A STUDY ON THE DYNAMIC ANALYSIS AND CONTROL OF 6-3 STEWART PLATFORM MECHANISM

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ABSTRACT

In the present study kinematics and dynamics as well as the control of a 6-3 SPM are considered. First, the dynamic model of the considered mechanism outlined and the solutions of the relevant equations given in great details. Then, in order to obtain a precise positioning and good-dynamic performance model, a control algorithm developed. Adequacy and success of the developed algorithm are numerically tested for various initial values, the results obtained indicate its high accuracy in capturing desired orientation.

I. INTRODUCTION

The Stewart platform (SPM) can be considered as a six-degree-of-freedom mechanism with two bodies connected together by six extensible legs. This parallel manipulating structure is obtained from generalization of the mechanism originally proposed by Stewart as a flight simulator [1]. After the instruction of Stewart platform as a flight simulator many variations have been introduced as fully parallel and serial-parallel manipulators.

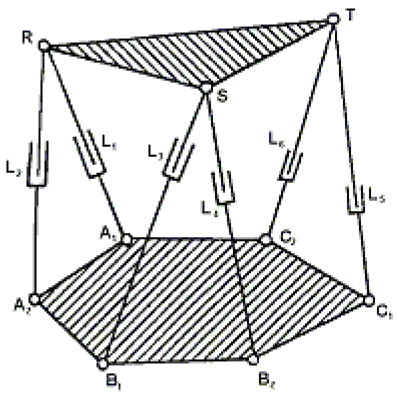

Fig.1. A General 6-3 Stewart Platform Mechanism

A glance at last two decades clearly indicates that the parallel manipulators have received more and more attention since Stewart published his famous article in [1965]. In the last few years the parallel manipulator have been to some extent kinematically and dynamically investigated by many researchers. The General Stewart Platform has a base and a moving platform connected by six extensible legs with spherical joints at the both ends or a spherical joint at one end and a universal joint at the other (Fig 1.) [2].

This fully parallel kinematics linkage system as a manufacturing manipulator has two fundamental characteristics by which it is apart from machine tools and industrial robots. It is a closed kinematics system with parallel links. This parallel mechanism link-ends are simply supported, making the manipulator system far more rigid in proportion to size and weight than any serial link robot. [3-5] In addition to that, the links of the Stewart Platform are arranged so that the major force components of all six actuator add together, resulting in a force output-to-manipulator-weight ratio more than one order of magnitude grater than most industrial robots.

In this study the one considered here is 6-3 SPM its extensible legs connected to the moving platform as a pair in order to get easy controllable and stronger manipulator as a parallel mechanism. The lower part of each legs connected to base platform by universal joint which has its fix axis perpendicular to the line which is through the lower connection points of pairs and also parallel to the base platform. Upper part–connection of each pair is performed by using revolute joint. And in each revolute joint which has the rotation center in the plane defined by pair of the legs there is a spherical joint. Connections of the revolute joints to the moving platform are performed by means of this spherical joint in each revolute joint.

The mass definitions of both platforms and inertia moments of legs and the methodology of derivation of the dynamic equations for the 6-3 UPS SPM dynamic model are adopted by ref. [6]. In that model, which is developed in the sense of Newton-Euler approach, frictions in all types of joints are considered

II. THE DYNAMIC MODEL

In recent years many practical and theoretical works have been conducted on the kinematics and dynamics of parallel mechanism. Several methods such as the Lagrange formulation, Newton-Euler formulation and principle of virtual work are proposed to derive the dynamic equations of parallel mechanism [7-11]. The method of virtual work is a more convenient approach to derive dynamics equations for the inverse dynamics of parallel mechanism [12,13]. On the other hand, the Lagrange formulation is well structured and can be expressed in closed form, but a large amount of symbolic computation is needed to find partial derivatives of the Lagrangian in this method [14]. However, the Newton-Euler approach requires computation of all constraint forces and moments between the links. And depending on the dynamic model developed these forces and moments may be related to the control parameters for the simulations that we have just succeeded in this study. To be clear in understanding of dynamic model the required definitions are as follows; the leg vectors in fixed reference axis are;

$$\mathbf{q}_i = \mathbf{R}\,\mathbf{p}_i \qquad (\,i:1,2,\ \ ,6\,) \tag{1}$$

$$\mathbf{S}_i = \mathbf{q}_i + \mathbf{t} - \mathbf{b}_i\,, \qquad (\,i:1,2,..,6) \tag{2}$$

In order to prevent indices ambiguity, the corners of moving platform are indicated by vectors $\{q_1, q_1, q_2, q_2, q_3, q_3\}$. From the kinematics point of view the velocities of pairs of legs must be the same.

$$\dot{\mathbf{S}}_i = \boldsymbol{\omega} \times \mathbf{q}_i + \dot{\mathbf{t}}\,, \quad (\,i:1,2,...,6) \tag{3}$$

The unit vectors and the lengths of the legs are respectively as follows,

$$\mathbf{s}_i = \mathbf{S}_i / L_i \tag{4}$$

$$L_i = \|\mathbf{S}_i\| \tag{5}$$

The geometric and the kinematics relations for the legs are given in their own axis. However the dynamic equations are written in the basis axis system or in the system, which are parallel to this system. The transformation matrix between two-axis system can be written.

$$\begin{aligned} \hat{\mathbf{x}}_i &= \mathbf{s}_i \ ; \\ \hat{\mathbf{y}}_i &= (\mathbf{k}_i \times \mathbf{s}_i) / \|\mathbf{k}_i \times \mathbf{s}_i\| \ ; \\ \hat{\mathbf{z}}_i &= \hat{\mathbf{x}}_i \times \hat{\mathbf{y}}_i \end{aligned} \tag{6}$$

$$\mathbf{T}_i = [\,\hat{\mathbf{x}}_i \quad \hat{\mathbf{y}}_i \quad \hat{\mathbf{z}}_i\,] \tag{7}$$

Where, $\mathbf{k}$ -direction is the direction of universal joint's fixed rotation axis. The upper and lower parts gravity vectors and their inertia matrices are given in the leg-axis system respectively,

$$\mathbf{r}_{di} = \mathbf{T}_i\,\mathbf{r}_{di0}\,, \tag{8}$$

$$\mathbf{r}_{ui} = \mathbf{T}_i\,(\mathbf{v}_i + \mathbf{r}_{ui0}) \tag{9}$$

$$\mathbf{v}_i = [\ L_i \quad 0 \quad 0\,]^T \tag{10}$$

$$\mathbf{I}_{di} = \mathbf{T}_i\,\mathbf{I}_{di0}\,\mathbf{T}_i^T, \tag{11}$$

$$\mathbf{I}_{ui} = \mathbf{T}_i\,[\,\mathbf{I}_{ui0} + m_{ui} L_i^2\,\mathbf{diag}(\,0\ 1\ 1)]\,\mathbf{T}_i^T \tag{12}$$

The relative velocity between two part of a considered leg is,

$$\dot{L}_i = \mathbf{s}_i \cdot \dot{\mathbf{S}}_i\,, \tag{13}$$

And the angular velocity is,

$$\mathbf{W}_i = \mathbf{s}_i \times \dot{\mathbf{S}}_i / L_i \tag{14}$$

III. ACCELERATION ANALYSIS

The acceleration of the upper part connection point of a leg can be given as two forms depending upon the equation in which it is used.

$$\begin{aligned} \ddot{\mathbf{S}}_i &= \ddot{L}_i\,\mathbf{s}_i + 2\,\mathbf{W}_i \times \dot{L}_i\,\mathbf{s}_i + \mathbf{A}_i \times \mathbf{S}_i \\ &+ \mathbf{W}_i \times (\mathbf{W}_i \times \mathbf{S}_i) \end{aligned} \tag{15}$$

$$\ddot{\mathbf{S}}_i = \mathbf{a}_{pi} + \mathbf{U}_{1i}\,, \tag{16}$$

Where A is the angular velocity and $\ddot{L}$ sliding acceleration of two parts of the leg considered,

$$\mathbf{a}_{pi} = \ddot{\mathbf{t}} + \boldsymbol{\alpha} \times \mathbf{q}_i\,, \tag{17}$$

$$\mathbf{U}_{1i} = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{q}_i) \tag{18}$$

$$\ddot{L}_i = \mathbf{s}_i \cdot \mathbf{a}_{Pi} + u_i \tag{19}$$

$$u_i = \mathbf{s}_i \cdot \mathbf{U}_{1i} - \mathbf{s}_i \cdot (\mathbf{W}_i \times \{\mathbf{W}_i \times \mathbf{S}_i\}) \tag{20}$$

The leg angular acceleration defined in equation (15) can be written as follows,

$$\mathbf{A}_i = \frac{1}{L_i}\,(\mathbf{s}_i \times \mathbf{a}_{Pi}) + \mathbf{U}_{2i} \tag{21}$$

$$\mathbf{U}_{2i} = \frac{1}{L_i}\left(\{\mathbf{s}_i \times \mathbf{U}_{1i}\} - 2\dot{L}_i \mathbf{W}_i\right) \tag{22}$$

Having the assumption that the lower part of the leg is only rotating while the upper part is not only rotating, but also translating. Since the lower part and the upper part accelerations are respectively,

$$\mathbf{a}_{di} = \frac{1}{L_i}\,(\mathbf{s}_i \times \mathbf{a}_{Pi}) \times \mathbf{r}_{di} + \mathbf{U}_{3i}\,, \tag{23}$$

$$\mathbf{a}_{Ui} = (\mathbf{s}_i \cdot \mathbf{a}_{Pi})\,\mathbf{s}_i + \frac{1}{L_i}(\mathbf{s}_i \times \mathbf{a}_{Pi}) \times \mathbf{r}_{Ui} + \mathbf{U}_{4i} \tag{24}$$

$$\mathbf{U}_{3i} = \mathbf{U}_{2i} \times \mathbf{r}_{di} + \mathbf{W}_i \times (\mathbf{W}_i \times \mathbf{r}_{di}) \tag{25}$$

$$\begin{aligned} \mathbf{U}_{4i} &= u_i \mathbf{s}_i + \mathbf{U}_{2i} \times \mathbf{r}_{ui} + \mathbf{W}_i \times (\mathbf{W}_i \times \mathbf{r}_{ui}) \\ &+ 2\dot{L}_i \mathbf{W}_i \times \mathbf{s}_i \end{aligned} \tag{26}$$

IV. DERIVATION OF DYNAMIC EQUATIONS

The force balance for each individual pairs-moving parts of legs under assumption of constant velocity, (fig.2)

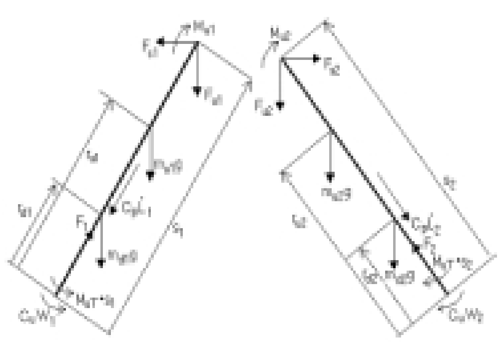

Fig. 2. All the forces and moments exerted on a pair of legs

$$\mathbf{F}_i - m_{Ui}\mathbf{g}.\mathbf{s}_i - C_p\dot{L}_i - \mathbf{F}_{Si} + (-1)^i \mathbf{F}_{Li}.\mathbf{s}_i = 0 \quad (27)$$

F_{Li} = Reaction force between two legs in the pairs,
F_i = Pressure applied to piston
F_{Si} = The force applied to the legs by the moving platform
$C_p\dot{L}_i$ = Friction force revealed by the sliding movement of the upper and the lower parts

The moment balance on the all legs can be generalized by using Newton-Euler Formulas as follows,

$$\begin{aligned} & m_{di}\mathbf{r}_{di} \times \mathbf{a}_{di} + m_{Ui}\mathbf{r}_{Ui} \times \mathbf{a}_{Ui} + (I_{di} + I_{Ui})\mathbf{A}_i \\ & + \mathbf{W}_i \times (I_{di} + I_{Ui})\mathbf{W}_i = (m_{di}\mathbf{r}_{di} + m_{ui}\mathbf{r}_{ui}) \times \mathbf{g} \\ & + \mathbf{S}_i \times (\mathbf{F}_{Si} + (-1)^{i+1}\mathbf{F}_i) + \mathbf{M}_{Ui} + \mathbf{M}_{UTi} \cdot \mathbf{s}_i \\ & + C_{Ui}\mathbf{W}_i \end{aligned} \quad (28)$$

M_{ui} = The moment revealed by the revolute joint at the connection point to the moving platform
CuW_i = Viscous friction reaction moment revealed by universal joint.
Mus_i = The moments revealed by fixing the leg's rotation axis
A_i = Angular acceleration of the legs
W_i = Angular velocities of the legs

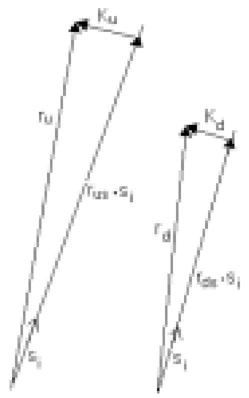

Fig. 3. The relations between gravity center the position vector of a leg (r_d, r_u) and the unit vector of the leg (s_i)

The center of the gravity vectors for the lower and the upper parts of a leg (Fig.3.) in terms of known quantities are,

$$r_{dsi} = \mathbf{r}_{di} . \mathbf{s}_i \quad (29)$$

$$r_{usi} = \mathbf{r}_{ui} . \mathbf{s}_i \quad (30)$$

$$\mathbf{K}_{di} = \mathbf{r}_{di} - \mathbf{r}_{dsi}\mathbf{s}_i \quad (31)$$

$$\mathbf{K}_{Ui} = \mathbf{r}_{Ui} - \mathbf{r}_{Usi}\mathbf{s}_i \quad (32)$$

The forces acted upon the moving platform by the legs can be written.

$$\begin{aligned} \mathbf{U}_{5i} &= m_{di}\mathbf{r}_{di} \times \mathbf{U}_{3i} + m_{Ui}\mathbf{r}_{Ui} \times \mathbf{U}_{4i} \\ & + (\mathbf{I}_{di} + \mathbf{I}_{Ui})\mathbf{U}_{2i} + \mathbf{W}_i \times (\mathbf{I}_{di} + \mathbf{I}_{Ui})\mathbf{W}_i \\ & - (m_{di}\mathbf{r}_{di} + m_{Ui}\mathbf{r}_{Ui}) \times \mathbf{g} + C_U\mathbf{W}_i \end{aligned} \quad (33)$$

$$\mathbf{V}_{5i} = [C_p\dot{L}_i + m_{Ui}\mathbf{g} \cdot \mathbf{s}_i]\mathbf{s}_i + \frac{\mathbf{U}_{5i} \times \mathbf{s}_i}{L_i} \quad (34)$$

$$\tilde{\mathbf{s}} = \begin{bmatrix} 0 & -s_z & s_y \\ s_z & 0 & -s_x \\ -s_y & s_x & 0 \end{bmatrix} \quad (35)$$

$$\mathbf{F}_{Si} = [\mathbf{Q}_i]\mathbf{a}_p + \mathbf{V}_{5i} - \mathbf{s}_i\ \mathbf{F}_i \quad (36)$$

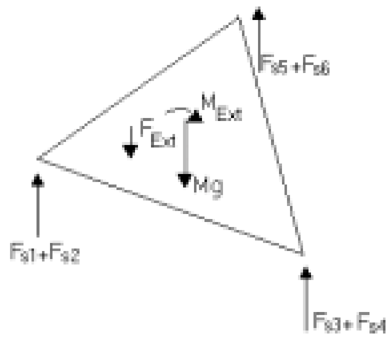

Fig. 3. The forces and the moments acted on the moving platform

V. THE DYNAMICS EQUATIONS FOR THE MOVING PLATFORM

With the similar manner, force and moment balance on the moving platform might be written

$$M\mathbf{g} + R\mathbf{F}_{Ext} - \sum_{i=1}^{6}\mathbf{F}_{Si} - M\mathbf{a} = 0 \quad (37)$$

These two equations are rearranged in order to obtain the final form of the dynamic equations for the moving platform.

$$\mathbf{J}\begin{bmatrix} \ddot{\mathbf{t}} \\ \boldsymbol{\alpha} \end{bmatrix} + \boldsymbol{\eta} = \mathbf{HF} + \begin{bmatrix} R\mathbf{F}_{ext} \\ R\mathbf{M}_{ext} \end{bmatrix} \quad (38)$$

Where, J is complicated inertia matrix, η is the resultant moment vectors, H is input-output force transformation, F is input vector and R is the rotation matrix; all these quantities are explicitly given in the appendix. Accelerations and the required velocity equations for the Runge-Kutta Method can be rearranged for the simulation.

$$\frac{\partial}{\partial t}\begin{bmatrix} \mathbf{t}_x \\ \mathbf{t}_y \\ \mathbf{t}_z \\ \boldsymbol{\theta}_x \\ \boldsymbol{\theta}_y \\ \boldsymbol{\theta}_z \end{bmatrix} = \begin{bmatrix} \dot{\mathbf{t}}_x \\ \dot{\mathbf{t}}_y \\ \dot{\mathbf{t}}_z \\ \dot{\boldsymbol{\theta}}_x \\ \dot{\boldsymbol{\theta}}_y \\ \dot{\boldsymbol{\theta}}_z \end{bmatrix} \quad (39)$$

VI. SIMULATION

After a tedious study the dynamic equations of system considered are obtained. The simulation of the system has been succeeded by the 4^{th} and 5^{th} order Runge-Kutta

Method (RK45) which is given as a routine the ode45 in Matlab. In the practical work, done in many studies, the determination of position and orientation of moving platform is generally performed measuring leg's length. These measurements, which can be managed by using linear sensors, are carried into the forward kinematic equations to simulate the precise positions and orientations. In this study, the leg-lengths are assumed to be measured. By using these known values, the required forces F_i needed to be applied to the pistons on the legs controlled by a developed PD algorithm.

$$F_i = (K_p)_i\{(L_i)_0 - L_i\} - (K_v)_i \dot{L}_i \qquad (40)$$

Where K_p is proportional gain and K_v is derivative gain.

The simulations of the dynamic system is depicted in fig.4, started the initial values $\dot{t}_0, \dot{\theta}_0, t_0, \theta_0$ and desired leg-lengths $(Li)_0$ and time step size h=0.01 s with the (K_p) = 4000, $(K_v)_i$=300 values, for 1000 time steps.

$$t_0 = [0.01 \quad 0.01 \quad 1]^T,$$

$$\dot{t}_0 = [0.01 \quad 0.01 \quad 0.01]^T$$

$$\theta_0 = [0.01 \quad 0.01 \quad 0.01]^T$$

$$\dot{\theta}_0 = [0.001 \quad 0.001 \quad 0.001]^T$$

$$(L_i)_0 = [1.9 \quad 1.9 \quad 1.6 \quad 1.6 \quad 1.7 \quad 1.7]^T$$

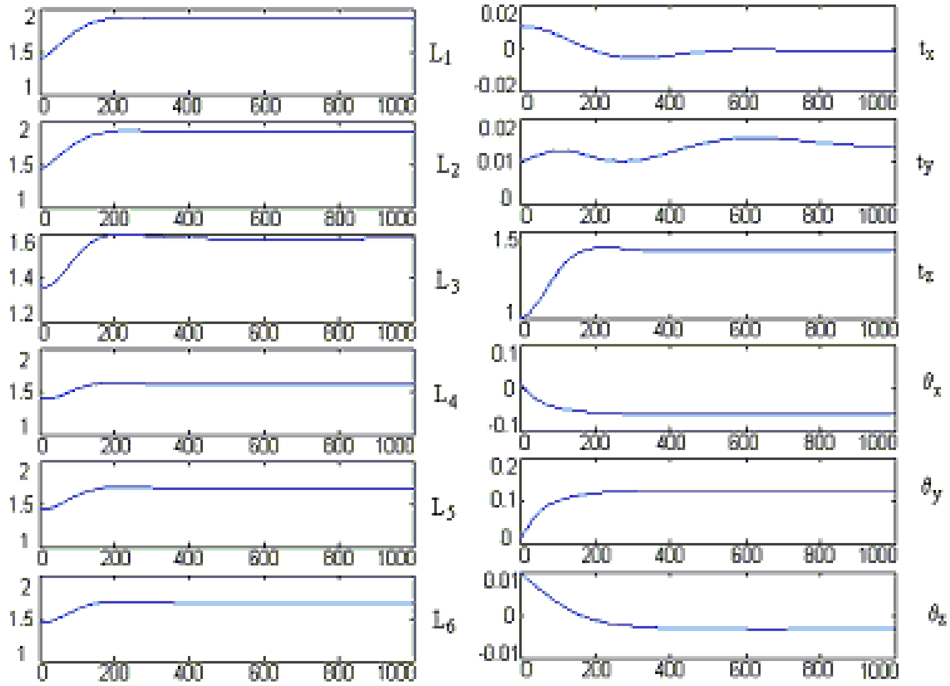

Fig. 4. Simulation results of the considered dynamical system including legs-length, position and orientation of moving platform

VII. RESULTS

Let have short review on the study in hand. Firstly, the dynamical equations are derived in great details. A leg-length-based algorithm is developed to control moving platform in the range of certain precision.

In order to obtain highly accurate positioning and orientation determining of a 6-3 SPM mechanism a PD Control algorithm is developed. This PD algorithm is implemented to the dynamical model by which it is possible to express all the trajectories of this SPM might have. The application of PD algorithm developed results in positioning error less than %1.

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