates them with the saved ones and estimates DOA. Reducing the mutual coupling between antennas is the most important advantage of correlative interferometer method over other interferometer methods. However, Correlative Interferometer method suffers from the ambiguity problem as the other interferometer methods when the half of the wavelength of the incoming signal is greater than the antenna aperture. On the other hand, increasing the antenna aperture increases the DF resolution and performance. If the ambiguity resulting from the greater antenna aperture than the half wavelength of the incoming signal can be solved, DF performance and bandwidth of the DF system with the same antenna array can be increased. One can also achieve these benefits by using asymmetric antenna array with different antenna apertures but have to face off with very large antenna array dimensions. We try to solve this ambiguity problem by using symmetric and reasonable antenna apertures.

#### 2. Correlative Interferometer Method

As discussed, above interferometer method depends on the phase differences of 2 or more antennas. Two antennas with spacing d and incident waveform coming from the angle  $\theta$ , take the signals with different phases.

Incident wavefront received at antenna 1 travels an additional distance  $(d^*sin \ \theta)$  to antenna 2. The time to travel this distance between antenna 1 and antenna 2 creates a phase difference of  $\Delta \Phi$  for the received signals at the antennas.

$$\Delta \Phi = \frac{2\pi d}{\lambda} \sin \theta \tag{1}$$

The angle of arrival  $\theta$  can be calculated from Eq. (2.1) is:

$$\theta = \arcsin\left[\frac{\Delta \Phi \lambda}{2\pi d}\right] \qquad ^{(2)}$$

where  $\lambda$  = wavelength of the received signal, *d*=distance between antennas.

If only 2 antennas are used, for the angle of arrival of incident waveform for  $\theta$  and  $\theta + 180^{\circ}$ , the resulting AOA will be same. To determine the correct AOA, the system needs 3 or more antennas correspond 2 or more baselines for the antennas.

Correlative interferometer is a method that works after phase measurements. Any technique can be used to find the phase differences of the antenna system. Correlative interferometer uses these phase differences to estimate the direction of arrival.

#### 3. Method Development

For a symmetric antenna array with N antenna elements, the received signal for each antenna element i is:

$$R(t,i) = s(t)m_i \tag{3}$$

the *i*th element response is:

$$\mathcal{M}_{i} = e^{j\frac{2\pi}{\lambda}\cos(-\frac{2\pi(i-1)}{N}+\Phi_{AOA})} \quad (4)$$

where  $m_i$  is the incoming signal,  $\lambda$  is the incoming signal wavelength,  $\Phi_{AOA}$  is the angle of arrival, r is the radius of the antenna array.



Fig. 1. Phase Differences of 4 Symmetric Antenna system Between Antennas 2-1, 3-2, 4-3 and 1-4 for  $d=\lambda/2$ 

Fig. 1 shows the expected phase differences between the each neighbor antenna pairs 2-1, 3-2, 4-3 and 1-4. Since the phase of the received signal each independent antenna differs from  $-180^{\circ}$  to  $180^{\circ}$ , the phase differences changes between  $-360^{\circ}$  to  $360^{\circ}$ . If antenna aperture is larger than the half wavelength of the incoming signal, these phase differences will increase and causes an ambiguity to resolve.

To find the DOA, first of all, the phase differences for the known directions are measured and saved in a table. Then, the phase differences for the unknown Angle of Arrival (AOA) are measured and correlated with the saved data. Maximum correlation gives the DOA. Euclidean distances of measurement data to lookup table are used as correlation method in this work. The minimum Euclidean distance means the maximum correlation. One can use or try another distance measurement techniques.

### 4. Numerical Analysis

For simulations, we can add white Gaussian noise to received signals or phase errors (also white Gaussian) to expected phase difference for hardware errors, multipath effects and measurement errors.

Let's use correlative interferometer method to estimate angle of arrival with using the expected phase differences with added white Gaussian noise of variance  $5^{\circ}$  to phase differences and lookup table in 0.1° resolutions for 4-8 antenna element systems.

 Table 1. Number of the symmetric antennas in a DF system vs

 the size of antenna array

Number of	Radius of the Antenna Array
antennas	(*wavelength)
4	0,353553391
5	0,425325404
6	0,5
7	0,576191218
8	0,653281482

Table 1 shows the size of the antenna array with respect to the antenna elements in the system. Antenna array dimensions are

increasing with the increase of the equally spaced antenna elements. An 8 antenna element system is 1.54 times greater than 5 antenna element system, where all antenna apertures are equal to the half of the wavelength of the incoming signal.

# 4.1. Antenna Aperture vs DF Accuracy

Fig. 2 shows the Measured vs Expected AOA with noise added to phase differences (variance of noise = 5) to 4 element symmetric antenna array at the half wavelength. After doing the 100 simulations, the RMS error of the system under this condition is " $1.84^{\circ}$ ".



Fig. 2. Measured vs Expected AOA with noise (variance of noise =  $5^{\circ}$ )



Fig. 3. Measured vs Expected AOA with noise (variance of noise = 5) for  $d=\lambda/4$ 

After doing the same simulation for an antenna aperture of  $\lambda/4$ , the RMS error of the system is "4.37°". It is approximately "2.4" times of the RMS error of  $\lambda/2$ . Fig. 3 shows the Measured vs Expected AOA with noise added to phase differences (variance of noise = 5) to 4 element symmetric antenna array at the 1/4 wavelength.



Fig. 4. RMS Error vs antenna aperture with different signal-tonoise ratios

Fig. 4 is the graph of the RMS Error vs antenna aperture with different signal-to-noise ratios (0-10-20dB). As it can be easily seen that increases in the antenna aperture and SNR (signal-to-noise ratio) provides increasing system performance.

## 4.2. Wide Aperture Correlative Interferometry

In this part, the case when the antenna aperture is greater than the half wavelength of the incident signal is investigated.



Fig. 5. Expected Phases of the signals at the antennas versus AOA of incoming signal for  $d = \lambda$ 

Fig. 5 shows the phase differences of the 4 antenna system with antenna aperture of  $\lambda$ . As easily seen, it can cause ambiguities at different AOAs.



Fig. 6. Expected vs Calculated AOA for d=0.75  $\lambda$  with an SNR of 5 dB (4 antenna system)

Fig. 6 shows the ambiguities resulted from the greater antenna aperture than  $\lambda/2$ .

The ambiguity caused from the greater antenna aperture from  $\lambda/2$  is shown at Fig. 7. Correlative interferometer can solve the ambiguity at some antenna apertures but it is also unsuccessful at some other apertures. As the SNR decreases, the number of ambiguity will increase as well.



Fig. 7. RMS Error vs antenna aperture from  $0.5 \lambda$  to  $2 \lambda$  with an SNR of 5 dB (4 antenna system)

In the 4 antenna case, we have actually 2 baselines. The other 2 antenna pairs are parallel to first antenna pairs. Antenna pair 1-2 is parallel to antenna pair 3-4 and antenna pair 2-3 is parallel to antenna pair 4-1. Due to this parallelism we have actually two baselines. Trying to increase baselines can resolve the ambiguity because the probability of the same phase difference for more than 1 angle of arrival will be decreasing with the increase of the baselines which are not parallel to each other.

Increasing the number of the symmetric antenna in the system can make possible to increase number of un-parallel baselines. The system with 5 antennas has 5 un-parallel baselines. Adding 1 antenna to 4 antennas increases number of un-parallel baselines from 2 to 5. Thus, this should give a better solving to ambiguity results from the antenna aperture greater than  $\lambda/2$ .



Fig. 8. RMS Error vs antenna aperture with different signal-tonoise ratios for 5 antenna system antenna aperture from 0.1 to  $2\lambda$ 

Fig. 8 shows the RMS error versus antenna aperture for SNR values of 0-10-20 dB for symmetric 5 antenna (pentagon) system. This shows that using 5 non-parallel baselines can solve the ambiguity resulted from the antenna aperture greater than  $\lambda/2$ .

Apply the same case for a 6 symmetric antenna system, adding a new antenna to antenna array. It should be noted that 6

antenna system has 3 non-parallel baselines. In the 5 antenna case it has 5 non-parallel systems.



Fig. 9. RMS Error vs antenna aperture with different signal-tonoise ratios for 6 antenna system

Fig. 9 shows the RMS error versus antenna aperture for SNR values of 0-10-20 dB for symmetric 6 antenna (hexagon) system. This shows that using 3 non-parallel baselines cannot solve the ambiguity resulted from the antenna aperture greater than  $\lambda/2$ . In contrast to 5 antenna system, 6 antenna system cannot solve the ambiguity.



Fig. 10. RMS Error vs antenna aperture with different signal-tonoise ratios for 7 antenna system

Fig. 10 shows the RMS error versus antenna aperture for SNR values of 0-10-20 dB for symmetric 7 antenna system. This shows that using 7 non-parallel baselines can solve the ambiguity resulted from the antenna aperture greater than  $\lambda/2$  as the 5 antenna system. In contrast to 4 and 6 antenna systems, 5 and 7 antenna systems can solve the ambiguity.

Fig. 11 shows the RMS error versus antenna aperture for SNR values of 0-10-20 dB for symmetric 8 antenna system. 8 antenna system has 4 non-parallel baselines. This shows that using 4 non-parallel baselines can solve the ambiguity resulted from the antenna aperture greater than  $\lambda/2$  as the 5 and 7 antenna systems. In contrast to 4 and 6 antenna systems; 5, 7 and 8 antenna systems can solve the ambiguity.

For 0 dB SNR at antenna aperture of 0.8 wavelength at one or 2 runs ambiguity has occurred but it is not very important compared to 4 and 6 antenna systems. Thus, it can be said that 5 and 7 antenna systems have more success in solving ambiguity than 8 antenna systems due to antenna aperture greater than 0.5 wavelength.



Fig. 11. RMS Error vs antenna aperture with different signal-tonoise ratios for 8 antenna system

# 4.3. Antenna Array Size vs DF Accuracy



**Fig. 12.** Error vs antenna aperture for 4, 5, 6, 7, 8 antenna systems at an SNR of 5 dB

Increasing the number of antennas in the symmetric antenna array achieves more DF accuracy. Fig. 12 shows the comparison of the 4, 5, 6, 7, 8 antenna systems versus antenna aperture. Simulations are done for antenna aperture changes from 0.1  $\lambda$  to 0.5  $\lambda$  because 4 and 6 antenna systems cannot resolve the ambiguity for greater apertures. It is observed that as the number of the antennas in DF system increases, the RMS error of the system decreases, in other words, the system performance increases with the increase of the symmetric antenna elements in a direction finding system.

# 5. Conclusion

It is shown that the performance of a DF system using Correlative Interferometer Method increases with the increase of the antenna aperture. On the other hand, if antenna aperture is greater than the half wavelength of the incoming RF (Radio Frequency) signal, an ambiguity occurs and it causes system to give wrong AOA outputs.

It is observed that, increasing the number of non-parallel baselines can provide system with solving the ambiguity at an antenna aperture greater than the half wavelength of the incoming RF signal. 5 non-parallel baselines actually solve the problem for an antenna aperture of 2 times of the incoming signal wavelength.

The second important result obtained from the simulations is that increasing the number of the antennas in a DF system increases the system performance as well. On the other hand, increasing the number of the antennas in the antenna array of the DF system also increases the size of the antenna array for the same frequency band.

As shown in Table 1, an 8 antenna element system is 1.54 times greater than 5 antenna element system, in spite of the fact that 5 antenna element system is more powerful to solve ambiguities. These results can be utilized to provide best DF accuracy on the desired bandwidth and antenna array size.

#### References

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