USING ROOT MOMENTS FOR MINIMUM PHASING IN DISPERSIVE BAND LIMITED DETECTOR AND INVERSE FILTER APPLICATIONS

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ABSTRACT

In this study, the transfer function of the distorting system has been made minimum phase by using the method of root moments. And inverse filtering process has been carried out. As application areas, band-limited detector has been investigated. Band-limited detector has been modeled as a FIR system. The algorithms, developed for inverse filter applications in the distorting system, has been coded in suitable programming language and applied to several examples. After the simulations, in the system output the desired undistorted input signal has been observed for distorting system.

I. INTRODUCTION

In some special systems, it is desired to obtain the undistorted input signal in the system output. But some systems cause distortion in input signal. Thus, distorted signal is obtained instead of required signal in the system output.

In many signal processing applications, inverse filters are applied to compensate distortions produced by physical media or measuring instruments [1].

Inverse filters work best when applied to a system that has the minimum-phase property. In order to apply an exact inverse filter, the transfer function of the distorting system must have minimum-phase property [1].

In this study, band limited detector that necessitates inverse filter application has been investigated. First of all, distorting system has been modeled as a digital filter. Transfer function of distorting system is obtained for inverse filter application. Then, it has been investigated whether transfer function is minimum phase or not. If transfer function doesn't have minimum phase property, minimum phase system is obtained from linear phase distorted system using root moments. Stable and causal inverse filter application is carried out.

II. INVERSE FILTER

In many signal processing applications, in observed signals distortion occurs. Band-limited detector acts like a filter that distorts observed signal. Inverse filters serve to compensate for filtering actions produced by detector [1],[2].

INVERSE FILTER PROPERTIES

Distorting system can be modeled as causal and stable. Poles of stable system are inside the unit circle, its zeros are anywhere around the unit circle [1].
Let transfer function of distorting system be H (z).

$$H(z) = \frac{N(z)}{D(z)} \tag{1}$$

In (1) equation roots of D(z) are poles of system, roots of N(z) are zeros of system. Transfer function of inverse filter is inverse transfer function of distorting system.

$$H_{if}(z) = \frac{D(z)}{N(z)} \tag{2}$$

This result indicates that poles of inverse filter are zeros of distorting system and zeros of inverse filter are poles of distorting system. Inverse filter must be causal and stable. Hence distorting system must have minimum phase property [1],[2].

LINEAR PHASE FIR FILTER

In order to have linear phase property group delay of FIR filter must be constant. Furthermore, all passband zeros of linear phase FIR filter are in conjugate reciprocal pairs around the unit circle [3[,[4].
Transfer function of linear phase FIR filter is given by

$$\begin{aligned} H(z) &= z^n + h_1 z^{n-1} + h_2 z^{n-2} + \ldots\ldots\ldots\ldots\ldots + h_n \\ &= \prod_{i=1}^{n} (z - r_i) \end{aligned} \tag{3}$$

To obtain a minimum phase (MP) version of the given a linear phase (LP) transfer function, $H_o(z)$ that contains all roots of H(z) that lie on the unit circle and $H_{min}(z)$ that has MP part of H(z) are determined. Then, $T(z) = [H_{\min}(z)]^2 H_o(z)$ is constructed that is the transfer function of MP system. And also $\left|T(e^{j\theta})\right| = \left|H(e^{j\theta})\right|$ must be.

In order to $H_{\min}(z)$ and $H_o(z)$ parts, firstly the roots of H(z) must be obtained. But root finding procedures are known to be inaccurate and unreliable for large order polynomials. Hence it can be found $H_{\min}(z)$ and $H_o(z)$ parts of LP filter using root moments of given polynomials without root finding forms [4].

III. DEFINITION OF ROOT MOMENTS

$H(z) = \prod_{i=1}^{n}(z - r_i)$ defined any polynomial, Newton defined a set of parameters that functions of roots of related polynomial. These parameters are given by

$$S_m = r_1^m + r_2^m + \ldots\ldots\ldots\ldots + r_n^m = \sum_{i=1}^{n} r_i^m \tag{4}$$

r_i is ith root of H(z). The parameters S_m are root moments of the polynomial of H(z). These parameters can be determined directly from the coefficients h_i through an iterative procedure [4],[5].

ITERATIVE ESTIMATION OF ROOT MOMENTS

Differentiation of (3) polynomial;

$$H'(z) = nz^{n-1} + (S_1 + nh_1)z^{n-2} + (S_2 + h_1 S_1 + nh_2)z^{n-3} + \ldots.$$
$$+ (S_m + h_1 S_{m-1} + h_2 S_{m-2} + \ldots\ldots nh_m)z^{n-m-1} + \ldots.$$

Direct differentiation of (3);

$$H'(z) = nz^{n-1} + (n-1)h_1 z^{n-2} + \ldots.. + (n-m)h_m z^{n-m-1} + \ldots.$$

By using equating these two equations the following fundamental relationship known as Newton Identities is obtained.

$$S_m + h_1 S_{m-1} + h_2 S_{m-2} + \ldots\ldots\ldots\ldots + mh_m = 0 \tag{5}$$

The (5) equation is used to calculate successive values of the root moments. If the signal is of finite duration then for m>n

$$S_m + h_1 S_{m-1} + h_2 S_{m-2} + \ldots\ldots\ldots\ldots + h_n S_{m-n} = 0$$

Essentially (5) equation it can be interpreted as a transformation of coefficients $\{h_r\}$ to parameter set $\{S_m\}$[4],[5].

NONITERATIVE ESTIMATION OF ROOT MOMENTS

The Newton Identities yield the root moments of entire polynomial. However, it is often the case, that a specific factor of a given polynomial H(z) is required, such as the MP or the maximum phase factor. In this case its root moments can be determined in a different manner, by using the Cauchy Residue Theorem.

Let a closed contour Γ defined $z = \rho(\theta)e^{j\theta}$. It is no zeros on Γ. This contour includes roots of required factor of polynomial H(z). Then it follows from Cauchy residue theorem that the root moments of the factor of the polynomial whose roots lie within Γ are given by

$$I_\Gamma(m) = S_m^\Gamma = \frac{1}{2\pi j}\oint_\Gamma \frac{H'(z)}{H(z)} z^m dz \tag{6}$$

This is evident from the fact

$$S_m^\Gamma = \frac{1}{2\pi j}\oint_\Gamma \sum_i \frac{1}{(z - r_i)} z^m dz \tag{7}$$

and the contribution to the integration are those due to those roots that lie within Γ [4],[5].

In practice the contour integration will have to be effected directly from the coefficients of H(z) and this can be done quite conveniently through the use of the DFT (Discrete Fourier Transform) as it shown below. Equation (6) becomes for $z = \rho(\theta)e^{j\theta}$

$$S_m^\Gamma = \frac{1}{2\pi j}\int_{-\pi}^{\pi} g(\theta)e^{j(m+1)\theta} d\theta \tag{8}$$

where

$$g(\theta) = \frac{H'(\rho(\theta)e^{j\theta})}{H(\rho(\theta)e^{j\theta})}\left(\frac{d\rho(\theta)}{d\theta} + j\rho(\theta)\right)\rho^m(\theta) \tag{9}$$

Discretization of (8) suitable for DFT use requires values $\theta_k = 2\pi k / N,\ k = 0,1,\ldots..N-1$ for an N-point transform. Note that the value of N has to be large enough in order to approximate sufficiently the integral by a summation and this is an issue for further investigation. Therefore, inverse DFT

$$S_m^\Gamma \approx \frac{1}{jN}\sum_{k=0}^{N-1} g(\theta_k)e^{j(m+1)\theta_k} \tag{10}$$

If the contour of integration is the unit circle $|z| = 1$ then the resulting root moments from the above, correspond to those of the MP component of H(z). In this case (10) is reduced to the form

$$S_m^{H_{\min}(z)} \approx \frac{1}{N}\sum_{k=0}^{N-1} \frac{H'(\theta_k)}{H(\theta_k)} e^{j(m+1)\theta_k} \tag{11}$$

DESIGN ALGORITHM OF MP FILTER

It is desired that MP Filter (T(z)) with the same amplitude response is obtained from a LP Filter (H(z)) given. H(z) transfer function has zeros on the unit circle. Hence it can not be obtained the MP part by integrating around the unit circle. This approach causes the steps below [4].

1) It is integrated around a circle centered at the origin of radius less than unity. With a careful choice of the contour radius, the integration gives the root moments $S_1(m) = S_{in}(m)$ that correspond to that part

of the original FIR transfer function which has its zeros inside the unit circle, namely MP part.
The radius of the contour is of crucial importance. The circle that includes all the roots of MP part of original polynomial has to be selected.

2) Integrate around a circle centered at the origin and of radius greater than unity. A good selection in step 1) yields a correspondingly good selection in step 2). The integration produces the parameters $S_2(m) = S_{in}(m) + S_o(m)$ where $S_o(m)$ are the root moments of that factor of the original FIR digital filter transfer function which has its zeros on the unit circle.
3) The required transfer function has the root moments $S(m) = 2S_{in}(m) + S_o(m)$, obtained as $S(m) = S_1(m) + S_2(m)$.
4) From step 3) and from Newton Identities the required MP FIR transfer function is formed.

IV. BAND LIMITED DETECTOR

Band limited detector distorts signals. Inverse filtering application has been carried out to suppress distortion effect of detector. Model of band limited detector and application of inverse filter is shown in figure 1.

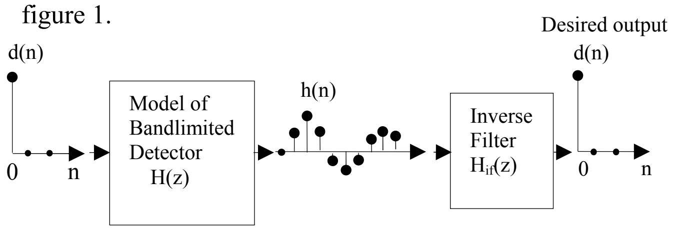


Figure 1. Model of band limited detector and application of inverse filter [1].

As shown figure 1. distorting signal is like a Sinc function in detector output. Hence, it is clear to be modeled detector as a low pass filter. If detector is modeled as a filter, the transfer function of detector is given by

$$H(z) = h(0)z^{n-1} + \dots\dots + h(n-2)z^1 + h(n-1) \qquad (12)$$

LP FIR low pass filter has been designed Fourier Series Method [3]. Thus transfer function of detector is obtained. After transfer function of detector is determined, detector that doesn't have MP property, is yielded minimum phase using root moments method [4],[5]. Auxiliary system is linked cascade detector ($H_1(z)$). System is shown fig. 2.

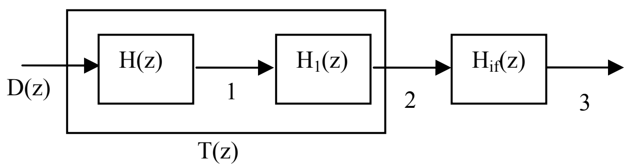


Figure 2. Demonstration of system of band limited detector and application of inverse filter

T(z) is MP filter with same amplitude response the detector. MP filter is given by

$$T(z) = T(0)z^{n-1} + T(1)z^{n-2} + \dots\dots\dots\dots + T(n-2)z^1 + T(n-1) \qquad (13)$$

Transfer function of inverse filter

$$H_{if}(z) = \frac{1}{T(z)} = \frac{1}{T(0)z^{n-1} + \dots\dots + T(n-2)z^1 + T(n-1)} \qquad (14)$$

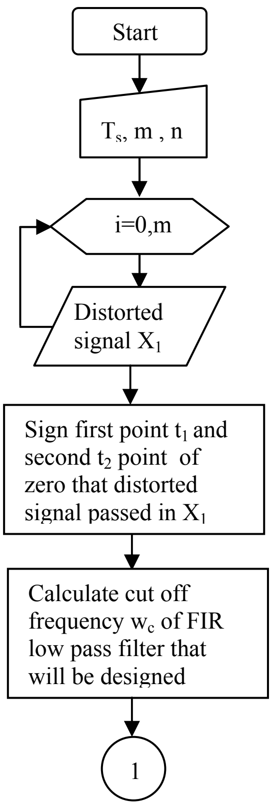
In figure 2, distorted signal not minimum phase is obtained in point 1, and distorted signal minimum phase is obtained in point 2, required output signal is obtained in point 3.

Transfer function of auxiliary system

$$H_1(z) = \frac{T(z)}{H(z)} = \frac{T(0)z^{n-1} + \dots\dots\dots\dots + T(n-2)z^1 + T(n-1)}{h(0)z^{n-1} + \dots\dots\dots\dots + h(n-2)z^1 + h(n-1)} \qquad (15)$$

V. OBTAINING TRANSFER FUNTION OF BAND LIMITED DETECTOR AND INVERSE FILTER USING COMPUTER PROGRAM

The algorithms developed for inverse filter applications in band limited detector, have been coded in Fortran Power Station 4.0 programming language and applied to several examples. Flowchart of computer program is shown below.



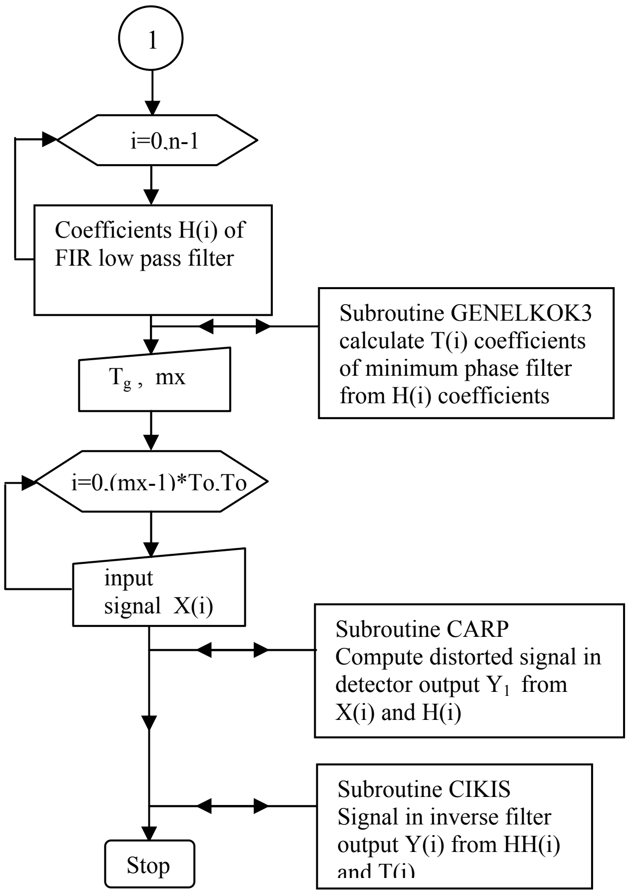


Figure 3. Flowchart of algorithm suggested

VI. EXPERIMENTAL RESULTS

Example: The period of transfer function of distorting detector $T_s=10^{-6}$ s, the number of data sequence obtained from distorted signal m=20 , system order of designed filter n=55, and input signal X(0)=-2, X(T_g)=3, period of input signal T_g=56µs, (input sequence number n=2), so in the program it is computed that coefficients of transfer function of inverse filter (T(z)) and coefficients of transfer function of detector (H(z)) at 55th order. Then as transfer function of detector and input signal are known distorted signal Y_1(z) in output detector is obtained. Because transfer function of inverse filter is known required output signal Y(z) is obtained.

Amplitude response of H(z) is shown in fig. 4, amplitude response of T(z) is shown in fig. 5, phase response of H(z) is shown in fig. 6, phase response of T(z) is shown in fig. 7.

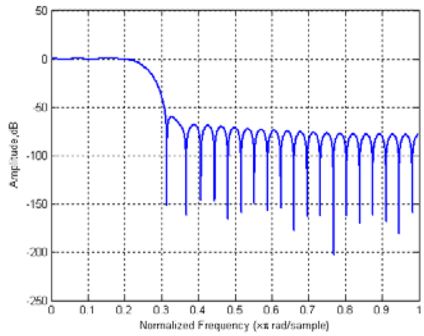


Figure 4. Amplitude response of H(z)

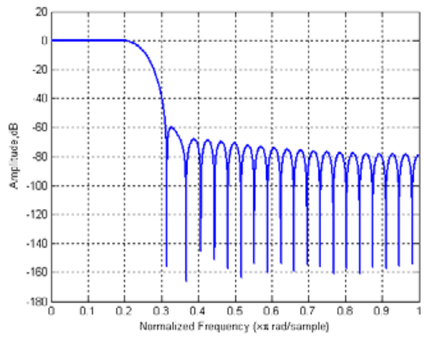


Figure 5. Amplitude response of T(z)

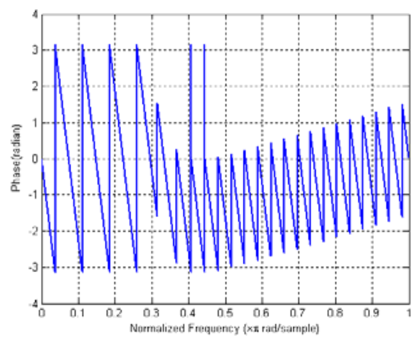


Figure 6. Phase response of H(z)

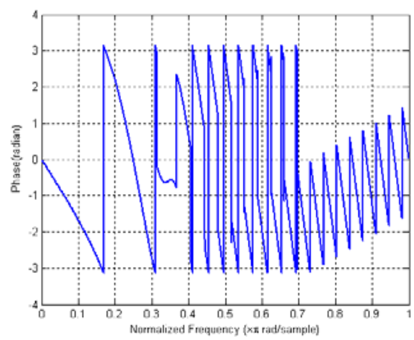


Figure 7. Phase response of T(z)

Input signal x(n) is shown in fig.8, impulse response of detector h(n) is shown in fig. 9, distorted signal $y_1(n)$ in detector output is shown in fig. 10, required signal y(n) in inverse filter output is shown in fig. 11.

In figure 8 and figure 11 , it is observed that input signal x(n) is same with signal y(n) in inverse filter output.

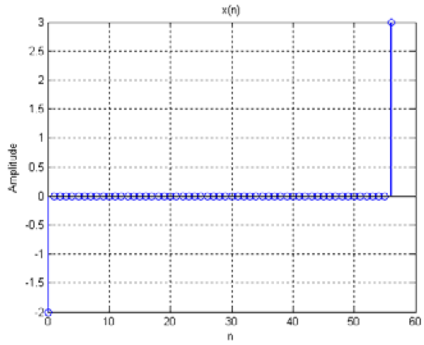


Figure 8. x(n) input signal

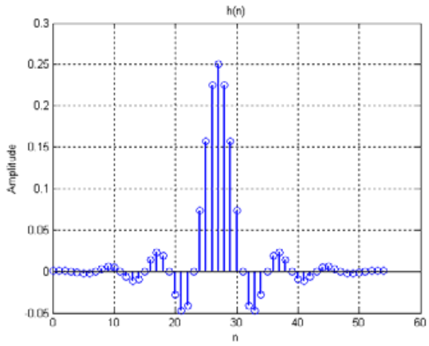


Figure 9. Impulse response of detector h(n)

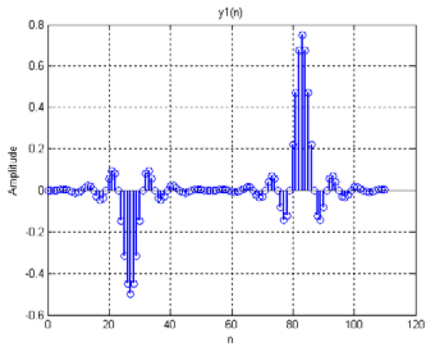


Figure 10. Distorted signal in detector output

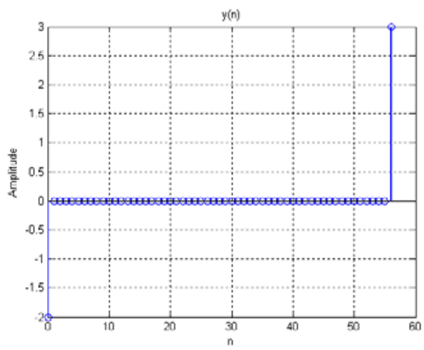


Figure 11. Required signal in inverse filter output

VII. CONCLUSIONS

In this study inverse filters and as application areas band limited detector has been investigated. For application of a stable and causal inverse filter distorting system has the minimum phase property.

In this condition firstly model of FIR low pass filter is obtained for detector. It is essential that auxiliary system is linked to the detector, which will make distorted system transfer function be minimum phase. Hence, with the help of root moments method MP transfer function with the same amplitude response has been obtained from transfer function of detector. So transfer function of auxiliary system is obtained. And then stable and causal inverse filter application is carried out from transfer function of detector and auxiliary system.

In this study examples of various inverse filter applications for band limited detector of computer program have been made and successful results are obtained.

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