On the First- and Second-order Statistical Properties of Different Rayleigh Fading Channel Models

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Abstract

This paper is a continuation of our previous works on performance of different simulation models for Rayleigh fading channels. Previously, we have compared and analyzed several Rayleigh fading simulation models in terms accuracy and complexity. Some of these models are non-ergodic and their statistical properties vary from one simulation trial to another. Thus, a focus on the convergence behavior of such models is highly recommended which is missing in our previous works. In this paper, the first- and second-order statistical properties of seven of the most popular Rayleigh fading models are reviewed. The quality of the envelope probability density function (PDF) of the different models is evaluated using a more convenient quantitative measure. The convergence of the non-ergodic stochastic models is inspected based on the variances of the time-averaged correlation properties. The simulation results reveal several important conclusions about the accuracy as well as the capacity of the different models.

1. Introduction

Radio Channel characterization is a key step in the design and assessment of any wireless communication system. This task can be made through site measurements, but this approach is usually very expensive and time-consuming. The radio engineers frequently call for channel models as low-cost and straightforward alternative. In mobile radio environments, where a mobile station is susceptible to move relative to a fixed transmitter with an obstructed line-of-sight (LOS) between them, the small-scale fading behavior caused by multipath propagation and Doppler effect is commonly modeled by a random process with Rayleigh distribution and time-correlated samples [1]. This outcome goes back to the seminal work of Clarke [2], who developed a mathematical description of the frequency non-selective mobile radio channels. Soon after the introduction of the Clarke's model, measurement campaigns have been conducted in typical urban environments (e.g., [3]) and the results confirmed the Clarke's work. Since then, the Rayleigh fading channel model has gained a great popularity and has shown to be the model of choice for many research applications.

Despite the widespread acceptance of the Rayleigh fading model – as originally introduced by Clarke – it is considered as computationally inefficient, which makes the computer simulation or the real-time hardware implementation of such model a very complex task requiring a large memory storage and a high computational effort [4]. This has prompted researchers to develop efficient Rayleigh fading simulation models, and the research toward this goal had really started in the mid-1970s with the introduction of the Jakes' popular model [5]. Mainly, the techniques used for generating time-correlated Rayleigh fading processes fall into three methods: the inverse discrete Fourier transform (IDFT) method [6], the filtering white Gaussian noise (FWGN) method [7] and the sum-of-sinusoids (SOS) method [4, 5, 8-11]. During the 2000s, this area has been investigated intensely and over time, each method has been the subject of several modifications and improvements leading to numerous computer simulation models.

Unfortunately, a little effort has been devoted to the comparison of these models. The present paper is a continuation of our previous work [see, 12] on the performance of different Rayleigh fading simulation models. In [12], the statistical properties of the most accepted and approved Rayleigh fading models are analyzed and compared using several quantitative measures. It was noted in [12] that the statistics of the non-ergodic stochastic SOS models may vary from one simulation run to another, but the convergence behavior of such models was not investigated. In this paper we return to this point in order to better conclude about the performance and the capability of the different models. For this purpose, the variance of the second-order time-averaged correlation functions is used as criterion to study the convergence of two of the most successful stochastic SOS-based models. Also, the quality of the first-order statistic, PDF of the envelope of the different models is evaluated using a more convenient quantitative measure than used in earlier studies (e.g., [12, 13]). Furthermore, in our former work the Doppler frequency is chosen to represent the fast fading scenario. Here, in order to get more out from this work, all the simulation results are driven with a different fading rate corresponding to the slow fading case. Eventually, it is noteworthy that all models investigated here are carefully chosen from previous studies on Rayleigh fading simulators [9, 12, 14], where they are shown to be superior in reproducing the frequency non-selective (i.e., flat) mobile radio channel characteristics with the classical – and often-assumed – Jakes power spectral density (PSD). Other Rayleigh fading models may be more appropriate for other forms of PSD (e.g., frequency-shifted Gaussian PSD [10] or non-symmetric PSD [7]) but they are out of the scope of the present paper and will be left for a future study.

The remainder of this paper is structured as follows. In Section 2, first we briefly review the Clarke’s reference model and its statistical properties and then, we present the Rayleigh fading simulation models that were investigated in this study. The simulation results are presented and discussed in Section 3. Finally, the conclusions are provided in section 4.

Table 1. The different Rayleigh fading simulation models adopted in our study.

Model Name	Authors (Year)	Method	Reference	Notes
Young's model	Young and Beaulieu (2000)	IDFT	[6]	-
Baddour's Autoregressive (AR) model	Baddour and Beaulieu (2005)	FWGN	[7]	-
MEDS	Pätzold *et al*. (1996)	SOS-deterministic	[10, pp. 128–133]	$N_Q = N_I + 1$
ZX-1	Zheng and Xiao (2002)	SOS-stochastic	[8]	$N_I = N_Q$
M-ZX-2	Patel *et al*. (2005)	SOS-stochastic	[9]	$N_I = N_Q$
ZX-3	Xiao *et al*. (2006)	SOS-stochastic	[4]	$N_I = N_Q$
WANG	Wang *et al*. (2012)	SOS-stochastic	[11]	$N_I = N_Q$

2. The reference model and simulation models

Under the flat fading assumption and adopting the Clarke's two-dimensional (2-D) isotropic scattering model, the low-pass fading process $g(t) = g_I(t) + jg_Q(t)$, in a typical mobile radio environment (i.e., with large number of scatters and an obstructed LOS between the transmitter and the mobile station), has been shown to be approximated by a complex wide sense stationary (WSS) Gaussian random process [1]. The in-phase component $g_I(t) = \Re\{g(t)\}$, and the quadrature component $g_Q(t) = \Im\{g(t)\}$ are independent, identically distributed zero-mean Gaussian processes with identical variances σ^2. Therefore, the envelope $|g(t)|$ is Rayleigh distributed at any time and its PDF is given by [1, 2]:

$$f_{|g|}(r) = \frac{r}{\sigma^2} e^{(-r^2/2\sigma^2)}, \quad r \geq 0 \tag{1}$$

Under these conditions, and considering the Jakes PSD, the normalized (i.e., $\sigma^2 = 1$) correlation properties of the fading process are [1,2]:

$$R_{g_I g_I}(\tau) = E[g_I(t) g_I(t+\tau)] = \mathcal{J}_0(\omega_d \tau) \tag{2a}$$

$$R_{g_Q g_Q}(\tau) = \mathcal{J}_0(\omega_d \tau) \tag{2b}$$

$$R_{g_I g_Q}(\tau) = R_{g_Q g_I}(\tau) = 0 \tag{2c}$$

$$R_{gg}(\tau) = E[g(t) g^*(t+\tau)] = 2\mathcal{J}_0(\omega_d \tau) \tag{2d}$$

where E is the statistical expectation operator, $\mathcal{J}_0(\cdot)$ is the zeroth-order Bessel function of the first kind, τ is the time lag and $\omega_d = 2\pi f_d$ is the maximum radian Doppler frequency due to the receiver motion. $R_{g_I g_I}(\tau)$ and $R_{g_Q g_Q}(\tau)$, are the autocorrelation functions (ACFs) of the in-phase and quadrature components of $g(t)$, respectively, $R_{g_I g_Q}(\tau)$ and $R_{g_Q g_I}(\tau)$ are the cross-correlation functions (CCFs) of these components and $R_{gg}(\tau)$ is the ACF of the low-pass complex fading process $g(t)$. Since the Doppler frequency characterizes the time-variation or fading rate of the radio channel, these ACFs are of major importance because they reflect the time-variability of the fading. The reference model is completely characterized by its statistical properties, thus the aim of any Rayleigh fading model is to reproduce the above statistics as faithfully as possible.

In the following, we present the different simulation models which will be investigated in this paper. We choose for this purpose seven of the most accepted models, which we listed in Table 1, and we make so that the three common methods of generating time-correlated Rayleigh fading are considered. The complete description of these models is omitted here due to space limitation, but we refer the reader to the corresponding references given in Table 1. For the SOS-based models, the principle consists in approximating each of the Gaussian processes $g_I(t)$ and $g_Q(t)$ by a sum of finite number of sinusoidal terms with amplitudes, frequencies and phases that are properly selected to achieve the desired statistical properties of the Clarke's reference model. In Table 1, N_I and N_Q designates respectively the number of sinusoids used in the in-phase and quadrature branches. In addition, two classes of SOS-based model are distinguished: the Frequency-Amplitude (FA) deterministic and stochastic models [14]. In the first class, the sinusoids amplitudes and frequencies are deterministic. The corresponding process is ergodic, and then a single simulation trial is generally sufficient for convergence. On the other hand, the sinusoids amplitudes and / or frequencies of the FA stochastic models are selected randomly. The statistical quantities of such models vary from one simulation trial to another, and one has to compute these properties by averaging over several simulation trials which increases the complexity of the model, but helps to improve the quality of generated processes. Further details about IDFT, FWGN and SOS methods can be found [12].

3. Performance analysis of the different simulation models

In this section, we analyze and compare the different models, given in Table 1, in terms of their first- and second-order statistical properties. All simulation results are driven here using a normalized maximum Doppler frequency $f_m = f_d T$ of 0.01 (where T is the sampling interval). As we mentioned above, the channel fading rate (rapidity) depends on the Doppler frequency. As a general rule-of-thumb, the channel is said to be slow fading if $f_d T < 0.02$, otherwise if $f_d T > 0.02$, it is referred to as medium to fast fading [15]. In this paper, we consider the slow fading scenario, and this choice is motivated by two practical reasons. First, we know that most of the present-day terrestrial mobile radio channels can commonly be characterized as slow fading. Second, in comparison with our previous work, where a fast fading condition is assumed ($f_d T = 0.05$), we present here a different fading environment in order to get more out from this work.

3.1. Analysis of the first-order statistics

We begin our analysis by evaluating the quality of the envelope PDF of the different simulators, which is one of the most important first-order statistics of Rayleigh fading channels. We use to this end the Jeffrey's divergence (J-div) criterion, popular in probability theory and statistics. This criterion measures the difference between a theoretical (reference) PDF and an empirical one. In previous works (e.g., [12, 13]), the authors have used the non-symmetric Kullback-Leibler Divergence (KL-div) criterion. The J-div is the symmetric version of the KL-div criterion, and it is more appropriate in this

Table 2. Evaluation based on the J-div of various models.

Model name	Method			
	IDFT method			
Young's model	$7.919 \cdot 10^{-6}$			
	FWGN method			
	Filter order			
	50	120	200	
Baddour's AR model	$8.623 \cdot 10^{-6}$	$7.601 \cdot 10^{-6}$	$7.649 \cdot 10^{-6}$	
	SOS method			
	Number of sinusoids, N_I			
	8	16	64	128
ZX-1	$2.879 \cdot 10^{-3}$	$6.318 \cdot 10^{-4}$	$3.724 \cdot 10^{-5}$	$1.262 \cdot 10^{-5}$
M-ZX-2	$5.907 \cdot 10^{-3}$	$1.221 \cdot 10^{-3}$	$6.126 \cdot 10^{-5}$	$2.096 \cdot 10^{-5}$
ZX-3	$5.416 \cdot 10^{-3}$	$1.232 \cdot 10^{-3}$	$6.445 \cdot 10^{-5}$	$2.697 \cdot 10^{-5}$
WANG	$2.872 \cdot 10^{-3}$	$6.105 \cdot 10^{-4}$	$3.944 \cdot 10^{-5}$	$1.373 \cdot 10^{-5}$
MEDS	$2.555 \cdot 10^{-3}$	$5.979 \cdot 10^{-4}$	$3.692 \cdot 10^{-5}$	$1.859 \cdot 10^{-5}$

kind of studies. The J-div, denoted D_J, is defined as follows [16, p. 251]:

$$D_J(P \parallel Q) = \sum_i (P(i) - Q(i))\, ln\, \frac{P(i)}{Q(i)} \quad (3)$$

where $P(i)$ and $Q(i)$ are the time-discrete versions of reference and the evaluated distributions respectively. The J-div is always non-negative with $D_J(P \parallel Q)$ equal to zero if and only if $P = Q$. Thus, the closer D_J is to 0, the better is the agreement of the empirical PDF with the theory. All the PDFs of the envelopes of the seven Rayleigh fading simulators are evaluated using this criterion and the results of simulation are shown in Table 2. The reference PDF is given by (1).

The empirical (simulated) PDFs were computed using a fading process of length 2^{20} samples. For each Rayleigh generator, the envelope PDF was averaged over 100 independent simulation trials before to be used in the J-div criterion. According to Table 2, the IDFT- and FWGN-based models achieve the best results in terms of the envelope PDF. For the AR model, increasing the filter order beyond 50 don't bring a significant improvement in the quality of the envelope PDF. The simulation results suggest that the SOS models also provide accurate results if a large number of sinusoids is used $(N_I \geq 64)$. However, increasing N_I leads to cumbersome computations due to the large number of expensive $\sin(\cdot)$ function calls needed in simulation. Generally, $N_I = 16$ is considered as a moderate choice for a good compromise between complexity and accuracy. For $N_I =$8, 16 and 64, the best results are seen for the MEDS. While, if 128 sinusoids are used, then the ZX-1 surpasses the MEDS. For the Zheng and Xiao's SOS family of models, the ZX-1 stands as the best model in terms of the envelope PDF statistic. Additionally, the WANG and ZX-1 models have approximately similar performances. Overall speaking, we conclude that ZX-1, WANG and MEDS are best SOS-based models regarding the J-div criterion. The worst cases are observed for the M-ZX-2 and ZX-3 with a slightly better performance for the M-ZX-2 model.

3.2. Analysis of the second-order statistics

Here, we evaluate the accuracy of the different models in terms of the ACF of the in-phase component. Two quantitative measures, which are commonly adopted in the evaluation of the quality of Rayleigh fading generators, are used for this purpose. Details about these measures are found in [17]. The first measure, called the mean power margin, is given by:

$$G_{mean} = \frac{1}{\sigma_x^2 L} trace\{\boldsymbol{C}_x \boldsymbol{C}_{\hat{x}}^{-1} \boldsymbol{C}_x\} \quad (4)$$

The second is the maximum power margin and is defined as:

$$G_{max} = \frac{1}{\sigma_x^2} max\left\{ diag\left\{\boldsymbol{C}_x \boldsymbol{C}_{\hat{x}}^{-1} \boldsymbol{C}_x\right\}\right\} \quad (5)$$

In (4) and (5), σ_x^2 is the variance of the reference distribution, $\boldsymbol{C}_{\hat{x}}$ is the $L \times L$ covariance matrix of any length-L subset of adjacent samples produced by the stationary random sequence generator, and $\boldsymbol{C}_x$ represents the desired (reference) covariance matrix of L ideally distributed samples. Though, for brevity reason, only real parts of the simulator outputs are analyzed here, similar results can be obtained for the imaginary parts. G_{mean} and G_{max} measure the similarity of a theoretical and simulated ACFs. Generally, these measures are expressed in dB and perfect performance corresponds to 0 dB for both quantities.

The variance σ_x^2 is set to unity and the time-average correlations needed for estimating the matrix $\boldsymbol{C}_{\hat{x}}$, were computed based on 2^{20} generated samples. An autocorrelation sequence length of 800 was considered. The computed quality measures were then averaged over 50 independent simulation trials. The simulation results are reported in Table 3.

The results presented in Table 3 reveal several important remarks. The major outcome that arises from this study is about the recently published WANG model. In terms of G_{mean} and G_{max}, the WANG model surpasses all the SOS stochastic models. It performs better than the ZX-1 model considered in some comparative studies as the best stochastic model [9, 14]. Moreover, for small N_I $(N_I = 8)$, the WANG model achieves better result than the deterministic MEDS, known by its quasi-optimal approximation of the ACF of (2a) [10]. On the other hand, for large N_I $(N_I \geq 64)$, this model outperforms the IDFT method that is always considered as accurate and efficient. Another important remark that comes from this study is about Zheng and Xiao's family of models. Despite ZX-3 and M-ZX-2 have been introduced after the ZX-1 model, they failed to realize

Table 3. Evaluation based on G_{mean} and G_{max}

			G_{mean}(dB)	G_{max}(dB)
Young's model			$1.08 \cdot 10^{-2}$	$1.15 \cdot 10^{-2}$
Baddour's model	Filter order	50	1.296	1.416
		120	1.664	1.854
		200	0.317	0.464
ZX-1	N_I	8	26.093	27.846
		16	0.127	0.163
		64	$4.38 \cdot 10^{-3}$	$4.49 \cdot 10^{-3}$
		128	$2.03 \cdot 10^{-3}$	$2.10 \cdot 10^{-3}$
M-ZX-2	N_I	8	30.569	32.605
		16	10.408	13.010
		64	$4.75 \cdot 10^{-2}$	$7.99 \cdot 10^{-2}$
		128	$4.54 \cdot 10^{-3}$	$9.91 \cdot 10^{-3}$
ZX-3	N_I	8	31.827	33.938
		16	17.015	20.779
		64	0.176	0.284
		128	$3.59 \cdot 10^{-2}$	$5.22 \cdot 10^{-2}$
WANG	N_I	8	24.11	26.536
		16	$5.03 \cdot 10^{-2}$	$8.08 \cdot 10^{-2}$
		64	$3.65 \cdot 10^{-3}$	$3.75 \cdot 10^{-3}$
		128	$3.41 \cdot 10^{-4}$	$3.82 \cdot 10^{-4}$
MEDS	N_I	8	24.856	27.047
		16	$4.72 \cdot 10^{-5}$	$7.22 \cdot 10^{-5}$
		64	$9.09 \cdot 10^{-4}$	$9.36 \cdot 10^{-4}$
		128	$-8.63 \cdot 10^{-4}$	$-8.18 \cdot 10^{-4}$

the performances reached by the latter model. As depicted in some previous works [9, 14], the ZX-1 stands as the best model among all Zheng and Xiao's models followed by the M-ZX-2. Additionally, the results confirm the good autocorrelation properties of the MEDS method. Compared with all methods, this model achieves the best approximation of $R_{g_I g_I}(\tau)$ provided that a sufficient number of sinusoids is used. Finally, we state that the AR model fails to accomplish the performances obtained in our previous work [12]. Therefore, this model is more suitable for approximating fast fading than slow fading channels. In comparison with [12], we conclude that IDFT- and SOS-based models approximately retain similar performances for both slow and fast fading scenarios.

The correlation functions involved in the computation of G_{mean} and G_{max} were computed based on time-averaging, a technique that is often used in simulation practice instead of ensemble averaging [4]. For the SOS stochastic models, the random frequencies make the time-average correlation functions to change from one simulation trial to another. To characterize this variability, we generally use the variance of these time-average correlations which measures the expected deviation of time-average correlation properties of a given simulation model (with finite N_I) from the ideal statistical correlation properties of the reference model. The time-average correlation functions are denoted here by $\hat{R}(\cdot)$, to distinguish them from the ideal (reference) statistical averages $R(\cdot)$. Then, the variance of $\hat{R}(\cdot)$ is defined as, $Var[\hat{R}(\cdot)] = E\left[\left|\hat{R}(\cdot) - R(\cdot)\right|^2\right]$. A lower variance indicates that a smaller number of simulation trials are needed to achieve desired statistical properties, and hence the convergence is better. Considering the results obtained until now, the ZX-1 and WANG models appear to be the most successful SOS stochastic models. In order to better conclude about the convergence of these models, we have chosen to analyze the variances of their time-averaged correlation functions. The variances of the ACF of the in-phase component, the ACF of the complex fading process and the CCF of the quadrature components are denoted by $Var\left[R_{g_I g_I}(\tau)\right]$, $Var\left[R_{gg}(\tau)\right]$ and $Var\left[R_{g_I g_Q}(\tau)\right]$, respectively. All these variances are computed by averaging over 500 independent simulation trials. In this study, we use fading processes with 10^5 samples to avoid both large memory storage and long processing time.

The variances of the ACF of the in-phase component for ZX-1 and WANG models are illustrated in Fig. 1(a). It can be seen clearly that the two models perform similar results regarding $Var\left[R_{g_I g_I}(\tau)\right]$ and their variances still approximately near zero up to a normalized time delay equal to 8, which means that the ACFs of the in-phase component of these models still unchanged over simulation trials until $f_d.\tau = 8$. From that value significant fluctuations are observed. If a good approximation of $R_{g_I g_I}(\tau)$ is needed beyound this value, then a single simulation trial is not sufficient, and one must average over several simulation runs to improve the quality. On the other hand, Fig. 1(b) shows the variances of the ACF of the complex process for ZX-1 and WANG. This time, the WANG model exhibits the

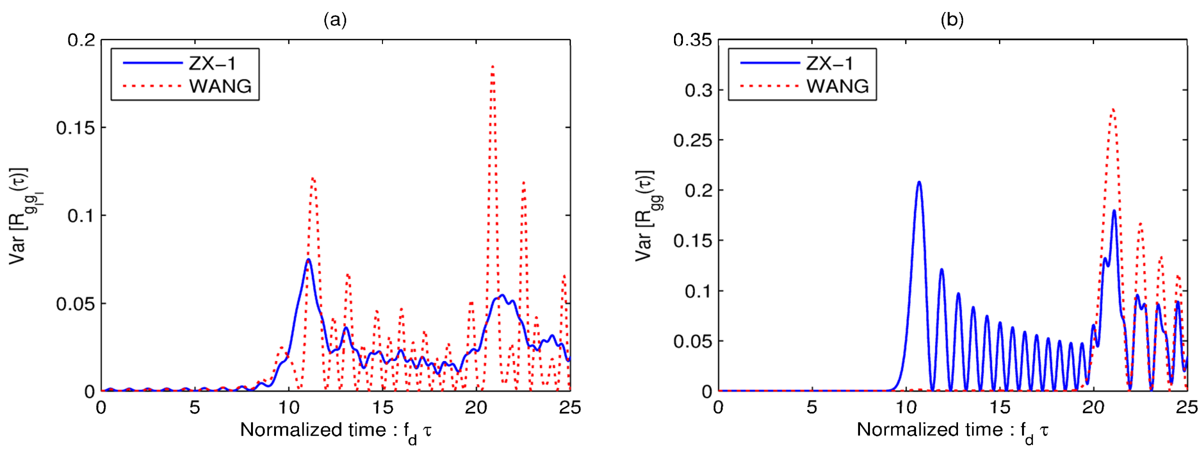

Fig. 1. (a) $Var\left[R_{g_I g_I}(\tau)\right]$ of ZX-1 and WANG models. (b) $Var\left[R_{gg}(\tau)\right]$ of ZX-1 and WANG models. ($f_m = 0.01$ and $N_I = 16$).

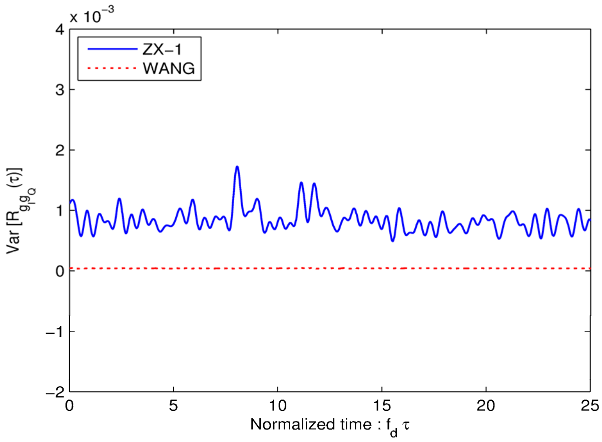

Fig. 2. $Var\left[R_{g_I g_Q}(\tau)\right]$ of ZX-1 and WANG models ($f_m = 0.01$ and $N_I = 16$).

better performance, and the variance of its ACF of the complex process is close to zero until a range that is roughly two times greater than the corresponding range of ZX-1 model. Therefore, the WANG model converges faster than ZX-1 and its correlation properties vary little from one simulation trial to another. By surpassing the well-accepted ZX-1 model, the WANG model shows a good capability in terms of the ACF of the complex process. Eventually, we presented in Fig. 2 the $Var[\hat{R}_{g_I g_Q}(\tau)]$ of ZX-1 and WANG models. Another time the WANG model proves its superiority over the ZX-1 model. Given the results found in the recent comparative study of [12], and regarding the simulation results of Fig. 2, we state that the WANG model provides the best approximation of $R_{g_I g_Q}(\tau)$ and it is a superior choice when a zero CCF is required. This model will be of a great interest when dealing with multiple uncorrelated fading waveforms (e.g., as in the case of frequency selective fading).

4. Conclusions

In this paper, seven of the most accepted Rayleigh fading simulation models have been analyzed and compared in terms of their first- and second-order statistical properties. The study has addressed the three common methods for generating time-correlated Rayleigh fading processes (IDFT, FWGN and SOS methods). The simulation results reveal that, in terms of the quality of the first-order PDF of the envelope, the best performances are observed for AR FWGN- and Young's IDFT-based models. Also, the SOS models provide a good approximation of the envelope PDF if a large number of sinusoids is used ($N_I \geq 64$). In this regard, the MEDS, ZX-1 and WANG models have been shown to be the best SOS-based models. On the other hand, the analysis of the second-order statistics has proved that the recently introduced WANG model has proven to be very successful. Its second order statistics outperform those of ZX-1 which has been considered for long time the most accurate SOS model. Furthermore, based on the study of the convergence behavior of the non-ergodic stochastic models, we conclude that in terms of ACF of the complex process and the CCF of the quadrature components, the WANG model converges faster than the ZX-1 model and it needs a small number of simulation trials to achieve the desired statistical properties. Although, the AR model has realized the best results in terms of the quality of the generated envelope PDF, its ACF fails to reach an acceptable accuracy level. Finally, in comparison with more recent models, the well-known Young's IDFT model still has accurate statistical properties but due to its incapability to ensure a sample-by-sample simulation, this method may be inappropriate for many applications.

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