INSTANTANEOUS POWER IN UNBALANCED THREE PHASE SYSTEMS

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Abstract: In this paper an investigation is carried out for the sinusoidal oscillations on the three-phase instantaneous power under unbalanced operating conditions. This study shows that under perfectly balanced conditions instantaneous power equals to active power, whereas under unbalanced conditions instantaneous power oscillates at twice the power system frequency due to the negative sequence component resulting from unbalanced system operation.

I. INTRODUCTION

An unbalanced condition usually deteriorates the performance of the power systems, causes voltage asymmetry that may be harmful for the loads connected to the system, and contributes to a deterioration of the supply quality. Therefore several research studies have been done on this subject [1-11]. For unbalanced loaded power systems power definitions and equations have been a matter of existing controversy and discussions for decades [1-11].

In recent years there has been a discussion on sinusoidal oscillations at twice the system frequency on three-phase instantaneous power. While the some of the publications [2,3] state that this is because of the unbalanced conditions, in some other publications [4,5,6] it is noted that these oscillations are not created by unbalanced conditions and the reason is not known yet.

On the other hand, more recently some researches [12-14] are going on for power based protection algorithms for the protection of power generation plants [12-14]. These studies have made some of the power-related events much more important for especially non-sinusoidal and unbalanced system conditions.

In this paper an investigation is carried out to find out what causes the sinusoidal oscillations on the instantaneous power. A mathematical equation is derived to calculate the instantaneous power using symmetrical component to clarify the relationships between these oscillations and unbalanced operation of the power systems.

II. POWER IN THREE PHASE SYSTEMS

Calculation of Instantaneous Power Using Direct Phase Quantities

When a voltage source supplies a linear load in a single phase sinusoidal system, the voltage source and the load current are represented by:

$$v = \sqrt{2}V\sin(\omega t) \tag{1}$$

$$i = \sqrt{2}I\sin(\omega t - \varphi) \tag{2}$$

Using the voltage and current equations the instantaneous power in a single-phase circuit can be obtained as follows.

$$p = vi = P[1-\cos(2\omega t)] - Q\sin(2\omega t) \tag{3}$$

Where
$P = VI\cos\varphi$, $\quad Q = VI\sin\varphi$

This equation shows that instantaneous power in the single-phase system can be separated in two parts. First part has an average value of $VI\cos\varphi$ and an alternating component on it oscillating at twice the system frequency. This term never becomes negative and therefore is a unidirectional power. Second term is also an alternating component, which oscillates at twice the system frequency. It has a peak value of $VI\sin\varphi$ and also an average value of zero.

On the other hand, the instantaneous power in a three-phase linear system [11,12] can be calculated as follows:

$$p = v_a i_a + v_b i_b + v_c i_c \tag{4}$$

Where, v_a, v_b, v_c, i_a, i_b, and i_c are the three phase voltages and currents respectively. When a three-phase voltage source supplies a three-phase asymmetrical linear load, the voltages and the load currents can be represented as:

$$\begin{aligned} v_a &= \sqrt{2}V\sin(\omega t) \\ v_b &= \sqrt{2}V\sin(\omega t - 120^\circ) \\ v_c &= \sqrt{2}V\sin(\omega t + 120^\circ) \end{aligned} \tag{5}$$

$$i_a = \sqrt{2} I_a \sin(\omega t - \varphi_a)$$
$$i_b = \sqrt{2} I_b \sin(\omega t - 120^\circ - \varphi_b) \quad (6)$$
$$i_c = \sqrt{2} I_c \sin(\omega t + 120^\circ - \varphi_c)$$

Using the equation (4), (5), and (6) the three-phase instantaneous power will be [12]:

$$p = P_{av} + P_{2m} \cos(2\omega t + \beta) \quad (7)$$

This equation shows that instantaneous power consists of a dc term called average power P_{av}, and a sinusoidal term oscillating at twice the power system frequency. When the system is balanced, the oscillating terms will disappear, and the instantaneous power will be equal to average power only.

Calculation of Instantaneous Three-Phase Power Using Symmetrical Components

If the balanced three phase voltage sources v_a, v_b, and v_c are supplying an unbalanced linear three-phase load, and three-phase current i_a, i_b, and i_c are considered as the superposition of instantaneous symmetrical sequence currents as in reference [2], then the three-phase instantaneous power can be calculated using,

$$p = \begin{bmatrix} v_a & v_b & v_c \end{bmatrix} \begin{bmatrix} i_a^+ & i_a^- & i_a^0 \\ i_b^+ & i_b^- & i_b^0 \\ i_c^+ & i_c^- & i_c^0 \end{bmatrix} = \begin{bmatrix} p^+ \\ p^{+-} \\ p^{+0} \end{bmatrix} \quad (8)$$

$$= p^+ + p^{+-} + p^{+0}$$

Where p^+, p^{+-}, and p^{+0} are the positive, negative and zero sequence components of instantaneous power respectively.

The first term of the instantaneous power is

$$p^+ = v_a i_a^+ + v_b i_b^+ + v_c i_c^+ = p_a^+ + p_b^+ + p_c^+ \quad (9)$$

If the symmetrical components of the currents are expressed as follows,

$$\underline{I}^+ = I^+ \angle\beta^+ \qquad \underline{I}^- = I^- \angle\beta^- \qquad \underline{I}^0 = I^0 \angle\beta^0$$

Then the positive sequence component of the instantaneous power (p^+) can be calculated as follows:

$$p_a^- = 2VI^+ \sin(\omega t)\sin(\omega t + \beta^+)$$
$$p_b^+ = 2VI^+ \sin(\omega t - 120^\circ)\sin(\omega t + \beta^+ - 120^\circ)$$
$$p_c^+ = 2VI^+ \sin(\omega t + 120^\circ)\sin(\omega t + \beta^+ + 120^\circ)$$

$$p^+ = P^+ - \frac{P^+}{3}(\cos(2\omega t) + \cos(2\omega t + 120^\circ)$$
$$+ \cos(2\omega t - 120^\circ)) + \frac{Q^+}{3}(\sin(2\omega t) + \sin(2\omega t + 120^\circ)$$
$$+ \sin(2\omega t - 120^\circ)$$

$$p^+ = P^+ - 0 + 0 = P^+ = 3VI^+ \cos\beta^+ \quad (10)$$

This result shows that the positive sequence component of the instantaneous power is equal to average power only as it is expected.

The instantaneous powers caused by negative sequence currents are:

$$p^{+-} = v_a i_a^- + v_b i_b^- + v_c i_c^- = p_a^{+-} + p_b^{+-} + p_c^{+-} \quad (11)$$

Where,

$$p_a^{+-} = 2VI^- \sin(\omega t)\sin(\omega t + \beta^-)$$
$$p_b^{+-} = 2VI^- \sin(\omega t - 120^\circ)\sin(\omega t + 120^\circ + \beta^-)$$
$$p_c^{+-} = 2VI^- \sin(\omega t + 120^\circ)\sin(\omega t - 120^\circ + \beta^-)$$

Then,

$$p^{+-} = VI^-\left[\cos\beta^- + \cos(\beta^- - 120^\circ) + \cos(\beta^- + 120^\circ)\right]$$
$$-VI^-\left[\cos(2\omega t + \beta^-) + \cos(2\omega t + \beta^-) + \cos(2\omega t + \beta^-)\right]$$

$$p^{+-} = 0 - 3VI^- \cos(2\omega t + \beta^-)$$
$$p^{+-} = -3VI^- \cos(2\omega t + \beta^-) \quad (12)$$

This equation shows that the net energy transfer caused by negative sequence current is zero. Negative sequence currents continuously delivered by one or two phases and returned to the source via the remaining one or two phases. These currents cause only active power losses on line resistance.

The instantaneous powers caused by zero sequence currents will be:

$$p^{+0} = v_a i^0 + v_b i^0 + v_c i^0 = p_a^{+0} + p_b^{+0} + p_c^{+0} \quad (13)$$

Where

$$p_a^{+0} = 2VI^0 \sin(\omega t)\sin(\omega t + \beta^0)$$
$$p_b^{+0} = 2VI^0 \sin(\omega t - 120^\circ)\sin(\omega t + \beta^0)$$
$$p_c^{+0} = 2VI^0 \sin(\omega t + 120^\circ)\sin(\omega t + \beta^0)$$

Then the instantaneous powers created by zero-sequence component:

$$p^{+0} = VI^0\left[\cos\beta^0 + \cos(\beta^0 + 120^\circ) + \cos(\beta^0 - 120^\circ)\right]$$

$$-VI^0(\cos(2\omega t+\beta^0)+\cos(2\omega t+\beta^0-120^\circ)$$
$$+\cos(2\omega t+\beta^0+120^\circ))$$

$$p^{\sim 0}=0-0=0 \qquad (14)$$

The equation (14) shows that the zero sequences current does not have any affect on instantaneous power, they only cause active power losses on line resistance.

Finally, using the above power components the three-phase instantaneous power with symmetrical components will be:

$$p=P^{+}-3VI^{-}\cos(2\omega t+\beta^{-}) \qquad (15)$$

This result shows that the sinusoidal term in instantaneous power seen equation (7) is due to the negative sequence current resulting from unbalanced operation. Also ($3VI^-$) defines the magnitude of the sinusoidal terms on instantaneous power. When the system is perfectly balances, the instantaneous power will be equal to the average (active) power only.

III. COMPUTER SIMULATION STUDIES

Extensive computer simulation studies have been undertaken using EMTP program to provide data representing the power system under different unbalanced operating conditions. Instantaneous power is the calculated using these data. The system simulated shown in Fig.1 was an 11 kV network containing a 18.5 MVA generator with 14.8 MW local load and operating in parallel with the utility power network. The three-phase voltages and currents are taken at the generator terminal connected to the 11 kV busbar.

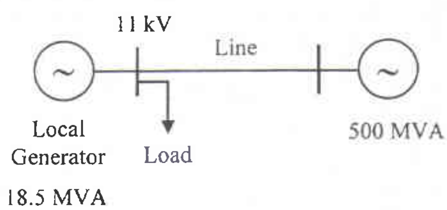

Figure 1. Model Power System Used for Simulation.

Figure 2 represents the case where while the system is operating under perfectly balanced conditions, an unbalanced situation is created via opening one of the phases. The changes in instantaneous power calculated both using the symmetrical components and direct-phase quantities.

Figure 3 shows the response to a phase to ground fault. Before the disturbance instantaneous power equals to average (active) power, immediately after disturbances

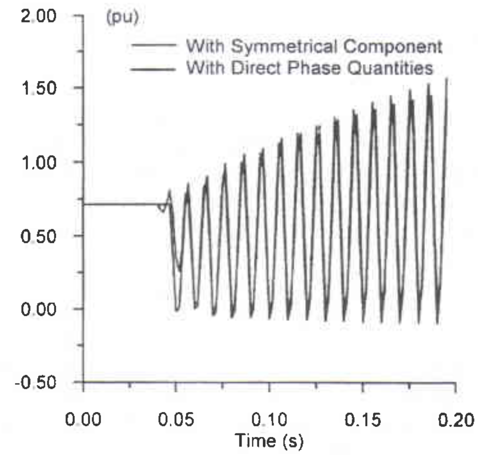

Figure 2. Instantaneous Power During Open- phase Condition

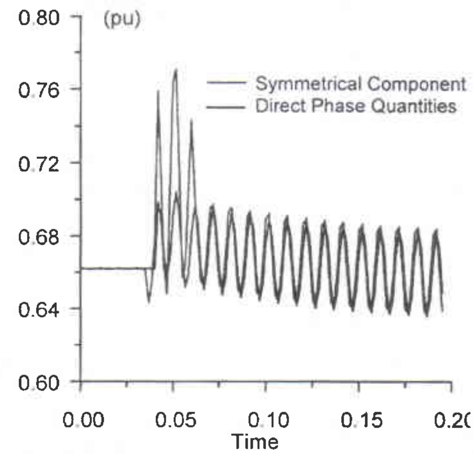

Figure 3. Instantaneous Powers During Phase to Ground fault.

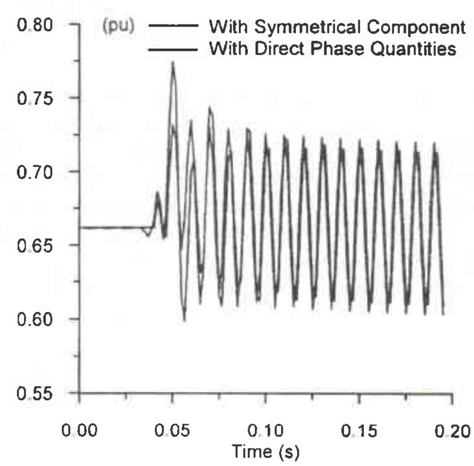

Figure 4. Instantaneous powers during unbalanced load condition

Power starts to oscillate at twice the power system frequency as it is in Figure 2.

System response to the case, where an unbalanced load condition has occurred, while the system is operating under balanced operating conditions, is illustrated in Figure 4. Immediately following the occurring of unbalanced condition, the generator output power starts to oscillate at twice the system frequency.

Figure 5. Shows the case where a three-phase load change has occurred following a three-phase switching operation. As it is clearly seen from the powers computed both using direct-phase quantities and symmetrical components, since the system is perfectly balanced before and after switching operation, oscillations in twice the system frequency is not seen on the power. Although there are some oscillations after the switching event, these oscillations are defined by system inertia and system time constant.

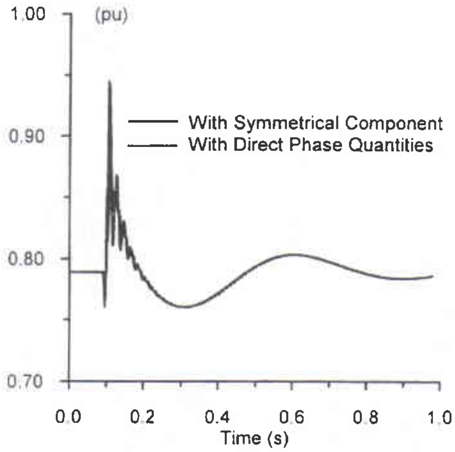

Figure 5. Instantaneous powers during symmetrical load change

IV. CONCLUSION

This paper has introduced a new equation to calculate the three-phase instantaneous power using symmetrical components. Computer simulation studies have been carried out to compare the instantaneous powers calculated using symmetrical components with those computed using direct phase quantities. Following conclusion can be taken from this study.

- Instantaneous power can be calculated using the symmetrical component, the only difference between the power calculated with the symmetrical components and that of computed using direct phase quantities can be seen in sub-transient period. This situation needs further investigation.
- Under balanced conditions, instantaneous power equals to the average (active) power as expected.
-Under unbalanced conditions instantaneous power oscillates at twice the power system frequency, and these oscillations is due to negative-sequence component resulting from the unbalanced system conditions. Therefore the magnitude of the sinusoidal oscillations can be used as a measure of system unbalance.

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