Determination of the Optimum Pilot Density for an OFDM System over Empirical Channels

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Abstract

In orthogonal frequency division multiplexing (OFDM) systems, in order to overcome distortion caused by multipath fading, channel estimation is performed. Pilot symbol assisted channel estimation is effectual way to obtain channel state information. However, use of pilot symbols, leads overhead and the number of pilot symbols should be kept as minimum as possible. In this study, in order to evaluate the optimum pilot density, channel state information is obtained by using two dimensional (2-D) pilot based channel estimation. The analytical and simulated optimum pilot intervals in frequency and time direction are determined in terms of mean squared error (MSE). Then these intervals related to channel's coherence time (T_c) and bandwidth (B_c) at 0.9 correlation coefficient. It is seen from the results that by using approximately 2.4 pilot symbols per $B_{c,0.9}$, and per $T_{c,0.9}$ the optimum pilot interval can be calculated easily.

1. Introduction

Orthogonal frequency division multiplexing (OFDM) allows an effective transmission over fading channels by eliminating frequency selective fading, and widely applied in wireless communication systems due to higher data rate transmission capability. Since radio channels nature is highly dynamic, mitigating capability with these detrimental effects is critical for efficient data transmission. In order to overcome these effects, and increase system performance channel estimation must be accomplished. In pilot aided channel estimation the quality of channel estimation depends on choice of proper pilot pattern. One of the most important aspects of pilot pattern is the number of used pilots which affects directly the spectral efficiency, and the system performance. Therefore, there have been many works on designing optimum pilot pattern.

A feedback technique in which the pilot allocation mechanism is adapted to the mobile channel is proposed in [1], general expression for the symbol error rate as a function of pilot allocation is derived, and with the use of two practical solutions using bounds on the objective function pilot locations are optimized. In [2] a windowed least-squares estimator for doubly selective fading channels is proposed, and the optimum pilot pattern for the estimator is designed. The frequency selective channel estimation problem in OFDM is investigated from the perspective of compressed sensing (CS), two criteria for optimizing the pilot pattern for CS-based channel estimation is proposed in [3], and better channel estimation mean squared error (MSE) and bit error rate (BER) performance is obtained by the use of the optimized pilot pattern without computational complexity. In [4] an optimal and suboptimal pilot cluster is proposed, and with the use of this proposed optimal pilot cluster sequence channel estimation quality and hence bit error rate performance significantly is improved. An optimal pilot-symbol pattern design using a post-equalization signal to interference and noise ratio framework is investigated in [5], distances between adjacent pilot-symbols and distribution of available power is determined, and by using the proposed system configuration significant capacity gain is obtained. In [6], optimal pilot symbol design for time invariant scenarios is investigated, analytical expressions for the optimal distance between adjacent pilot-symbols and the optimal power distribution between pilot-symbols and data symbols are obtained, and with the use of this system approximately 30% capacity gain is achieved. The analytical formulation of the channel estimation MSE as a function of the pilot density for a given power efficiency is presented in [7], and it is confirmed that the optimum density is the one that closely fulfills the 2-D sampling theorem via simulations and analytical results.

In this study in order to find the optimum pilot density; the impact of the pilot distribution on the channel estimation error is assessed using MSE analysis. In order to do so, rectangular pilot pattern is used as 2-D pilot pattern, channel estimate at pilot positions are obtained by least squares (LS), then frequency and time domain channel coefficients are obtained through low pass interpolation which yields the best performance [8]. The loss in signal to noise ratio (SNR) due to pilot usage is evaluated and taken into account. ITU empirical channel models are used to simulate the different multipath fading environments. The pilot symbols that give the minimum analytical [7] and simulated MSE in frequency and time domain are determined as the optimum pilot symbols. The intervals between these optimum pilot symbols are defined as the optimum pilot intervals, denoted Δf_{opt} and Δt_{opt}, in frequency and time domain respectively. Different from previous studies, relationships between Δf_{opt} and channels' coherence bandwidths (B_c), Δt_{opt} and channels' coherence times (T_c) are determined.

2. OFDM System Description

The baseband OFDM system with pilot based channel estimation is given in Fig. 1. In this system the input data divided into blocks called OFDM symbol. Pilots are inserted into data according to chosen pattern, a cyclic prefix during guard interval is added to each symbol in order to prevent inter symbol interference (ISI) and inter carrier interference (ICI) then the OFDM signal is sent to channel. At the receiver discrete

Fourier transform (DFT) is performed on guard interval removed symbol. The received symbol is as follows:

$$Y(n,k) = H(n,k)X(n,k) + W(n,k) \quad n = 0,...,N_c - 1, k = 0,...,N_s - 1 \tag{1}$$

where H(n,k) is complex channel coefficients for n^{th} subcarrier and k^{th} symbol, X(n,k) is modulated input data, W(n,k) is Additive White Gaussian Noise (AWGN), N_c is the number of subcarriers per OFDM symbol, N_s is the number of OFDM symbols per OFDM frame.

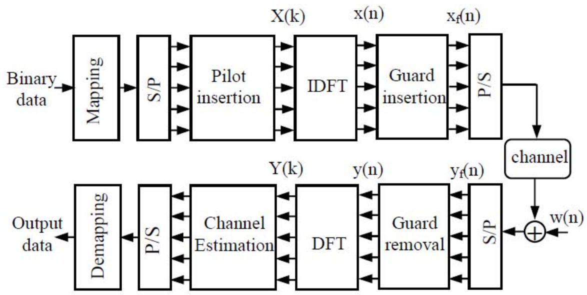

Fig. 1. Baseband OFDM system

In wireless communication systems the transmitted signal is distorted by detrimental effects such as reflection, reflection, scattering, diffraction and movement which limit system performance. In order to overcome these effects, and to design reliable systems the characteristic of the channel must be known. Time varying multipath channel can be characterized in time domain by T_c, and in frequency domain by B_c,

The T_c can be obtained from time correlation function (TCF) [9]. TCF for a wide sense stationary uncorrelated scattering (WSSUS) channel is given by:

$$R(\xi) = \int_{-\infty}^{+\infty} S_h(f)e^{j2\pi\xi f}df \tag{2}$$

where ξ difference time variable and $S_h(f)$ is Doppler power spectrum. T_c is defined as the time over which the normalized TCF is above 0.5, 0.75, 0.9 ($T_{c,0.5}$, $T_{c,0.75}$, $T_{c,0.9}$). An example for time correlation function is given, and $T_{c,0.5}$, $T_{c,0.75}$, $T_{c,0.9}$ are shown in Fig. 2.

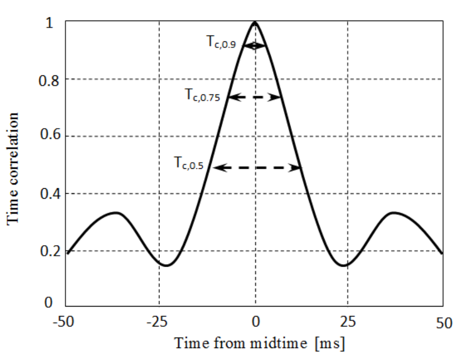

Fig. 2. An example for time correlation function

The B_c can be obtained from frequency correlation function (FCF) [9]. The FCF for a WSSUS channel is given as:

$$R(\Omega) = \int_{-\infty}^{+\infty} P_h(\tau)e^{-j2\pi\Omega\tau}d\tau \tag{3}$$

where Ω is difference frequency variable, and $P_h(\tau)$ is power delay profile. B_c is defined as the bandwidth over which the normalized FCF function is above 0.5, 0.75 or 0.9 ($B_{c,0.5}$, $B_{c,0.75}$,$B_{c,0.9}$). An example for frequency correlation function is illustrated, and $B_{c,0.5}$, $B_{c,0.75}$, $B_{c,0.9}$ are shown in Fig.3.

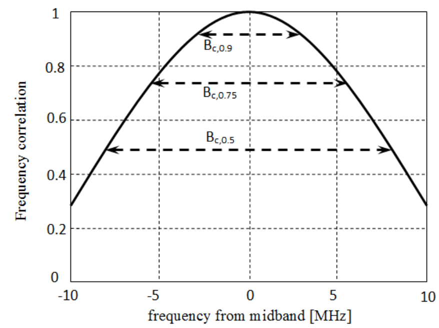

Fig. 3. An example for frequency correlation function

2.1. Channel Estimation

In pilot aided channel estimation known pilot sequence is inserted into data symbol, with the interval N_t in time direction and the interval N_f in frequency direction which should fulfill Nyquist sampling theorem in (4).

$$f_m.T_s.N_t \le \frac{1}{2}, \qquad \tau_{max}.F_s.N_f \le \frac{1}{2} \tag{4}$$

where T_s is OFDM symbol duration and F_s is subcarrier spacing. f_mT_s and $\tau_{max}F_s$ are the normalized one-sided channel bandwidths in time and frequency domains respectively [10]. The estimate of channel at pilot positions can be based on least square (LS) that is given by;

$$\hat{H}(n',k') = \frac{Y(n',k')}{X(n',k')} = H(n',k') + \frac{W(n',k')}{X(n',k')} \tag{5}$$

where X(n',k') denote the pilots, at pilot locations n' in frequency direction, and k' in time direction The final estimate of CSI obtained from LS estimate via interpolation. The performance of channel estimate can be measured by MSE that is defined as follows:

$$MSE = E\left\{\left|H(n,k) - \hat{H}(n,k)\right|^2\right\} \tag{6}$$

where $\hat{H}(n,k)$ is the final estimate of channel at n^{th} subcarrier and k^{th} symbol, E{.} denotes the expected value.

3. Analysis and Results

The impact of pilot density and spacing on channel estimation error performance is evaluated by analytical [7] and simulation results. Pilot symbols are inserted as 2-D rectangularly. Channel estimate at pilot positions are obtained by LS then complete channel coefficients in frequency and time domain are obtained by using low pass interpolation. Pilot symbols are inserted as equi-spaced and equi-powered in specific intervals both in frequency and time domain. QPSK modulation scheme, N_c=1024 subcarriers, N_s=2048 OFDM symbols, $N_c/4$ cyclically extended guard interval, 20MHz transmission bandwidth, 100 ms frame duration, 5 and 70 Hz Doppler frequency are used in simulations. ITU Pedestrian A and ITU Vehicular A channel models are used to simulate the different multipath fading environments [11] which referred to as channel 1 (Ch1) and channel 2 (Ch2) respectively. The parameters for the channels are given in Table 1.It is assumed that channel impulse response does not change within one OFDM symbol duration, perfect synchronization and linear power amplification is done. The loss in signal to noise ratio due to pilot symbol and cyclic prefix usage is also taken into account, and given by [12];

$$SNR_{loss} = 10\log_{10}\left(\frac{1}{1-\Lambda}\right) \quad (7)$$

where $\Lambda = N_{grid}/N_c N_s$ and N_{grid} is the number of pilot symbols in an OFDM frame.

Table 1. Relative power and delay for the channels

Path	Pedestrian-A		Vehicular-A	
	Delay [ns]	Relative Power [dB]	Delay [ns]	Relative Power [dB]
1	0	0	0	0
2	110	-9,7	310	-1,0
3	190	-19,2	710	-9,0
4	410	-22,8	1090	-10,0
5			1730	-15,0
6			2510	-20,0

Time selectivity of the channels are created through the simulator that uses Clarke and Gans model [13]. The Rayleigh fading envelopes are obtained for 5 and 70 Hz Doppler frequency and carrier frequency of 2 GHz. Then these fadings are applied to all paths. The Rayleigh fading envelopes for 5 and 70 Hz are shown in Fig. 4.

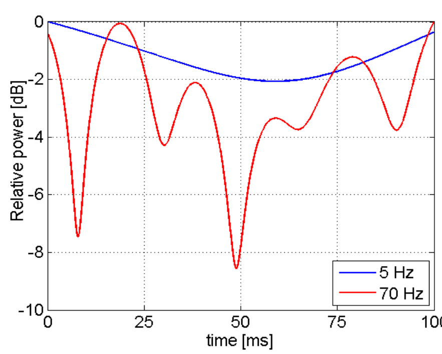

Fig. 4. Rayleigh fading envelopes

The effect of pilot density on channel estimation MSE performance is evaluated for various N_f and N_t, then the N_f and N_t values that gives the minimum MSE are defined as the optimum pilot symbols ($N_{f,opt}$, $N_{t,opt}$). The changes in channel estimation MSEs versus N_f and N_t, for Ch1 and Ch2, 10 dB E_b/N_0 are given in Figs. 5*a* and *b* respectively. The MSEs are not given, and shown as they are not satisfactory when N_t and N_f are outside the observation interval. Doppler frequencies are set to 5 Hz for Ch1, 70 Hz for Ch2. Analytical results are represented by blue surface, simulation results are given in red surface, and the ($N_{f,opt}$, $N_{t,opt}$) values are shown with black dot. Since Ch1's time and frequency selectivity is low, optimum pilot density is obtained at larger N_t and N_f values, and there is no significant change in MSE near the optimum value. However, in the case of the higher time and frequency selectivity as in Ch2, more pilots should be inserted in both directions in order to lower MSE. As seen from the Fig. 5*b* an increase in N_f values causes significant decrease in MSE.

The optimum pilot symbol interval in frequency and time direction can be obtained from $N_{f,opt}$, $N_{t,opt}$ as follows;

$$\Delta f_{opt} = N_{f,opt}.F_s \quad (8)$$

$$\Delta t_{opt} = N_{t,opt}.T_s \quad (9)$$

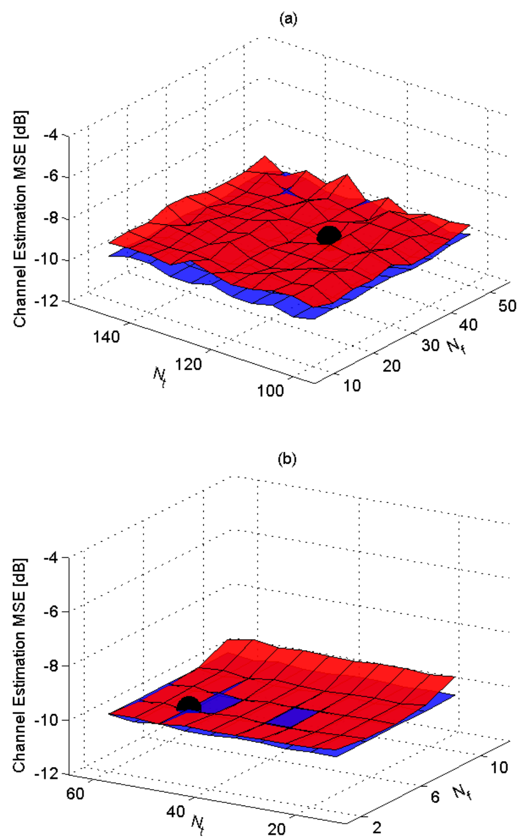

Fig. 5. Channel estimation MSE for a) Ped.-A b) Veh.-A

Similar analyses are performed for 20 dB E_b/N_0, analytical and simulated optimum pilot intervals are determined. The relationship between the calculated values of (Δf_{opt}, Δt_{opt}) for Ch1, Ch2 and channels' coherence bandwidth and coherence time is investigated and the results are indicated in Table 2. It is seen from the results the analytical and simulated optimum values are very close to each other, pilot density, and channel's type of selectivity effect channel estimation error performance.

As seen from the Table $B_{c,0.9}$ is equal to $2.25\Delta f_{opt}$ -$2.72\Delta f_{opt}$ and $T_{c,0.9}$ is equal to $2.38\Delta t_{opt}$-$2.56\Delta t_{opt}$. A more general expression can be obtained by averaging these values, and the ratio between them is calculated as 2.40 and 2.47 respectively.

Although similar investigations are performed for coherence bandwidth and time at 0.5 and 0.75 correlation the results are not given in the Table since the interval of the ratio is much wider. The ratios have shown good agreement with the results in [7].

Table 2.Δf_{opt}, Δt_{opt}, B_c and T_c for Ch1 and Ch2 at 5 and 70 Doppler frequencies

Channel	E_b/N_0[dB]	Doppler Frequency [Hz]	Δf_{opt} [kHz]	Δt_{opt} [ms]	$\frac{B_{c,0.9}}{\Delta f_{opt}}$	$\frac{T_{c,0.9}}{\Delta t_{opt}}$
Ch1	10	5	780	6.00	2.39	2.38
	20	5	682	6.00	2.72	2.38
Ch2	10	70	78	2.50	2.25	2.56
	20	70	78	2.50	2.25	2.56

4. Conclusions

In this study, channel estimation MSE performances are assessed for pilot density, and different types of fading via simulations and analytically [7]. The pilot intervals that give the lowest MSE in frequency and time direction are determined, and related to $B_{c,0.9}$ and $T_{c,0.9}$ respectively. The results demonstrate that, approximately 2.4 pilot symbols per $B_{c,0.9}$, and per $T_{c,0.9}$ can be used for 2-D pilot placement. Therefore with the use of this relationship the optimum pilot density is determined with reduced computational complexity.

5. References

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