

TIME-FREQUENCY CHANNEL EQUALIZATION FOR WIRELESS OFDM SYSTEMS

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ABSTRACT

In wireless OFDM systems, estimation of channel parameters is required to design coherent receivers. In this paper, we present a time-frequency based channel estimation method for wireless OFDM channels. Discrete evolutionary transform of the noisy channel output is used to obtain a complete model of a multi-path, fading and frequency selective channel. In order to estimate the sent data symbols, two forms of receiver are proposed; one in the time and the other in the frequency domain. Performance of the proposed method is tested on different channel environments.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is considered an effective method for broadband wireless communications because of its great immunity to fast fading channels and inter-symbol interference (ISI). It has been adopted in several wireless standards such as digital audio broadcasting (DAB), digital video broadcasting (DVB-T), the wireless local area network (W-LAN) standard; IEEE 802.11a, and the metropolitan area network (W-MAN) standard; IEEE 802.16a [1, 2]. OFDM partitions the entire bandwidth into parallel subchannels by dividing the transmit data bitstream into parallel, low bit rate data streams to modulate the sub-carriers of those subchannels. However, inter-carrier interference (ICI) due to Doppler shifts, phase offset, local oscillator frequency shifts, and multi-path fading severely degrades the performance of multi-carrier

communication systems [1, 3]. For fast-varying channels, especially in mobile systems, large fluctuations of the channel parameters are expected. Estimation of the channel parameters is required to employ coherent receivers. Most of the channel estimation methods assume that the communication channel is either completely time-invariant or it remains time-invariant over an OFDM symbol, which is not valid for fast varying environments [4]. A complete time-varying model of the channel can be obtained by employing TF representation methods. A new time-varying channel estimation method based on the Discrete Evolutionary Transform (DET) is recently presented for OFDM systems [5]. The DET provides a TF representation of the received signal by means of which the time-varying frequency response of the multi-path, fading channel with Doppler shifts can be estimated. In this work, we improve above method and present a sub-sampled technique where the time-varying channel information for each OFDM symbol can be estimated by using channel response to known pilot symbols.

2. WIRELESS CHANNEL MODEL

In wireless communications, the multi-path, fading channel with Doppler frequency shifts may be modeled as a linear time-varying system with the following discrete-time impulse response [6, 7]

$$h(m, \ell) = \sum_{i=0}^{L-1} \alpha_i e^{j\psi_i m} \delta(\ell - N_i) \quad (1)$$

where L is the total number of transmission paths, ψ_i represents the Doppler frequency, α_i is the relative attenuation, and N_i is the time delay caused by path

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i. The Doppler frequency shift ψ_i , on the carrier frequency ω_c , is caused by an object with radial velocity v and can be approximated by $\psi_i \cong \frac{v}{c} \omega_c$ where c is the speed of light in the transmission medium [7]. In wireless mobile communication systems, with high carrier frequencies, Doppler shifts become significant and have to be taken into consideration. Time-varying channel parameters can be estimated in the TF domain by means of the so called spreading function which is related to the time-varying frequency response and the bi-frequency function of the channel. Time-varying transfer function of this linear channel is calculated by taking the discrete Fourier transform (DFT) of the impulse response with respect to ℓ , i.e.,

$$H(m, \omega_k) = \sum_{i=0}^{L-1} \alpha_i e^{j\psi_i m} e^{-j\omega_k N_i} \quad (2)$$

where $\omega_k = \frac{2\pi}{K}k$, $k = 0, 1, \dots, K-1$. The bi-frequency function of the channel $B(\Omega_s, \omega_k)$, is found by computing the DFT of $H(m, \omega_k)$ with respect to m .

$$B(\Omega_s, \omega_k) = \sum_{i=0}^{L-1} \alpha_i e^{-j\omega_k N_i} \delta(\Omega_s - \psi_i). \quad (3)$$

and $\Omega_s = \frac{2\pi}{K}s$, $k = 0, 1, \dots, K-1$. Furthermore, the spreading function of the channel is obtained by calculating the DFT of $h(m, \ell)$ with respect to m , or by taking the inverse DFT of $B(\Omega_s, \omega_k)$ with respect to ω_k ;

$$S(\Omega_s, \ell) = \sum_{i=0}^{L-1} \alpha_i \delta(\Omega_s - \psi_i) \delta(\ell - N_i) \quad (4)$$

which displays peaks located at the TF positions determined by the delays and the corresponding Doppler frequencies, and with α_i as their amplitudes [7]. If we extract this information from the received signal, we will be able to equalize for the channel and estimate the transmitted data symbol.

3. OFDM SYSTEM MODEL

In an OFDM communication system, the available bandwidth B_d is divided into K subchannels. The input data is also divided into K -bit parallel bit streams, and then mapped onto some transmit symbols $X_{n,k}$ drawn from an arbitrary constellation points where n is the time index, and $k = 0, 1, \dots, K-1$, denotes the frequency or subcarrier index. We then insert some pilot symbols, $p_{n,k}$ at some pilot positions (n', k') , known to the receiver: $(n', k') \in \mathcal{P} = \{(n', k') | n' \in \mathcal{Z}, k' = id + (n' \bmod d), i \in [0, P-1]\}$ where P is the number of pilots, and the integer $d = K/P$ is the distance between adjacent pilots in an OFDM symbol [8].

The n^{th} OFDM symbol $s_n(m)$ is obtained by taking the inverse DFT and then adding a cyclic prefix (CP) of length L_{CP} where L_{CP} is chosen such that $L \leq L_{CP} + 1$, and L is the time-support of the channel impulse response. This is done to mitigate the effects of intersymbol interference (ISI) caused by the channel time spread [1, 2].

$$s_n(m) = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} X_{n,k} e^{j\omega_k m} \quad (5)$$

$m = -L_{CP}, -L_{CP}+1, \dots, 0, \dots, K-1$ where again $\omega_k = \frac{2\pi}{K}k$, and each OFDM symbol has $N = K + L_{CP}$ length. The channel output suffers from multipath propagation, fading and Doppler frequency shifts:

$$\begin{aligned} y_n(m) &= \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} X_{n,k} \sum_{i=0}^{L-1} \alpha_i e^{j\psi_i m} e^{j\omega_k(m-N_i)} \\ &= \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} H_n(m, \omega_k) e^{j\omega_k m} X_{n,k} \end{aligned} \quad (6)$$

The transmit signal is also corrupted by additive white Gaussian noise $\eta(m)$ over the channel. The received signal for the n^{th} frame can then be written as $r_n(m) = y_n(m) + \eta_n(m)$. At the receiver the CP is discarded and the signal is demodulated using a K -point DFT:

$$\begin{aligned} R_{n,k} &= \frac{1}{K} \sum_{s=0}^{K-1} X_{n,s} \sum_{i=0}^{L-1} \alpha_i e^{-j\omega_s N_i} \\ &\times \sum_{m=0}^{K-1} e^{j\psi_i m} e^{j(\omega_s - \omega_k)m} + Z_{n,k}. \end{aligned} \quad (7)$$

If the Doppler effects in all the channel paths are negligible, $\psi_i = 0$, $\forall i$, then the channel can be assumed time-invariant within one OFDM symbol. In that case, channel output above becomes,

$$\begin{aligned} R_{n,k} &= X_{n,k} \sum_{i=0}^{L-1} \alpha_i e^{-j\omega_k N_i} + Z_{n,k} \\ &= X_{n,k} H_{n,k} + Z_{n,k} \end{aligned} \quad (8)$$

where $H_{n,k}$ is the channel frequency response, and $Z_{n,k}$ is the Fourier transform of the channel noise for the n^{th} OFDM frame. By estimating the channel frequency response coefficients $H_{n,k}$, data symbols, $X_{n,k}$, can be recovered by a simple equalizer, $\hat{X}_{n,k} = R_{n,k}/H_{n,k}$. However, if there are large Doppler frequency shifts in the channel, then the above time-invariance assumption is no longer valid. Here we consider a completely time-varying channel that might change during an OFDM symbol and approach the problem from a TF point of view [7, 9].

4. CHANNEL ESTIMATION FOR OFDM SYSTEMS

In the following we briefly explain the Discrete Evolutionary Transform (DET) as a tool for the TF representation of wireless channel output.

4.1. Time-Frequency Analysis by DET

A non-stationary signal, $x(n), 0 \leq n \leq N - 1$, may be represented in terms of a time-varying kernel $X(n, \omega_k)$ or its corresponding bi-frequency kernel $X(\Omega_s, \omega_k)$. The TF discrete evolutionary representation of $x(n)$ is given by [10],

$$x(n) = \sum_{k=0}^{K-1} X(n, \omega_k) e^{j\omega_k n}, \quad (9)$$

where $\omega_k = 2\pi k/K$, K is the number of frequency samples, and $X(n, \omega_k)$ is the TF kernel. A similar representation can be given in terms of the corresponding bifrequency kernel $X(\Omega_s, \omega_k)$:

$$x(n) = \sum_{k=0}^{K-1} \sum_{s=0}^{K-1} X(\Omega_s, \omega_k) e^{j(\omega_k + \Omega_s)n} \quad (10)$$

where $X(\Omega_s, \omega_k)$ is the Fourier transform of the above TF kernel with respect to n . The discrete evolutionary transformation (DET) is obtained by expressing the kernel $X(n, \omega_k)$ in terms of the signal. This is done by using conventional signal representations [10]. Thus, for the representation in (9), the DET that provides the kernel $X(n, \omega_k), 0 \leq k \leq K - 1$, is given by

$$X(n, \omega_k) = \sum_{\ell=0}^{N-1} x(\ell) w_k(n, \ell) e^{-j\omega_k \ell}, \quad (11)$$

where $w_k(n, \ell)$ is, in general, a time and frequency dependent window. Specifically, for the representation of multipath wireless channel output, signal-dependent windows that are adapted to the Doppler frequencies of the channel are presented in [5, 9].

4.2. OFDM Channel Estimation

We will now consider the computation of the spreading function by means of the evolutionary transformation of the received signal. The output of the channel, after discarding the cyclic prefix, for the n^{th} OFDM symbol is given in equation (6). Now calculating the DET of $y_n(m)$, we get

$$y_n(m) = \sum_{k=0}^{K-1} Y_n(m, \omega_k) e^{j\omega_k m} \quad (12)$$

The above equation can also be given in a matrix form as, $\mathbf{y} = \mathbf{A}\mathbf{x}$ where

$$\begin{aligned} \mathbf{y} &= [y_n(0), y_n(1), \dots, y_n(K-1)]^T, \\ \mathbf{x} &= [X_{n,0}, X_{n,1}, \dots, X_{n,K-1}]^T, \\ \mathbf{A} &= [a_{m,k}]_{K \times K}, \quad a_{m,k} = H_n(m, \omega_k) e^{j\omega_k m}. \end{aligned}$$

If $H_n(m, \omega_k)$ is known, then $X_{n,k}$ may be estimated by

$$\hat{\mathbf{x}} = \mathbf{A}^{-1}\mathbf{y}. \quad (13)$$

A TF procedure to estimate $H_n(m, \omega_k)$ is explained in the following. Comparing the representations of $y_n(m)$ in (6) and (12), we require that the kernel is,

$$Y_n(m, \omega_k) = \frac{1}{\sqrt{K}} \sum_{i=0}^{L-1} \alpha_i e^{j\psi_i m} e^{-j\omega_k N_i} X_{n,k}. \quad (14)$$

Finally, the channel frequency response for n^{th} OFDM symbol can be obtained as,

$$H_n(m, \omega_k) = \frac{\sqrt{K} Y_n(m, \omega_k)}{X_{n,k}}. \quad (15)$$

Calculation of $Y_n(m, \omega_k)$ in such a way that it satisfies equation (14) is achieved by using signal-dependent windows that are adapted to the Doppler frequencies in [9].

According to (15), we need the input data symbols $X_{n,k}$ to estimate the channel frequency response. This is achieved in [5], by using pilot symbols and a two step procedure. Here we present a computationally efficient method by eliminating above intermediate step. Equation (15) can be given in a matrix form,

$$\mathbf{H} = \sqrt{K} \mathbf{Y} \mathbf{X}^{-1}, \quad (16)$$

where

$$\begin{aligned} \mathbf{H} &\triangleq [h_{m,k}]_{K \times K}, \quad h_{m,k} = H_n(m, \omega_k) \\ \mathbf{Y} &\triangleq [y_{m,k}]_{K \times K}, \quad y_{m,k} = Y_n(m, \omega_k) \\ \mathbf{X} &\triangleq \mathbf{I} \mathbf{x}, \end{aligned} \quad (17)$$

where \mathbf{I} denotes a $K \times K$ identity matrix. Above relation is also valid at the preassigned pilot positions $k = k'$;

$$H'_n(m, \omega_p) = H_n(m, \omega_{k'}) = \frac{\sqrt{K} Y_n(m, \omega_{k'})}{X_{n,k'}} \quad (18)$$

where $p = 1, 2, \dots, P$ and $H'_n(m, \omega_p)$ is a decimated version of the $H_n(m, \omega_k)$. Note that P is again the number of pilots, and $d = K/P$ is the distance between adjacent pilots. Taking the inverse DFT of $H'_n(m, \omega_p)$ with respect to p and DFT with respect to m , we obtain the sub-sampled spreading function $S'(\Omega_s, \ell)$,

$$S'(\Omega_s, \ell) = \frac{1}{d} \sum_{i=0}^{L-1} \alpha_i \delta\left(\frac{\Omega_s - \psi_i}{d}\right) \delta(\ell - N_i) \quad (19)$$

By comparing $S'(\Omega_s, \ell)$ and $S(\Omega_s, \ell)$, we observe that the channel parameters α_i, ψ_i , and N_i calculated from $S(\Omega_s, \ell)$ in [5], can also be estimated from the sub-sampled spreading function $S'(\Omega_s, \ell)$. Hence, the evolutionary kernel $Y_n(m, \omega_k)$ can be calculated directly from $y_n(m)$ and channel parameters can be obtained according to (18) and (19).

4.3. Time-Frequency Receivers

After estimating the spreading function and the corresponding frequency response $H_n(m, \omega_k)$ of the channel, data symbols can be detected using either of the following two receivers:

1. Time domain receiver: $X_{n,k}$ may be estimated using the time domain detector given in (13).

2. Frequency domain receiver: The channel output in the frequency domain can be rewritten as

$$\begin{aligned} R_{n,k} &= \frac{1}{K} \sum_{s=0}^{K-1} \sum_{m=0}^{K-1} H_n(m, \omega_k) e^{j(\omega_s - \omega_k)m} X_{n,s} \\ &+ Z_{n,k} \\ &= \frac{1}{K} \sum_{s=0}^{K-1} B_n(\omega_s - \omega_k, \omega_k) X_{n,s} + Z_{n,k} \end{aligned}$$

where $B_n(\Omega_s, \omega_k)$ is the bi-frequency function of the channel given in (3), and above indicates a circular convolution with the data symbols. It is possible to write above equation in a matrix form as

$$\mathbf{r} = \mathbf{B}\mathbf{x} + \mathbf{z} \quad (20)$$

where $\mathbf{B} = [b_{s,k}]_{K \times K}$, $b_{s,k} = B_n(\omega_s - \omega_k, \omega_k)$ is a $K \times K$ matrix whose rows are shifted versions of the bi-frequency function and, received signal \mathbf{r} , sent data symbol \mathbf{x} and channel noise \mathbf{z} are $K \times 1$ vectors defined by $\mathbf{r} = [R_{n,1}, R_{n,2}, \dots, R_{n,K}]^T$, $\mathbf{x} = [X_{n,1}, X_{n,2}, \dots, X_{n,K}]^T$, and $\mathbf{z} = [Z_{n,1}, Z_{n,2}, \dots, Z_{n,K}]^T$ respectively. Finally, data symbols $X_{n,k}$ can be estimated by using a simple frequency domain equalizer,

$$\hat{\mathbf{x}} = \mathbf{B}^{-1} \mathbf{r}$$

which is an extension of the LTI channel equalizer to the time-varying channel model given in (1).

5. SIMULATIONS

First, we show the estimated spreading function of a 3-path channel shown in Fig. 1 by using 16 pilot symbols in an OFDM frame. We then test the performance of the proposed method over a noisy wireless channel. The number of paths, $1 \leq L \leq 5$, the delays, $0 \leq N_i \leq L_{CP} - 1$ and the Doppler frequency shift $0 \leq \psi_i \leq \psi_{\max}$, $i = 0, 1, \dots, L - 1$ of each path are

picked randomly. Input data is 4-PSK coded and modulated onto $K = 128$ sub-carriers, 8 or 16 of which are assigned to the pilot symbols. The OFDM symbol duration is chosen to be $T = 200 \mu\text{s}$, and $T_{CP} = 50 \mu\text{s}$. Frequency spacing between the sub-carriers is $F = 5 \text{ kHz}$. Signal-to-Noise Ratio (SNR) of the channel noise is changed between 0 and 15 dB, and the BER is calculated by four approaches: 1) No Channel Estimation, 2) Proposed Approach with 8 pilots, 3) with 16 pilots, and 4) Known Channel parameters. Fig. 2 shows the BER versus SNR for $\psi_{\max} = 500 \text{ Hz}$. Notice that the proposed sub-sampled method is able to track the channel parameters even with only 8 pilots. Furthermore, we fix the channel noise to 10 dB, while changing the Doppler shift in each path and calculate the BER performance given in Fig. 3. Note that the proposed method performs well under harsh channel conditions.

6. CONCLUSIONS

In this work, we present a time-varying modeling of the multi-path, fading OFDM channels with Doppler frequency shifts by means of discrete evolutionary transform of the channel output. This approach allows us to obtain a representation of the time-dependent channel transfer function from the noisy channel output. At the same time, using the estimated channel parameters, a better detection of the input data can be achieved. Examples show that, proposed method has a considerably better performance than previous two-step method and PSA channel estimation [5].

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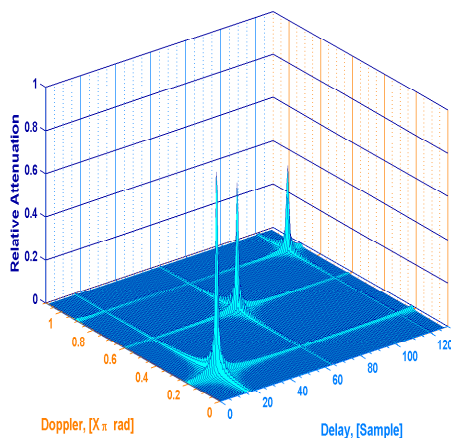


Figure 1: Estimated spreading function for a 3-path channel.

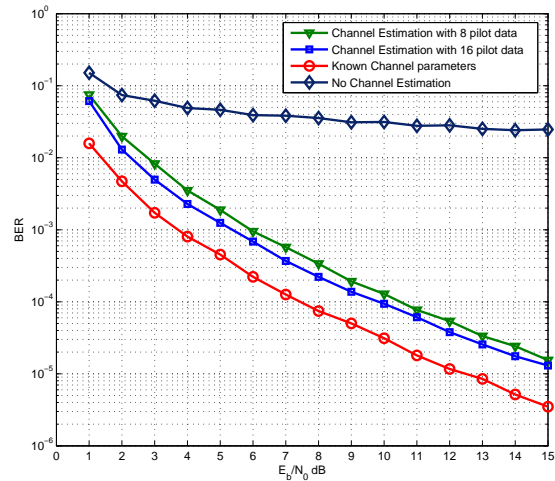


Figure 2: BER versus SNR for 4 different receivers.

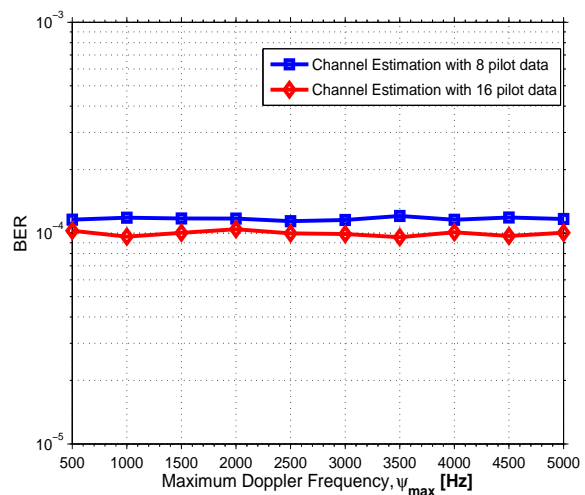


Figure 3: BER versus ψ_{max} for SNR=10 dB.