Speed control of a DC motor using a fractional order sliding mode controller

Mohammad Tabatabaei[1], Saeed Heidarpoor[2]

[1]Department of Electrical Engineering, Khomeinishahr Branch, Islamic Azad University, Isfahan, Iran
tabatabaei@iaukhsh.ac.ir
[2]Department of Electrical Engineering, Khomeinishahr Branch, Islamic Azad University, Isfahan, Iran
saeed.heidarpoor@ iaukhsh.ac.ir

Abstract

In this paper, a fractional-order sliding mode controller for a DC laboratory velocity servo system is established. First, the permanent magnet DC motor is identified through step response data. Then, a fractional-order sliding mode controller based on a fractional-order sliding surface is constructed. The experimental results demonstrate the performance of the proposed non-integer sliding mode controller in both command following and load disturbance rejection.

1. Introduction

Direct Current (DC) motors have been widely used in industrial applications due to their reliability and simple modelling. Permanent Magnet DC (PMDC) motors could be utilized as actuators in control loops and robot manipulators. A lot of control strategies have been proposed in the literature to construct position or velocity servo systems based on DC motors. Position tracking of a DC motor by an adaptive Proportional-Integral-Derivative (PID) controller implemented on a Field-Programmable-Gate-Array (FPGA) chip has been performed [1]. State and output feedback based adaptive fuzzy controller was utilized to control position of DC motors [2].

The simple structure and robustness of sliding mode controllers has made them as popular choices to control real plants [3]. Thus, these controllers have been employed in DC motor servo systems. In [4], a sliding mode controller has been utilized to control a DC servo mechanism with unmodeled stator and sensor dynamics. Fuzzy model based sliding mode controller has been employed in this regard, too [5]. Two different sliding mode approaches for angular position tracking of a DC motor have been provided based on the Slotine's and Utikn's works [6]. A discrete sliding mode controller with disturbance compensation has been proposed to control a velocity servo system [7].

On the other hand, fractional-order controllers have made a major contribution in modelling and control of real plants [8]. Fractional-order Sliding Mode Controller (FO-SMC) as the non-integer version of ordinary sliding mode controller is one of these controllers. The performance of the sliding mode controller for multi-input multi-output nonlinear systems has been improved through the mentioned FO-SMC [9]. In a recently published work, the fractional-order sliding surface was utilized for robust control of perturbed integer-order Linear-Time-Invariant (LTI) systems [10]. Due to this non-integer sliding surface, the chattering phenomenon was significantly reduced. The efficiency of FO-SMC for velocity control of a permanent magnet synchronous motor [11] and position control of a DC motor has been verified [12].

In the current paper, a permanent magnet DC motor is modeled through a parametric identification approach. Then, a fractional order proportional integral (FO-PI) sliding surface is considered to attain a fractional order sliding mode controller. Then, the armature voltage is determined to guarantee the reaching condition in this FO-SMC structure. The designed controller is applied to a real modular servo mechanism system through an Advantech hardware interface. The experimental results in MATLAB Real-Time toolbox are given to show the speed tracking efficiency of the proposed method in the presence of external load disturbance.

The outline of this paper is given as follows. The mathematical model of the DC motor is given in section 2. Section 3 introduces a brief review on preliminary concepts in the fractional calculus. The proposed FO-SMC for velocity control of DC motor is illustrated in section 4. Experimental results are provided in section 5. Finally, section 6 concludes the paper.

2. Mathematical model of DC motor

In this paper, a permanent magnet DC (PMDC) motor is considered. This means that the field current is constant and the electric torque ($T_e(t)$) is only proportional to the armature current ($i_a(t)$). Or:

$$T_e(t) = k_m i_a(t) \tag{1}$$

where k_m is the permeability constant of the magnetic material. The armature current ($i_a(t)$) and the applied armature voltage ($v_a(t)$) are related to each other through the following relation

$$v_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + v_b(t) \tag{2}$$

where R_a and L_a are the armature resistance and the armature inductance, respectively. Moreover, $v_b(t)$ is the back-electromotive force voltage related to the motor angular velocity ($\omega(t)$) according to the following expression

$$v_b(t) = k_b \omega(t) \tag{3}$$

where k_b is the back-electromotive force coefficient. The following relation between the electric torque ($T_e(t)$) and the motor shaft speed ($\omega(t)$) could be written

$$T_e(t) = J\frac{d\omega(t)}{dt} + B\omega(t) + T_L(t) \quad (4)$$

where J is the moment of the inertia, B is the damping coefficient and $T_L(t)$ is the load torque. The transfer function for armature controlled DC motor ($G(s)$) with input $v_a(t)$ and output $\omega(t)$ is obtained according to relations (1), (2), (3) and (4) (considering $T_L(t) = 0$) as follows

$$G(s) = \frac{\omega(s)}{V_a(s)} = \frac{k_m}{(L_a s + R_a)(Js + B) + k_b k_m}\,. \quad (5)$$

Neglecting the armature time constant ($\tau_a = \frac{L_a}{R_a}$), yields the following approximated version of the transfer function (5)

$$G(s) = \frac{\omega(s)}{V_a(s)} = \frac{k}{\tau s + 1} \quad (6)$$

where

$$k = \frac{k_m}{R_a B + k_b k_m} \quad , \quad \tau = \frac{R_a j}{R_a B + k_b k_m}\,. \quad (7)$$

The parameters of the first-order transfer function (6) could be estimated through a parametric system identification approach.

3. Brief review on fractional calculus

Fractional calculus deals with the generalization of ordinary derivative and integral operators to their corresponding non-integer ones. The popular Caputo definition for the fractional order derivative of an arbitrary function $f(t)$ is given by [8]

$$D^{\rho} f(t) = \frac{1}{\Gamma(n-\rho)}\int_0^t \frac{f^{n}(\tau)}{(t-\tau)^{1+\rho-n}}d\tau, \qquad (n-1 < \rho \le n) \quad (8)$$

where ρ is the fractional order, $\Gamma(.)$ denotes the Gamma function, and n is an arbitrary integer number. If $\rho = 1$, then the ordinary derivative will be obtained.

To implement fractional order controllers, several methods have been proposed in the literature. Approximating these operators with high order integer transfer functions is one of these methods [13]. The “Ninteger” MATLAB toolbox constructed based on this idea is utilized here to implement fractional order derivatives and integrals [13].

4. Fractional-order sliding mode controller design

To design FO-SMC, a fractional order sliding surface and an appropriately control signal are designed so that the state trajectories reach to the sliding surface and remain on it. Let to define the speed error (e) as

$$e = \omega_r - \omega \quad (9)$$

where ω_r is the constant command speed. According to relations (6) and (9), the following speed error dynamic is obtained

$$\frac{de}{dt} = -\frac{e}{\tau} + \frac{\omega_r}{\tau} - \frac{k}{\tau}v_a(t)\,. \quad (10)$$

Now, the following fractional-order sliding surface is chosen

$$s = e + cD^{-\alpha}e \quad (11)$$

where c is a positive constant determinates the sliding speed and α is the fractional order belongs to $(0,1]$. If $\alpha = 1$, then ordinary sliding mode controller is obtained. Increasing the value of c could increase the sliding speed of state trajectories through sliding surface to zero. To ensure the reaching condition, the following inequality should be satisfied

$$s\dot{s} < 0\,. \quad (12)$$

The following theorem gives the control signal ($v_a(t)$) satisfying the reaching condition (12).

Theorem 1: the reaching condition (12) is fulfilled if the armature voltage is chosen according to the following equation

$$v_a(t) = \frac{\omega_r}{k} - \frac{e}{k} + \frac{c\tau}{k}D^{1-\alpha}e + \frac{F\tau}{k}sgn(s) \quad (13)$$

where F is an arbitrary positive constant and $sgn(.)$ is the sign function defined as

$$\mathrm{sgn}(x) = \frac{|x|}{x}\,. \quad (14)$$

Proof: differentiating sliding surface equation (11) yields

$$\dot{s} = \dot{e} + cD^{1-\alpha}e\,. \quad (15)$$

Substituting relations (10), (13) and (14) into (15) gives

$$\dot{s} = -F\,\mathrm{sgn}(s)\,. \quad (16)$$

According to (14) and (16), the left side of inequality (12) becomes

$$s\dot{s} = -F|s|\,. \quad (17)$$

It is obvious that the right side of expression (17) is negative. Thus, the proof is completed.

The parameter F could determine the reaching speed. The next section shows the ability of the so-designed FO-SMC in velocity control of a DC servo motor.

5. Experimental results

The DC modular servomechanism system components are shown in Fig.1. A permanent magnet DC motor coupled with a tachometer to measure angular velocity are utilized. An Advantech A/D and D/A interface is used for hardware implementation of the FO-SMC in MTLAB real time environment. A magnetic brake is considered as an external load disturbance.

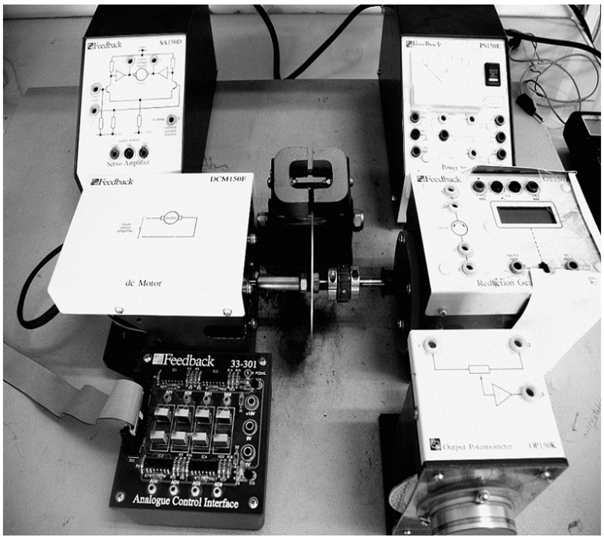

Fig. 1. The DC modular servomechanism system

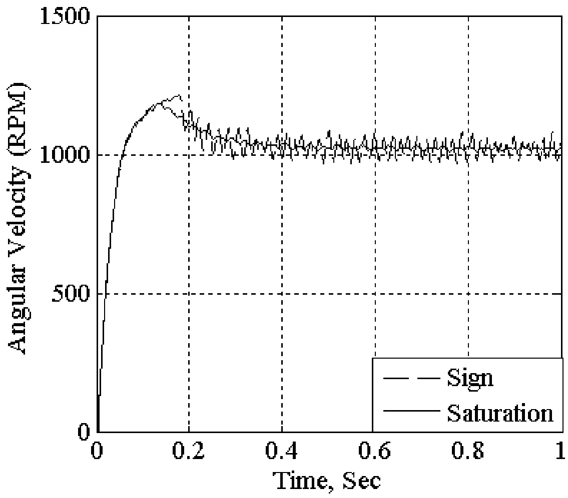

Fig. 2. The angular velocities obtained with sign and saturation functions

The corresponding parameters for the first-order plant transfer function (6) are estimated through its step response. The steady sate gain k and the time constant τ are equal to the steady state and the time constant of the DC motor unit step response, respectively. The obtained parameters are

$$k = 3.45, \quad \tau = 0.1 . \tag{18}$$

The sign function in control signal (13) could be replace with the following saturation function to decrease the chattering phenomenon in sliding mode control structure.

$$sat(x) = \begin{cases} sgn(x) & |x| > \lambda \\ \dfrac{x}{\lambda} & |x| < \lambda \end{cases} \tag{19}$$

where λ is a boundary layer considered around the switching surface to decrease the chattering effect. It could be easily investigated that the reaching condition is satisfied in this case, too.

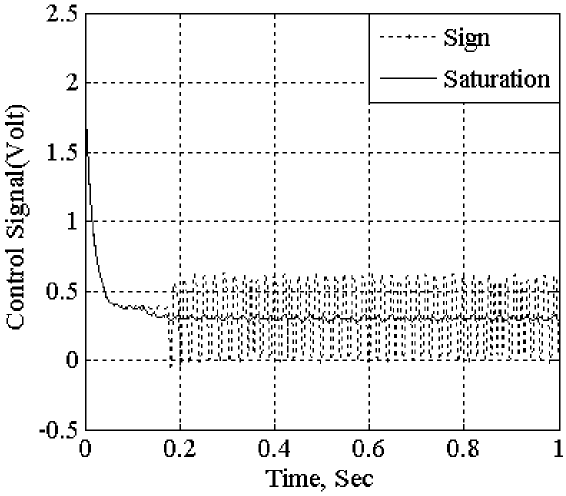

Fig. 3. The control signals obtained with sign and saturation functions

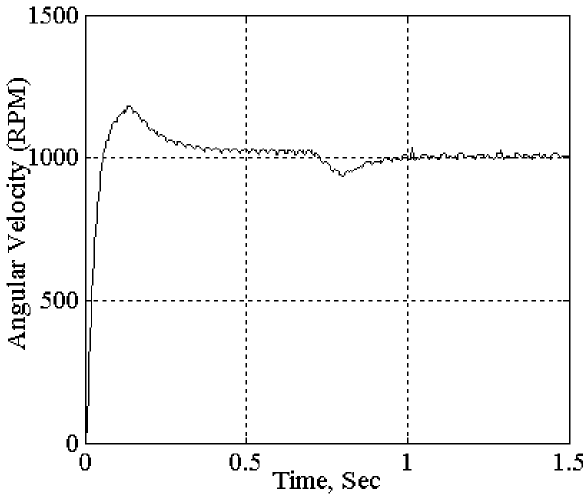

Fig. 4. The load disturbance effect on angular velocity

The sliding mode parameters are chosen as $F = 20, c = 25, \lambda = 1$. Moreover, the optimal value of the fractional order is obtained based on minimization of the following performance index.

$$J = \int_0^T e^2(t)dt \tag{20}$$

where T is the settling time. Due to this optimization approach, $\alpha = 0.85$ is obtained. The reference velocity is considered as $\omega_r = 1000\, RPM$ in all tests. Fig. 2 compares the effect of the sign and saturation functions in sliding mode controller structure. As could be seen from Fig. 2, the chattering phenomenon is significantly reduced by the saturation function. The corresponding control signals shown in Fig. 3 confirm this fact, too. To investigate the load disturbance rejection capability, the magnetic brake is applied in $t = 0.7\,\text{sec}$. Fig. 4 shows that the effect of load disturbance is quickly eliminated.

6. Conclusions

Incorporating fractional order integrator in sliding surface dynamic converts the sliding mode controller to its fractional-order version. The performance of this FO-SMC scheme is verified through experimental tests applied to a laboratory PMDC motor. The load disturbance rejection and reference velocity tracking capabilities makes the proposed method as a good choice for the mentioned speed servo mechanism system. Employing fractional-order adaptive sliding mode controller in DC speed servomechanism could be considered as a future work in this area.

7. References

[1] C. F. Hsu, and B. K. Lee, "FPGA-based adaptive PID control of a DC motor driver via sliding-mode approach", *Expert Syst Appl*, vol. 38, no. 9, pp. *11866-11872,* Sept., 2011.

[2] G. G. Rigatos, "Adaptive fuzzy control of DC motors using state and output feedback", *Electr Pow Syst Res*, vol. 79, no. 11, pp. *1579-1592,* Nov., 2009.

[3] A. Pisano, and E. Usai, "Sliding mode control: A survey with applications in math", *Math Comput Simulat*, vol. 81, no. 5, pp. *954-979,* Jan., 2011.

[4] J. X. Xu, T.H. Lee, and Y.J. Pan, "On the sliding mode control for DC servo mechanisms in the presence of unmodeled dynamics", *Mechatronics*, vol. 13, no. 7, pp. *755-770,* Sept., 2003.

[5] M. J. Knight, and R. Sutton, "Fuzzy model based sliding-mode control of a d.c. servomechanism", *P I Mech Eng I-J Sys*, vol. 218, no. 3, pp. *211-225,* May., 2004.

[6] A. Susperregui, G. Tapia, and A. Tapia, "Application of two alternative sliding-mode control approaches to DC servomotor position tracking ", *IET Electr Power Appl* , vol. 1, no. 4, pp. *611-621,* July., 2007.

[7] C. Milosavljevic, B. Perunicic-Derazenovic, and B. Veselic, "Discrete-time velocity servo system design using sliding mode control approach with disturbance compensation", *IET Trans Industr Inform*, vol. 9, no. 2, pp. *920-927,* May., 2013.

[8] I. Podlubny, "Fractional Differential Equations", Academic Press, San Diego, CA, USA, 1999.

[9] M. O. Efe, and C. Kasnakoglu, "A fractional adaptation law for sliding mode control", *Int J Adapt Control Signal Process*, vol. 22, no. 10, pp. *968-986,* Dec., 2008.

[10] M. L. Corradini, R. Giambo, and S. Pettinari, "On the adoption of a fractional-order sliding surface for the robust control of integer-order LTI plants", *Automatica*, vol. 51, no. A1-A4, pp. *364-371,* Jan., 2015.

[11] B. Zhang, Y. Pi, and Y. Luo, "Fractional order sliding-mode control based on parameters auto-tuning for velocity control of permanent magnet synchronous motor", *ISA Trans*. vol. 51, no. 5, pp. *649-656*, Sept., 2012.

[12] W. Khan, "A comparison between integer and fractional order algebraic approaches for on-line parameter estimation and position control of DC motor", in *11th International Bhurban Conference on Applied Sciences and Technology.*, Islamabad., 2014, pp. *103-107.*

[13] D. Valério, and J. S. Da Costa, " Ninteger: a non-integer control toolbox for Matlab", in *Proceedings of the 1st IFAC Workshop on Fractional Differentiation and its Applications*, Bordeaux, 2004.