A Fuzzy Decision Model for Optimal Shunt Allocation on Unbalance Radial Distribution with Nonlinear Loads
Part II : The Numerical Results

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Abstract: This paper presents the numerical results obtained throughout the application of the fuzzy decision model for optimal location of shunt capacitors on unbalanced distribution networks with harmonic sources (which are described and formulated in the previous paper, Part I. Tests on a realistic unbalanced radial system with typical harmonic sources are presented with promising results. Sensibility analysis to show the effect of harmonic distortion limits and different forms of membership functions on final results also are presented.

I. INTRODUCTION

In Part I of this paper [1], it is shown that the capacitor placement problem for distribution systems with nonlinear loads is formulated by a fuzzy decision model which the objective function and set constraints incorporate the inherent vagueness and uncertainty on the way of operation of the system. By definition of membership functions for power losses, voltage deviation and harmonic distortion which is based on knowledge and experience of planner about operation system, the optimal location of shunt capacitor corresponds the intersection operation among those fuzzy functions for the set *candidate buses*. This search method for optimal solution greatly reduces the effort via an exhaustive search and the system loss and harmonic distortion can be reduced very effectively by this model as will be demonstrated in what follows. A three-phase harmonic load flow program which take into account the skin effect on resistance and presence of multiple harmonic sources is incorporated to fuzzy model as basis for calculation of performance indices, as three power losses, r.m.s voltages and harmonic distortion factor on each phase.

The membership functions values from those indices depend on planner needs related to the objective and constraints that are expressed by membership functions forms. The optimal sizes of capacitors at each location are obtained by local variations method from initial configuration of reactive power and standard capacitor units. The constraint of maximum number of capacitor units at optimal bus is crisp and it follows some heuristics rules.

The fuzzy model of optimal reactive compensation allocation is tested on realistic Brazilian network [2] which some data were modified and added for effect of harmonic analysis. Only six-pulse converters are treated as harmonics sources this paper for lack of simplicity of harmonic analysis but, of course, other harmonic sources as arc furnaces, controller reactors by tiristors, small groups of motors can be easily simulated by equivalent currents injection models. The bare system is the system without capacitors. The reduction in power losses and the improvement in voltage and harmonic distortion profiles for the compensated are shown for comparison with the corresponding values for the bare system. By simulation of different values of harmonic distortion factor limit and forms of membership functions, it's observed that these factors are essentials on final results.

II. TEST DISTRIBUTION SYSTEM

In order to evaluate the proposed fuzzy model for the optimal placement of shunt capacitors at systems with distorted voltages and currents, a realistic 21kV radial distribution system with thirty-two buses, thirty-two branches and three harmonic sources of the type six-pulse convert is used. The one-line diagram of this system is shown in Fig. 1.

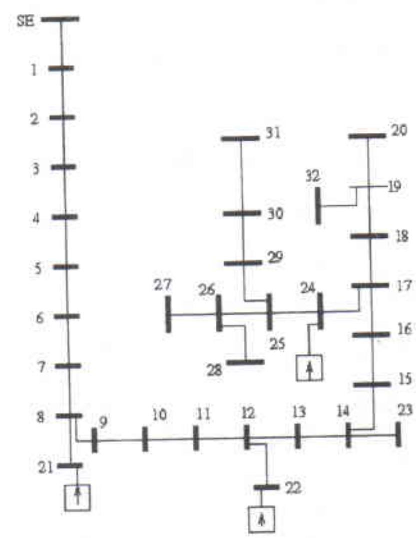

Fig.1 – The on-line diagram of the test distribution system.

The data and characteristics of the three harmonic sources located on buses 21, 22 and 24 for effect of load flow analysis is shown in Table 1. The per-phase total power load is:

Phase a: 17806.3 kW + j 7009.1 kVAr
Phase b: 12735.5 kW + j 7027.1 kVAr
Phase c: 12716.9 kW + j 7154.6 kVAr

This load profile corresponds to highest load level on which we established the maximum capacitor reactive power to be installed per phase. A real power loss cost of $ 450/kW was used and the unit cost of 3-phase capacitor sizes commercially available is chosen in agreement with representation of Fig.2. By notation of Part I, V_{min} = 0.958 p.u /phase, V_{max} = 1.05 p.u /phase and the maximum harmonic distortion factor permissible is HDF_{max} = 4% /phase.

With these data and the initial configuration network shown in Fig. 1 (non-compensated system), a harmonic load flow analysis is realized. The results are resumed at Table 2. In agreement with the set constraints imposed to problem, it was observed that buses {15,16,..,20,23,24,25,...,32} have a voltage violation problem. Therefore, this is the set buses on which the capacitors could be installed is, i.e., it's the "*candidate buses*". The cost function and total three-phase power losses are $ 116640 and 259.2 kW respectively.

Table 1 – Harmonic sources data

Bus	Type	Power (kW)	Harmonic Current Injected orders	firing angle
21	Convt. 6-pulse	1600 (3ϕ)	5,7,11,13,17,19,23....	15
22	Convt. 6-pulse	900 (3ϕ)	5,7,11,13,17,19,23....	15
24	Convt. 6-pulse	100 (3ϕ)	5,7,11,13,17,19,23....	40

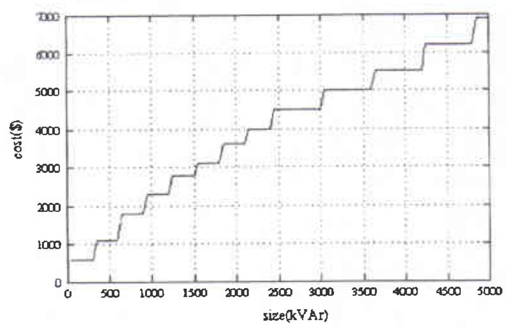

Fig.2 - Cost of 3-phase capacitors

The least r.m.s voltage is 0.9434p.u at phase A of bus 31, and the maximum harmonic distortion is 4.77% at bus 21.

By using the membership functions defined in Part I, for the candidate buses, with k_l = 10, k_v = 10 and k_h = 10, an optimal solution for capacitor locations, which gives minimum cost power losses, without violates voltage constraints, was achieved by fuzzy decision model. The results for the fuzzy decision at each test bus are presented in Table 2. Bus 24 is selected to be the optimal location of the shunt capacitor due to the smallest fuzzy decision value among candidates buses. (μ_{D24} = 0.4207). The optimal size of the capacitor at bus 24 is then solved by the method of local variations. The solution obtained is that the lowest cost without violates the r.m.s voltages and harmonic distortion will be achieved if a capacitor of 3000 kVAr is installed at this location. Fig.3 shows the variation of harmonic distortion with the capacitor units added on bus 24 until to reach optimal size. This procedure is repeated until all the voltage violates are completely solved. For this particular system, the optimal reactive compensation is solved as {bus 24, bus 31} and corresponding capacitor sizes per phase are {3000 kVAr, 300 kVAr}. Table 3 shows the resultant system voltage profile and total harmonic distortion. These results show that a better r.m.s voltage profile, lower power losses and less harmonic distortion have been obtained by proposed fuzzy decision model.

III. SENSIBILITY ANALYSIS

It is obvious from formulation presented in Part I, that the resulting optimal reactive compensation depends on limits of harmonic total distortion and voltages well as on forms of membership functions of losses, voltage deviation and harmonic distortion, which have been chosen somewhat arbitrarily. The aim of this section is to show the effect of harmonic distortion limit and different forms of membership functions on final results.

Sensibility to Harmonic Distortion Limits

In addition to the value of 5% more two cases were imposed on the total harmonic distortion: *i*) $HDF_{max} \leq 4\%$ and *ii*) constraint of HDF_{max} ignored. Table 4 shows the optimal cost function and corresponding capacitor sizes and locations for each of theses cases under investigation. The power losses, minimum and maximum r.m.s voltages, and maximum total harmonic distortion are also shown in same table.

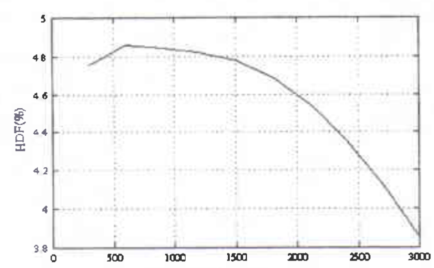

Fig. 3 – Harmonic distortion x capacitor size at bus 24

Table 2 – Harmonic Load Flow Solution before compensation and membership function values

Bus	r.m.s voltage (p.u)			Harmonic Distortion (%)			μ_p	Membership function values for violate buses					
								μ_V			μ_H		
	Phase A	Phase B	Phase C	Phase A	Phase B	Phase C		Phase A	Phase B	Phase C	Phase A	Phase B	Phase C
01	0.9816	0.9958	0.9826	1.77	1.85	1.84							
02	0.9755	0.9932	0.9776	2.50	2.61	2.59							
03	0.9731	0.9918	0.9757	2.85	2.98	2.96							
04	0.9708	0.9906	0.9740	3.23	3.36	3.34							
05	0.9691	0.9896	0.9726	3.52	3.66	3.64							
06	0.9678	0.9889	0.9717	3.74	3.66	3.87							
07	0.9661	0.9880	0.9703	4.05	3.89	4.19							
08	0.9649	0.9874	0.9695	4.27	4.21	4.41							
09	0.9638	0.9868	0.9686	4.30	4.43	4.44							
10	0.0624	0.9861	0.9675	4.35	4.46	4.49							
11	0.9613	0.9855	0.9667	4.40	4.51	4.54							
12	0.9605	0.9851	0.9661	4.43	4.56	4.57							
13	0.9596	0.9847	0.9654	4.39	4.59	4.53							
14	0.9580	0.9839	0.9641	4.32	4.55	4.46							
15	0.9564	0.9831	0.9629	4.27	4.47	4.41	0.8533	0.9969	1.000	1.000	1.000	1.000	1.000
16	0.9551	0.9825	0.9618	4.24	4.41	4.37	0.8838	0.9901	1.000	1.000	1.000	1.000	1.000
17	0.9540	0.9821	0.9609	4.22	4.37	4.34	0.9069	0.9811	1.000	1.000	1.000	1.000	1.000
18	0.9537	0.9819	0.9607	4.21	4.35	4.33	0.9929	0.9787	1.000	1.000	1.000	1.000	1.000
19	0.9532	0.9815	0.9604	4.20	4.34	4.32	0.9885	0.9735	1.000	1.000	1.000	1.000	1.000
20	0.9531	0.9814	0.9603	4.20	4.32	4.32	0.9992	0.9727	1.000	1.000	1.000	1.000	1.000
21	0.9650	0.9875	0.9696	4.60	4.32	4.75							
22	0.9606	0.9852	0.9662	4.59	4.77	4.74							
23	0.9580	0.9839	0.9641	4.32	4.76	4.46	0.9999	1.000	1.000	1.0000	1.000		
24	0.9514	0.9807	0.9590	4.22	4.47	4.34	0.8492	0.9509	1.000	1.000	0.5692	0.4207	0.4238
25	0.9489	0.9794	0.9572	4.21	4.34	4.32	0.8523	0.9111	1.000	0.9992	1.000	1.000	1.000
26	0.9472	0.9779	0.9557	4.21	4.32	4.31	0.9502	0.8791	1.000	0.9937	1.000	1.000	1.000
27	0.9471	0.9778	0.9556	4.21	4.31	4.31	0.9904	0.8763	1.000	0.9930	1.000	1.000	1.000
28	0.9466	0.9774	0.9551	4.21	4.31	4.31	0.9921	0.8665	1.000	0.9904	1.000	1.000	1.000
29	0.9458	0.9787	0.9552	4.20	4.30	4.31	0.8795	0.8514	1.000	0.9911	1.000	1.000	1.000
30	0.9443	0.9788	0.9545	4.20	4.29	4.30	0.9455	0.8194	1.000	0.9856	1.000	1.000	1.000
31	0.9434	0.9793	0.9541	4.20	4.29	4.30	0.9745	0.7977	1.000	0.9826	1.000	1.000	1.000
32	0.9521	0.9805	0.9594	4.20	4.32	4.31	0.9797	0.9600	1.000	1.000	1.000	1.000	1.000

Table 3 – Resultant r.m.s voltage and harmonic distortion after compensation.

Bus	r.m.s voltages (p.u)			Total harmonic distortion (%)			Bus	r.m.s voltages (p.u)			Total harmonic distortion (%)		
	Phase			Phase				Phase			Phase		
	A	B	C	A	B	C		A	B	C	A	B	C
1	0.9855	0.9989	0.9862	1.01	1.05	1.04	17	0.9679	0.9936	0.9739	3.10	3.19	3.19
2	0.9810	0.9976	0.9826	1.42	1.48	1.47	18	0.9676	0.9934	0.9737	3.10	3.19	3.18
3	0.9792	0.9967	0.9813	1.62	1.69	1.67	19	0.9671	0.9931	0.9734	3.09	3.18	3.17
4	0.9777	0.9961	0.9803	1.83	1.90	1.89	20	0.9671	0.9930	0.9733	3.09	3.18	3.17
5	0.9765	0.9956	0.9795	2.00	2.07	2.06	21	0.9737	0.9945	0.9776	2.68	2.77	2.75
6	0.9756	0.9952	0.9789	2.12	2.20	2.19	22	0.9710	0.9937	0.9758	2.53	2.63	2.62
7	0.9745	0.9948	0.9781	2.30	2.38	2.36	23	0.9696	0.9935	0.9749	2.52	2.61	2.60
8	0.9737	0.9945	0.9776	2.42	2.50	2.49	24	0.9667	0.9935	0.9733	3.50	3.60	3.59
9	0.9730	0.9942	0.9771	2.41	2.49	2.48	25	0.9644	0.9924	0.9716	3.54	3.63	3.62
10	0.9721	0.9940	0.9765	2.41	2.50	2.49	26	0.9627	0.9909	0.9702	3.54	3.62	3.62
11	0.9714	0.9938	0.9761	2.43	2.52	2.51	27	0.9625	0.9908	0.9700	3.54	3.62	3.62
12	0.9710	0.9937	0.9758	2.45	2.54	2.53	28	0.9621	0.9903	0.9696	3.54	3.62	3.62
13	0.9705	0.9936	0.9755	2.45	2.54	2.53	29	0.9616	0.9919	0.9700	3.66	3.74	3.74
14	0.9697	0.9935	0.9749	2.52	2.61	2.60	30	0.9604	0.9922	0.9694	3.76	3.84	3.84
15	0.9689	0.9934	0.9745	2.68	2.77	2.76	31	0.9596	0.9929	0.9693	3.87	3.94	3.95
16	0.9683	0.9935	0.9741	2.88	2.97	2.96	32	0.9660	0.9921	0.9724	3.09	3.18	3.17

Table 4 – Sensibility to Harmonic Distortion Limits

	$HDF_{max} \le 4\%$	$HDF_{max} \le 3.5\ \%$	HDF_{max} ignored
Total Cost ($)	107280	111393	117015
Power Losses (kW)	238.4	236.4	250.7
Optimal Location Buses	{24,31}	{24}	{30,31}
Capacitor Sizes (kVAr)	{3000,300}	{3600}	{300,2100}
Min $V_{r.m.s}$ (p.u)	0.9596 at bus 31	0.9601 at bus 31	0.9584 at bus 28
Max HDF (%)	3.95 at bus 31	3.36 at bus 28	4.2 at bus 31

Table 5 – Sensibility to Membership Functions

	$k_l = 0.5;\ k_v = 1.0;\ k_h = 0.1$	$k_l = 10;\ k_v = 10;\ k_h = 10$
Total Cost ($)	117450	107280
Power Losses (kW)	251.0	238.4
Optimal Location Buses	{31}	{24,31}
Capacitor Sizes (kVAr)	2700	{3000,300}
Min $V_{r.m.s}$ (p.u)	0.9612 at bus 28	0.9596 at bus 31
Max HDF (%)	3.16 at bus 31	3.95 at bus 31

Sensibility to Membership Functions

To see how different forms of membership functions affect the optimal solution, was simulated a case in which $k_l = 0.5$; $k_v = 1.0$ and $k_h = 0.1$. With these parameters, the corresponding membership functions have larger values than the original case. The results obtained are showed in Table 5 together with the results of the original solution for effect of comparison.

By analyzing the results from Tables 2,3,4 and 5, the following conclusions are derived:

i) The decisive factor in the choice of the bar 24 as the optimal bus, among the selected candidate buses, it is the largest percent of existent nonlinear load in this bar providing smaller membership function value of $\mu_{H\phi}$ than μ_P and $\mu_{V\phi}$.

ii) The resultant r.m.s voltage and total harmonic distortion profiles presented in Table 3 confirms that by shunt capacitors strategically located and sized it's possible to control generation of harmonics.

iii) When limits on the total harmonic distortion are ignored, the cost function found by neglecting harmonic frequencies is $ 9735 higher than that of case in which harmonic components are taken into account. This is due to the fact that harmonic frequencies naturally increase the r.m.s voltages. Thus, less kvars are needed to bring the voltage levels up to the minimum permissible level.

iv) To maintain a limit of harmonic distortion below 4% the investment in capacitors it should be larger and consequently the optimal solution is less economical.

v) Depending of the coefficients values k_l, k_v and k_h for membership functions, the optimal location and size of capacitors can to change. The decrease of coefficient k_h from 10 to 0.1 it facilitated that the decision criterion for chosen optimal bus has been of tension deviation. In this case, an only one capacitor is necessary for optimal compensation and the solution is less economical than original case.

vi) The saving with reduction of losses, that is $ 9360,0 in this example, shows that the fuzzy decision based on intersection of membership functions is enough of this type of application.

VII. CONCLUSIONS

This paper demonstrates the application of the theory and solution method presented in Part I [1]. A thirty-two unbalanced radial distribution system with three harmonic sources of type six-pulse convert is chosen to exemplify, via numerical results, the validity of the proposed fuzzy decision model. The optimal solution, which consists in find the optimal location for shunt capacitors, is found using the property of intersection of fuzzy sets. For each phase of radial network are defined membership functions for deviation voltage and total harmonic distortion that represent the constraints set problem. Another membership function is defined to represent the total power losses including the harmonic components. A three-phase harmonic load flow program based on admittance matrix method was used for obtain the state of network in all stages of fuzzy programming which results are used for calculates the membership functions. The local variation method was used for to solve the optimal size of capacitors at each optimal location by fuzzy decision model. As result, the buses location and sizes of units of capacitor of 300kVAr that provided better saving with loss reduction without violates the r.m.s voltage and harmonic distortion profiles is found for the test network.

The validity of the fuzzy decision criterion for optimal location of shunt capacitors can be clearly observed comparing the results of r.m.s voltage and harmonic distortion of Tables 2 and 3 further general results of losses and cost function presented in the second column of the Table 3. Note that only 8% of loss reduction provided a saving of $ 9360 in investment that could be used to buy others equipments as regulators and filters. The economic benefits depend on the maximum total harmonic distortion allowed and it was proven that the solution may not be economical if strict regulations on waveforms distortion levels are applied.

It's fundamental to take into account the constraint of harmonic distortion in the reactive compensation problem in distribution systems with nonlinear loads because in this case the installation of shunt capacitors to improve the voltage profile provided serious problems of parallel resonance in agreement the results shown in fourth column at Table 4.

The property of convexity of the three membership functions guaranties the application of the property of intersection among those functions as basis of fuzzy decision model.

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