POTENTIALITY OF CHANNEL ASSIGNMENT FOR CELLULAR COMMUNICATIONS

R.E.Goot and V.A.Fainstein

Department of Computer Science
Center for Technological Education
52 Golomb St., P.O.B. 305, Holon, 58102, Israel
E-mail <valentina@barley.cteh.ac.il>

ABSTRACT

This report is devoted to the method of how to determine maximal potentially possible number of users having suitable communication quality under the frequency reuse and inevitable co-channel interference. The method can be use as benchmark to compare the possibilities of practical allocation schemes.Since the method is very complex, the a suboptimal allocation procedure is proposed and it is shown, that its perfomance is closed to the potentiality.

I. INTRODUCTION

In multi-user wireless networks it is desirable to maintain communication quality above a chosen minimum. The quality is quantified by a carrier-to-noise ratio (*CIR*) and this value should be maintained above the prescribed minimal level h_0^2 by the allocation of frequency channels. The main idea behind the majority of frequency allocation algorithms is to reduce the co-channel interference and thus to increase both the radio spectrum reuse efficiency and the probability of fulfillment of inequality $CIR \geq h_0^2$ for each user [1]. In most cases these algorithms are introduced heuristically and the questions remain open how much the performance is close to the potentially possible one and what are these potential bounds. In this report we formulate some extremal problem and obtain the allocation algorithm as the result of its solution. The solution yields the allocation procedure, allowing to obtain maximal, potentially achievable number of users with suitable communication quality.

Thus, we obtain the potential performance bounds. The values of this sort are of a great both theoretical and practical importance, as they allow to determine possibilities of an object and provide a useful benchmarks to compare of real objects and allow to reveal resources of its improvement and perfection. Since however it is difficult to reach these bounds at present time, we introduce a more realistic allocation procedure and show that its performance is closed to the potential bound.

II. THE OPTIMAL ALLOCATION PROCEDURE

Let a group of M radiolinks operates at a common pool of N frequencies. Each member of the group can use any, but only one frequency and any frequency must be allocated to each user.

Formally it may be expressed by variables a_{mn} $\left(m=\overline{1,M},\ \ n=\overline{1,N}\right)$ such, that $a_{mn}=1$ if the m-th user occupies the n-th frequency and $a_{mn}=0$ otherwise and

$$\sum_{n=1}^{N} a_{mn} = 1; \quad m = \overline{1,M}. \qquad (1)$$

The presented model is adequate to frequency reuse in cellular communication systems [1]. The *CIR* value for *m*-th user under the condition that it occupies the *n*-th frequency is

$$CIR(m|n) = \frac{\mu_{mm}^{(n)} Q_m}{\sum_{\substack{k=1 \\ k\neq m}}^{M} a_{kn} \mu_{km}^{(n)} Q_k + P_{nm}}, \qquad (2)$$

where $\mu_{lm}^{(n)}$ is the power transient coefficient from *l*-th user's transmitter to the *m*-th user's receiver at the *n*-th frequency, Q_l is the power of *l*-th transmitter, P_{nm} is the power of environment noise for the *m*-th user receiver at the *n*-th frequency. The presence of variables a_{kn} in the denominator shows what *k*-th user transmits (or not) at the same *n*-th frequency and consequently, interferes with the *m*-th one.

The value showing suitability of the quality for the *m*-th user under the given *n* is $q(m|n) = u\left[CIR(m|n) - h_0^2\right]$ with *u*(*x*) is unit step function. Since a_{mn} and $q(m|n)$ are equal to 1 or 0, the expression for number of users with suitable communication quality can be written as follows:

$$S = \sum_{m=1}^{M}\sum_{n=1}^{N} a_{mn}\, q(m|n) = \sum_{m=1}^{M}\sum_{n=1}^{N} a_{mn} u\left[CIR(m|n) - h_0^2\right]. \qquad (3)$$

The natural objective is to maximize the value *S* and thus the procedure of optimal allocation consists in solution of the extremal problem $S \Rightarrow \max = S_{\max}$ under the conditions (1). The result of the solution will be a vector **a** $= \{a_{mn}\}$, supplying maximum to *S* and yielding the allocation algorithm. The probability that one user obtains the suitable communication quality is $P_u = E(S_{\max})/M$ with $E(\bullet)$ be the symbol of mathematical expectation, and we will accept this value as the figure-of-merit for performance of the allocation procedure.

Performance: We treated the performances for the case of homogeneity, that is, for all *m*, *n*, $Q_m = Q$, $\overline{\mu}$ and $\overline{\mu}_{\text{int}}$ are average values of $\mu_{mm}^{(n)}$ and $\mu_{km}^{(n)}$ correspondingly ($k \neq m$), $P_{nm} = P$, $\overline{\mu}/\overline{\mu}_{\text{int}} = d$, and $H = \overline{\mu} Q/P$. The values *d* and *H* are the average signal-to-interference (*SIR*) and signal-to-noise (*SNR*) ratios for one user. The obtained algorithm is generated by the extremal problem, belonging to *NP*-complete ones with essential difficulties for the solution. Owing to *NP*-completeness and since our aim does not consist in surmounting of computation difficulties, we restricted ourselves to simple exploration example solved by simulation. The simulation was performed for the case of log-normal shadowing (slow) fading, averaged over fast fading [2]. The results of the simulation are presented in Fig 1.

III. THE SUBOPTIMAL ALLOCATION PROCEDURE

Since the optimal procedure is *NP*-complete and hardly feasible practically we introduce the following heuristical allocation procedure

$$a = \left\{a_{mn} : a_{mn} = 1, a_{mk} = 0 \ \textit{for n such that}\ \mu_{mm}^{(n)} \geq \mu_{mm}^{(k)}; n,k \in \left\{\overline{1,N}\right\},\ k \neq n\right\}_{m=1}^{M} \qquad (4)$$

According to the procedure each user occupies its best frequency regardless of pretensions and intentions of other users.As for characterization of performance here unlike the optimal procedure, we can obtain the expression for P_u analytically.

Let all random variables $\mu_{mm}^{(n)}$ and $\mu_{km}^{(n)}$ are statistically independent with the same density function $f_\mu(\mu)$ (for variables $\mu_{mm}^{(n)}, m=\overline{1,M}, n=\overline{1,N}$) and $f_{\mu_{int}}(\mu)$ (for variables $\mu_{km}^{(n)}, k=\overline{1,M}$, $k \neq m$).

Denote $\eta = \max_n \mu_{mm}^{(n)}, \vartheta_l = \sum_{k=1}^{l} \mu_k$,

where random variables $\mu_l \; (l=\overline{1,M-1})$ posses the same density $f_{\mu_{int}}(\mu)$.

Then the CIR value for m-th user under the condition, that l other users also chose the same frequency is

$$CIR(l) = \frac{\eta Q}{\vartheta_l Q + P} = \frac{\eta}{\vartheta_l + P/Q} \qquad (5)$$

Now we can write

$$P_u = \sum_{l=0}^{M-1} P(l) \Pr\{CIR(l) > h_0^2\}, \qquad (6)$$

where $P(l)$ is the probability that l other users chose the same frequency,

$$P(l) = \frac{1}{N^l} \; (l=\overline{1,M-1}),$$

$$P(0) = 1 - \sum_{l=1}^{M-1} P(l) = \qquad (7)$$

$$1 - \frac{N^{M-1}-1}{N^{M-1}(N-1)}$$

The density function of variable η is equal [3] $f_\eta(\eta) = N F_\mu^{N-1}(\eta) f_\mu(\eta)$.

With use of the density functions and formula (6) we obtain

$$P_u = P(0) \int_{h_0^2 P/Q}^{\infty} f_\eta(\eta) d\eta +$$

$$\sum_{l=1}^{\infty} P(l) \int_0^{\infty} f_{\vartheta_l}(\vartheta) d\vartheta \int_{h_0^2(\vartheta_l - P/Q)}^{\infty} f_\eta(\eta) d\eta =$$

$$\left(1 - \frac{N^{M-1}-1}{(N-1)N^{M-1}}\right)\left(1 - F_\mu^N(h_0^2 P/Q)\right) +$$

$$\sum_{l=1}^{M-1} \frac{1}{N^l} \int_0^{\infty} f_{\vartheta_l}(\vartheta\;) \cdot \qquad (8)$$

$$\left[1 - F_\mu^N[h_0^2(\vartheta \; + P/Q)]\right] d\vartheta \; =$$

$$1 - \left\{\left(1 - \frac{N^{M-1}-1}{(N-1)N^{M-1}}\right) F_\mu^N\left(h_0^2 P/Q\right) + \right.$$

$$\left. \sum_{l=1}^{M-1} \frac{1}{N^l} \int_0^{\infty} f_{\vartheta_l}(\vartheta) F_\mu^N\left(h_0^2\left(\vartheta + P/Q\right)\right) d\vartheta \right\}$$

where $f_{\vartheta_l}(\vartheta)$ is density function of variable ϑ_l .

Note that the term in the braces gives the expression for probability of outage (quality of communication is unsuitable). The computational example was performed for the condition of the previous case with usage of approximation of density $f_{\vartheta_l}(\vartheta)$ by log- normal one [4].

The results are presented in Fig.2

IV. CONCLUSION

1. The obtained potential performance bounds demonstrate the strong dependence on co-channel interference in the case of number of users M is more than the number of frequencies N. If $N = M$ the users prefer to work at different frequencies and the possibility to work also at the same frequency for several users does not improve the performance, essentially even if the value H is not so great.

2.The suboptimal procedure performance is closed to the results for optimal one, at least for the treated model and parameters. Note, that the prepared heuristic allocation procedure is of "local" character, unlike the

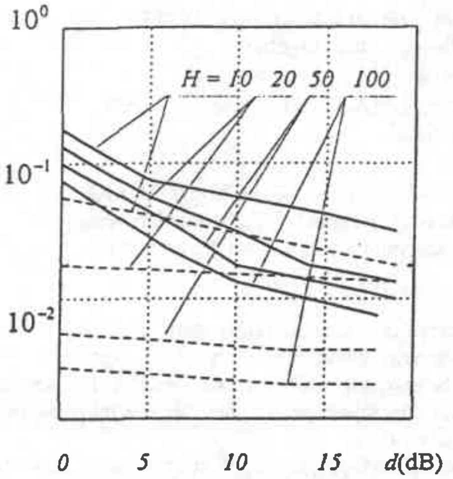

Fig.1. Probability of outage versus *SIR* for *N* =3 (optimal allocation,—*M*=4,- - -*M*=3).

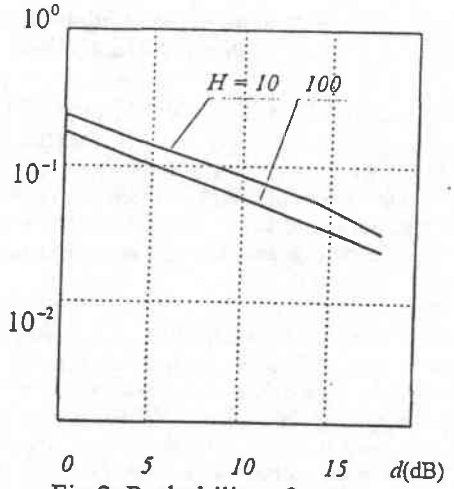

Fig.2. Probability of outage versus *SIR* for *N* =3 (suboptimal allocation, *M*=4).

optimal procedure which must be centralized. It means, that for suboptimal procedure each user chooses its channel irrespective of other users. Thus the procedure are more realistic and easier implemented.

3. The presented optimal procedure refers to class of centralized allocation schemes [5] and provides a useful benchmark to compare performance of real allocation schemes.

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