ILLUSTRATION OF ROTOR POLE ALIASING EFFECT IN INDUCTION MOTOR LINE CURRENT SPECTRUM

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Abstract

In this study a general method for obtaining the induction motor line current spectra is developed. The airgap harmonic frequencies of a three phase squirrel cage induction motor are studied analytically using mmf-permeance method. This approach gives detailed information about the airgap harmonic frequencies, corresponding pole pairs and the originating mechanism such as slotting, saturation or excitation. Using the flux density harmonics the associated voltage and current harmonics are obtained. The spatial aliasing effect is explained and illustrated and the equivalent circuits used to cover pole aliasing are given. In order to verify the analysis experimental measurements are carried out on a 30 kW three phase induction machine under different operating conditions. Good correlation is found between theoretical and experimental results.

I. Introduction

The source of harmonic components in an induction machine line current waveform can be divided into two main areas: the space harmonic components and the time harmonic components. Space harmonics are produced by sinusoidal currents received from the supply. They result when the stator windings do not possess perfect distribution. Several authors have studied the harmonics produced by stator-rotor slotting [1, 2]. If the voltage supplied to a poly-phase induction motor contains harmonics, the airgap flux may have components that rotate at speeds other than corresponding to fundamental frequency. These induce fundamental voltages. The non-linear effect of saturation can produce harmonics of different pole number rotating in synchronism with the fundamental flux [3, 4].

The line current spectrum may also contain harmonics at a frequency determined by supply voltage harmonics [5]. The time harmonic voltages present in the mains supply are usually small in proportion to the fundamental voltage. The reactance of the motor to time harmonics is high. Consequently the amplitudes of the time harmonic currents are small. Conners et al. [5] are among many authors who discuss and analyse the effects of supplying induction motors from a solid state adjustable frequency controller which results in the input voltage being far from sinusoidal.

Even when the voltage input to an induction motor is sinusoidal, the line current will not normally be sinusoidal. When saturation is absent, the motor line current will still contain a number of harmonics of varying amplitudes and frequencies from a variety of different sources. The two main sources of these harmonics are mmf variations and permeance variations. Each of these sources has a number of causes and produces a number of different harmonics. The sources of some of these flux density harmonics and their effect on performance is discussed by Binns et al [2].

Since all the harmonics are associated with variations in the physical airgap of the machine, the airgap flux density is modulated and stator currents are generated at predictable frequencies related to electrical supply, motor rotational speed and source of the harmonics. The prediction of the complete harmonic components in the line current spectrum of an induction motor requires an accurate description of the distribution of the flux density around the motor and its variation both in space and time.

2. Method of Analysis

The general form of the analysis in this study is based on the concept of expressing the mmf distribution acting on the airgap as a series of Fourier components and the airgap permeance as a constant term plus a series of Fourier components which are multiplied together to produce the harmonic series of airgap flux density distribution.

The three phase currents in induction motor set up an mmf varying with time, alternating around the circumference, so in addition to the fundamental pole pair *p*, space harmonics of *np* pole pair are produced. The permeance of the airgap is not uniform being altered by slotting, saturation and eccentricity. The mmf waves interact with the constant term of the permeance and produce stator fundamental wave and the winding flux density harmonics. The number of pole pairs of these harmonics is equal to the number of pole pairs of the mmf waves. In addition all the mmf waves interact with each permeance wave, inducing further flux density waves of pole number and frequency equal to the sum or difference of the corresponding orders of the stator mmf and permeance waves. These flux density waves passing through the airgap, act on the rotor bars inducing in it voltages. The induced voltages circulate currents through the rotor bars and end rings producing rotor mmf waves. As a result of the interaction between these mmf waves and permeance waves of the airgap a new set of flux density waves are generated. The component of these flux density waves which is of the same order as the stator flux density wave inducing the current and which rotates at the same speed and in the same direction reacts directly with that stator flux density wave. The other rotor fields, which have different pole numbers to the inducing stator flux density wave develop voltages in the stator windings with frequencies different from that of the supply frequency if the rotor is moving.

The rotor and stator are both slotted and magnetically operate in a complex manner. The process of how the flux density distribution is calculated in the airgap is shown in Figure 1 which outlines the development of the algorithm for computation.

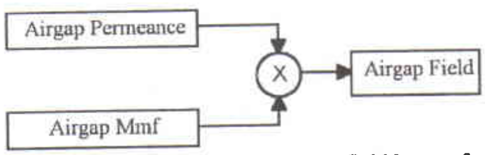

Figure 1: Computation of airgap field by mmf permeance wave technique

If the permeance wave is denoted by $\Lambda_T(\theta,t)$, the flux density in the airgap for a given scalar magnetic potential difference $F_T(\theta,t)$ is obtained by the expression

$$B(\theta,t) = \Lambda_T(\theta,t)F_T(\theta,t) \quad (1)$$

Where $\Lambda_T(\theta,t)$ is the total permeance wave including rotor and stator slotting, eccentricity and saturation which are represented as series normalised to the mean permeance value. The harmonic series coefficients are then multiplied together and the resulting modulation process generates further frequencies and pole numbers of permeance waves.

3. The Induced Voltage and Current

With the flux density given in equation (1), the rms value of the harmonic voltage induced in a winding is given by [6]

$$v_n = N_{ph}\frac{DL}{n}B_n kw_n\frac{f_n}{\sqrt{2}} \quad (2)$$

where N_{ph} is the number of turn per phase, D is diameter, L is the length of the machine kw is the winding factor and f is the frequency of the harmonic.

This voltage will result in current of the same frequency. Therefore, it can clearly be seen that the harmonics with different frequencies contained in the flux density are reproduced in the line voltage and current.

If the supply impedance is defined as Z_{so} then the expression for the fundamental current is the voltage divided by the stator self impedance ($R^*_s + jX^*s$) and the supply impedance. Therefore the expression for current can be written as

$$i = \frac{v}{R^*_s + R_{so} + j(X^*_s + X_{so})} \quad (3)$$

For the harmonics with frequency other than fundamental the induced voltage with np poles appears across the n^{th} magnetising branch. For these voltages, the stator circuit is virtually short circuited via the supply and the resulting current can be calculated using the equivalent circuit as shown in Figure 2. In this equivalent circuit all the impedances, which appear as inductive reactance for higher order harmonics except for supply frequencies, are corrected by the frequency of the voltage. In Figure 2 s_h is the slip of the harmonic and f_h is the frequency of the harmonic voltage induced in the stator and f is the fundamental frequency of the supply.

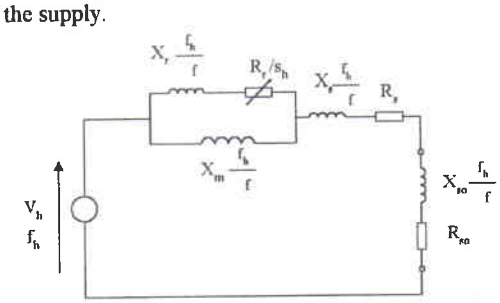

Figure 2: The equivalent circuit for the current harmonics

4. Spatial Aliasing Effect

Spatial aliasing is a feature not considered previously in the mmf-permeance approach but included automatically in time stepping FE or permeance network methods. Aliasing is familiar in digital systems when signals or noise are sampled below the Nyquist rate. The signal frequency is apparently changed by being folded about the Nyquist frequency of the actual sampling rate. The rotor slots (bars) acts as the sampling mechanism to the field distribution spatially sampling the flux density waves produced by interaction of say the constant, or dc, permeance term with the stator mmf harmonic series. That is the normal space harmonics of airgap flux density. Once these resulting stator, airgap, flux density waves have an harmonic pole number exceeding half the number of rotor slots per pole pair (the equivalent in spatial terms of the Nyquist frequency) there will be spatial aliasing on the rotor. That is, the rotor will respond to a given pole number of flux wave by producing a different pole number of induced emf distribution and mmf distribution in the bars. This leads to an harmonic chain equivalent circuit which differs from the traditional one.

The modified equivalent circuit for the fundamental time harmonic is presented in Figure 3. The circuit has the usual stator winding resistance and leakage reactance. The chain format is similar to the traditional equivalent circuit. The chain circuit of impedance only covers harmonics to which the rotor can react by producing the same number of harmonic poles.

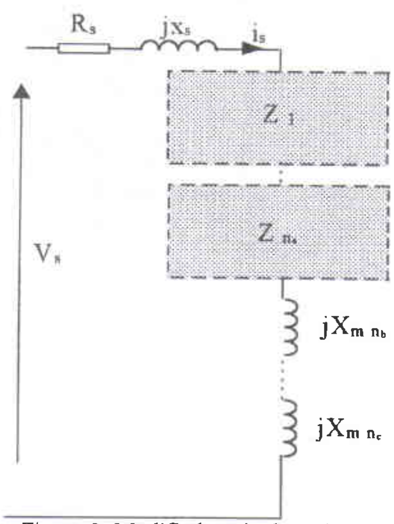

Figure 3: Modified equivalent circuit

The order of spatial harmonic that the rotor can react to is determined by the number of samples that are sampled in one cycle of the field distribution which is in this case determined by the number of rotor bars and number of poles of the machine. The upper harmonic for this range, n_a is the highest harmonic which does not exceed half the number of rotor bars (R) per pole pair (p). Thus:

$$n_a \leq \frac{R}{2p} \tag{4}$$

The remaining space harmonics above n_a up to the highest stator slot harmonic, are represented by their magnetising reactances only since the rotor cannot react directly to produce a similar opposing flux wave with the same pole number.

4.1 Time Harmonic Circuit to Cover Spatial Aliasing

Space harmonics greater than n_a do induce voltage in the rotor bars. The pole number of the mmf induced in the rotor is not the same as the inducing stator flux wave. The induced rotor voltages produce rotor currents which in turn produce mmfs and an airgap flux which induces new frequencies in the stator. These new frequencies are dependent upon rotor angular velocity. The equivalent circuit for this interaction is shown for the n^{th} harmonic referred to the stator with h poles after aliasing in Figure 4. The stator produces the flux to induce the generated voltage in the rotor but the energy for the rotor and induced stator current flow comes from the drag torque imposed on the rotor. This is the same as for generation in a synchronous machine. Accordingly the equivalent circuit is similar to that of a non salient synchronous machine with damper windings operating asynchronously. Obviously when spatial aliasing yields space harmonics (h) which are multiples of 3 no interaction takes place and Figure 4 does not apply.

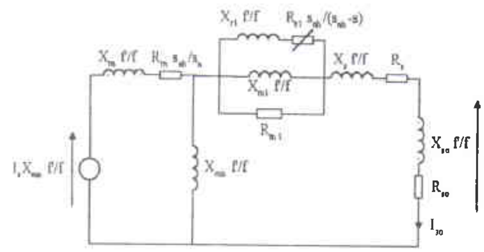

Figure 4: Harmonic Circuit for Spatial Aliasing Harmonics

The harmonic reactances depend upon the frequency of the induced currents. However, these are generally small since although the stator can react to produce a pole number equal to that of the inducing rotor flux (the common magnetising branch of Figure 4), unlike the rotor, it is primarily distributed to produce a fundamental pole number. The fundamental pole number airgap inductance thus forms part of the leakage inductance but is shunted of course by a high slip rotor circuit. The remainder of the circuit comprises the stator resistance and leakage inductance and the power system or source resistance and inductance. The interaction is obviously complex. However it is most significant for the case of rotor pole aliasing where space harmonics of fundamental pole number are produced.

The voltage V_{so} and current i_{so} are respectively induced across and flow through the harmonic impedance of the supply. The slip of the harmonic is given by

$$s_{nh} = 1-(1-s)(n-h) \tag{5}$$

which is the slip of the n^{th} harmonic induced h^{th} harmonic. The frequency f' is given by

$$f' = [1-(1-s)(n-h)]f \tag{6}$$

where s is the fundamental slip, f is the fundamental frequency, n is the order of the harmonic and h is the order of the harmonic after aliasing. It is clear that when there is no pole aliasing these equations become the usual equations for n^{th} harmonic as

$$s_n = 1-(1-s)n \tag{7}$$

which is the slip of the n^{th} space harmonic.

Table 1 tabulates the frequencies due to pole aliasing resulting from an induction motor with a 48 slot stator and 56 slot rotor under full load. Without pole aliasing, only the slot harmonic frequency and saturation harmonic frequency side bands to the slot harmonics appear as speed dependent in the line current spectrum. However, when spatial aliasing is considered then many frequency components are spread over the spectrum as in Table 1.

150	1134.6
245.8	1230.4
343	1330.4
345.8	1430.4
443	1530.4
543	1626.2
837.4	1723.4
934.6	1726.2
1034.6	1823.4

Table 1: Frequencies due to spatial aliasing

4.2 Representation of Spatial Aliasing

Spatial aliasing is best illustrated with the examples of Figure 5 and 6. Here, in Figure 5 a 5^{th} harmonic flux wave is interacting with a rotor of 20 bars per pole pair which yields a fifth harmonic pole distribution (i.e. no aliasing). However, in Figure 6, 19^{th} harmonic flux wave is interacting with the same number of rotor bars, the resulting sampled flux waves can be seen to provide a first harmonic harmonic pole distribution.

The harmonic pole number after aliasing (h) is given in terms of the spatial harmonic n and the number of rotor slots R in the p pole pair machine by [6]

$$h=\frac{R}{p}-n \tag{8}$$

This equation covers the harmonics of order up to $3R/2p$. The more general expression which covers all the harmonics is

$$h=(k+1)\frac{R}{2p}-n \tag{9}$$

where

$$k\frac{R}{2p} < n \le (k+2)\frac{R}{2p}\ , \quad k=1,3,5... \tag{10}$$

Two harmonics of order n_1 and n_2 can be seen as the same by the rotor bars if (n_1-n_2) or (n_1+n_2) is a multiple of rotor bar per pole pair $cR/2p$, c is an integer.

The effect does not change the frequency of the currents induced in the bars just the distribution of the current magnitudes in space. The aliasing effect of the rotor bar sampling is obviously directly related to the spatial position of those bars and therefore the mechanical speed of rotation. Consequently, harmonic voltages induced in the stator by the rotor flux waves consequent upon aliasing will have frequencies which are functions of the mechanical speed of rotation.

The phase sequence of the aliased harmonic is the same as that of the harmonic which induced the spatial aliasing. This is clearly seen in Figure 7 which illustrates the 19^{th} harmonic interacting with 40 slot rotor and the resulting aliased harmonic at 3 different phase angle. As can be seen both 19^{th} and aliased harmonics move forward in the same direction with increasing phase angle.

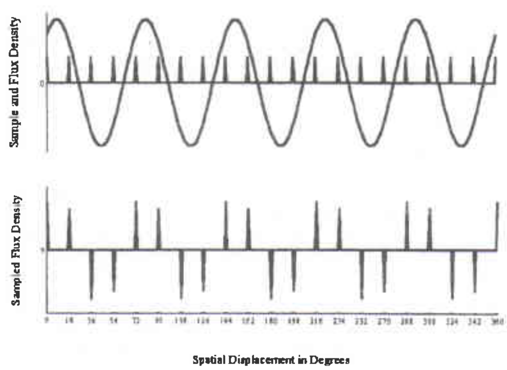

Figure 5: Fifth harmonic interacting with 20 bars and spatially sampled flux wave

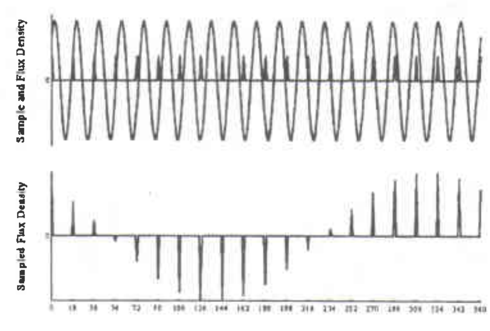

Figure 6: Nineteenth harmonic interacting with 20 bars and spatially sampled flux wave

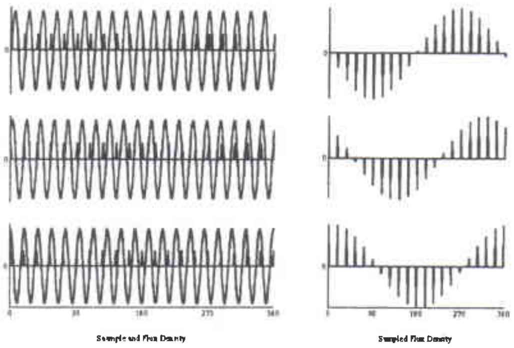

Figure 7: 19th harmonic field interacting with 40 slot rotor and resulting aliased harmonic at different phase angles

5. Results

Figure 8 shows a typical spectrum from an unloaded induction machine. The frequencies f1 and f2 are the principle slot harmonics, one of which (f1) is very small having a pole number of its principle harmonic flux source which results in zero sequence induced voltages.

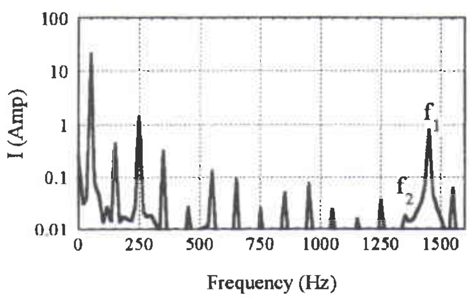

Figure 8: Spectrum of induction motor under no load

The Figure 9 shows part of the predicted spectrum centred around the rotor slot harmonics for the unskewed 56 closed slot rotor for three different loading conditions.

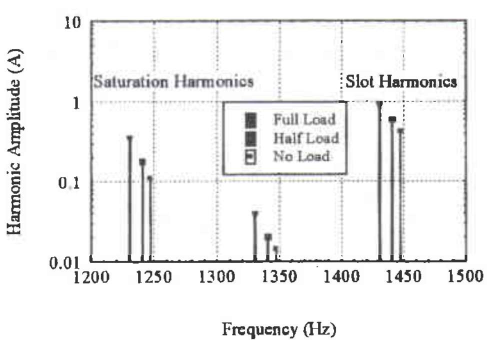

Figure 9:Predicted part spectrum 56 closed slot rotor

The spectral line frequencies decrease with increasing load in each group of harmonic components. There is very good agreement between the predicted and measured slot harmonic amplitudes Figure 10. If the normal classical theory is taken into account the set of smaller harmonics around 1350 Hz should not exist. These come from pole aliasing effects. The set of harmonics around 1250 Hz region occurs as a consequence of interaction of harmonics due to slotting with those arising from saturation.

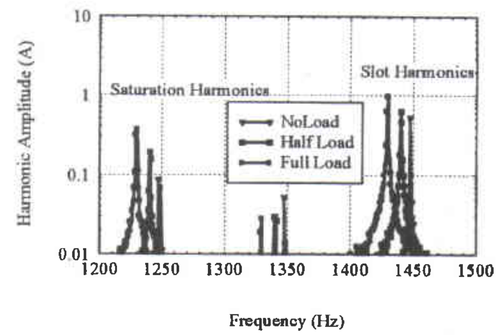

Figure 10:Measured part spectrum 56 closed slot rotor

6. Conclusion

The complex nature of the interactions of mmf and permeance harmonics in the airgap field of induction machines has been clearly demonstrated. The spatial aliasing effect has been explained and illustrated and the equivalent circuits used to cover pole aliasing have been given. The inclusion of the spatial aliasing has been shown to introduce additional speed dependent harmonics in to the line current spectrum.. An experimental investigation has been conducted on a 30 kW induction machine with 56 closed slot unskewed rotor.

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