USING GENETIC ALGORITHM TO IMPROVE THE ROBUST PERFORMANCE OF A FLEXIBLE TRANSMISSION SYSTEM

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ABSTRACT

This paper addresses the problem of selecting appropriate weights in mixed sensitivity controller design, when various specifications are the design objectives. Genetic algorithm is used for tuning weight functions in order to improve the performance satisfactory of design. Application of the resulting method is used to design a controller for flexible transmission system.

I. INTRODUCTION

There has been a great interest in the control community, using mixed sensitivity controller design. Such an interest is motivated by the fact that the controller design is simply based on nominal plant model P(s) and bounds or weights on sensitivity and complementary sensitivity functions. The success of achieving performance constraint in an optimal mixed sensitivity control design depends, in large part, on the selection of appropriate weights used in optimization process. In [1], the problem of weight selection for sinusoidal tracking performance has been addressed. However, there is no proved method for selection of weights according to various specified specifications in problem.

In this paper, we outline a guideline for selecting the parameters of the weighting functions in the mixed sensitivity design when multiple specifications in time and frequency domain are considered. The method is based on selecting a structure for weight functions and tuning their parameter in order to maximize a satisfactory performance function through an optimization method which in this paper is genetic algorithm. Selecting a structure for weight functions is not difficult according to robust control literature [2] but fine tuning the parameters have been always a point of confusion in mixed sensitivity problem. The strength of this method is that it is applicable to all problems including the ones with multiple specifications and MIMO systems. The method is presented by designing a controller for flexible transmission system. This system has been the subject of a benchmark on robust control at the European Control Conference in Rome 1995. Very low damped vibration modes and their large variation with loads along with several specifications make this system in general very difficult to control. Several controller design methods have been presented for this system, including adaptive control, QFT, GPC, CRONE, pole placement with sensitivity loop shaping, and mixed sensitivity optimization with multiplicative uncertainty [2-3]. However, only one of the proposed approaches, has satisfied all the specifications. In this paper, two controllers are constructed using mixed sensitivity optimization and genetic algorithm is used to improve their performance.

II. BENCHMARK PROBLEM STATEMENT

Flexible Transmission system consists of three horizontal pulleys connected by two elastic belts. The first pulley is driven by a DC motor whose position is controlled by a local feedback. Since the dynamic of this feedback loop is much faster than the mechanical parts, it can be neglected in the analysis of the system. The objective is to control the position of the third pulley which may be loaded by small disks. The schematic diagram of this system is shown in Figure 1.

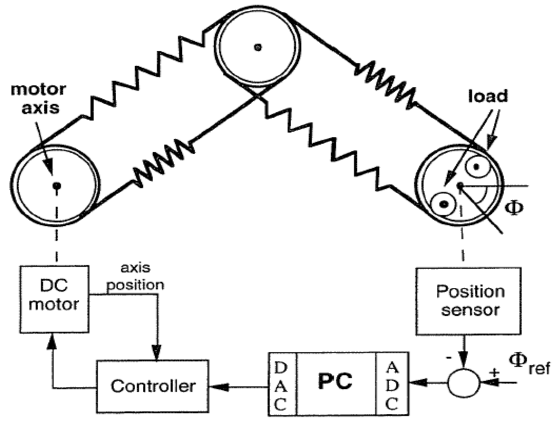

Figure 1. Flexible transmission system.

The plant model is given as a set of three transfer functions depending on the loads, as given below.

No load:

$$G_{NL}(s)=\frac{11.56}{714\times10^{-7}s^4+1225\times10^{-7}s^3+0.08607s^2+0.07395s+11.56} \quad (1)$$

Half load:

$$G_{HL}(s)=\frac{11.56}{2159\times10^{-7}s^4+2458\times10^{-7}s^3+0.2017s^2+0.07395s+11.56} \quad (2)$$

Full load:

$$G_{FL}(s)=\frac{11.56}{3604\times10^{-7}s^4+369\times10^{-6}s^3+0.3173s^2+0.07395s+11.56} \quad (3)$$

This system is characterized by two low damped vibration modes, and their large variation with loads as shown in Figure 2. The goal is to tune the parameter of the weight function in the mixed sensitivity problem to achieve the following specifications:

a) Time domain specification:
   1- Rise time (t_r) of less than 1 sec.
   2- Overshoot (M_p) of less than 10 %.
   3- Settling time (t_s) of less than 1.2 sec.

b) Frequency domain specifications:
   1- Modulus margin greater than 0.5, $\|S\|_\infty < 6$ dB.
   2- A maximum value of less than 6 dB of the input sensitivity function.
   3- Disturbance attenuation in the low frequency band from 0 to 0.2 Hz.

In the benchmark defined in European Control Conference in Rome, there was no condition on settling time (specification 3). Adding this specification makes the problem of designing the controller more difficult.

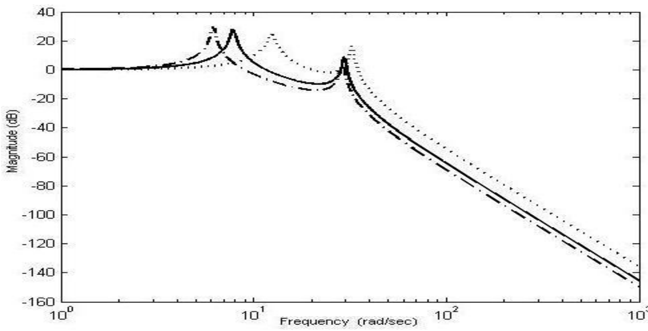

Figure 2. Freuency characteristic of the system, full load (dash-dot), half load (solid), no load (dot).

III. CONTROLLER SYNTHESIS

A. *MIXED SENSITIVITY PROBLEM*

We initiate the discussion by considering the feedback system shown in Figure 3 Let $S(s)=(1+GK)^{-1}$ and $T(s)=1-S(s)$ be sensitivity and complementary sensitivity transfer functions, respectively. In the mixed sensitivity problem the objective is to minimize the infinity norm shown below:

$$\left\| \begin{bmatrix} W_P S & W_R KS & W_T T \end{bmatrix}^T \right\|_\infty \quad (4)$$

where W_P , W_R and W_T are weight functions.

B. *WEIGHTING FUNCTIONS STRUCTURE*

According to robust control literature, there are several methods for selecting weight functions; we just outline clues used for weight selection in our benchmark problem. Due to changing the plant transfer function with loads, a type of perturbation must be used to model the system. According to the benchmark, it is assumed that G_{HL} is the nominal plant transfer function and G_{NL} is perturbed plant transfer function. To emphasize the effect of selecting weights structure on final results, two types of perturbation have been considered.

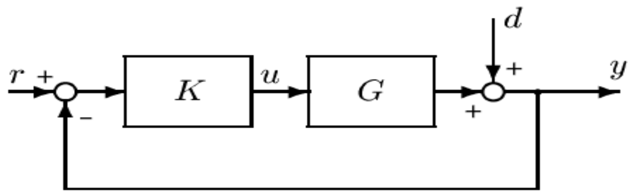

Figure 3. Closed loop system.

STRUCTURE 1: MULTIPLICATIVE UNCERTAINTY

Condition to achieve robust performance for a plant with multiplicative uncertainty is [4]:

$$\left\| |W_T T| + |W_P S| \right\|_\infty \prec 1 \quad (5)$$

where W_T is robust stability weight and should satisfy the following equation:

$$\left| \frac{G_{NL}}{G_{HL}} - 1 \right| \le |W_T(jw)|, \quad \forall w \quad (6)$$

A little time with bode magnitude plot, W_T can be calculated as (6) which satisfies (5).

$$W_T = \frac{(s+0.1)^2}{5(0.0001s+1)^2} \quad (7)$$

Note that weight functions wisely selected to be proper and stable, in order to use Robust Control Toolbox of MATLAB [5]. The term $(0.0001s+1)^2$ is added to meet this limitation. Gain and zero of W_T should be tuned, in order to achieve best performance. So the structure of W_T can be selected as bellow:

$$W_T = \frac{(s+a_1)^2}{a_2(0.0001s+1)^2} \quad (8)$$

The initial value for a_1 and a_2 will be 0.1 and 5 respectively, which will be used in optimization algorithms. W_P is nominal performance weight and can be determined by shaping closed-loop transfer function. The closed-loop transfer function which meets second and third specification is:

$$T(s) = \frac{(5.55)^2}{s^2+6.66s+(5.55)^2} \quad (9)$$

which is a standard second order system with t_s=1.2 and M_P=10 %. Then sensitivity function will be:

$$S(s) = 1 - T(s) = \frac{s(s+6.66)}{s^2 + 6.66s + (5.55)^2} \quad (10)$$

The weight function can be considered as $S^{-1}(s)$ which should be modified to be stable and strictly proper for better performance as follows:

$$W_P(s) = \frac{s^2 + 6.66s + (5.55)^2}{(s+0.00001)(s+6.66)(0.0001s+1)} \quad (11)$$

Therefore structure of W_P will be:

$$W_P(s) = \frac{s^2 + a_3 s + (a_4)^2}{a_5(s+0.00001)(s+a_6)(0.0001s+1)} \quad (12)$$

Initial values are taken as in (11). W_R is weight function on controller output and is taken to be constant with initial value of 1.

$$W_R = a_7 \quad (13)$$

STRUCTURE 2: ADDITIVE UNCERTAINTY

Condition for robust performance for additive perturbation is [4]:

$$\| |W_R KS| + |W_P S| \|_\infty \prec 1 \quad (14)$$

where W_R should satisfy the following equation:

$$|G_{NL} - G_{HL}| \le |W_R(jw)|, \quad \forall w \quad (15)$$

Again, a little time with bode magnitude plot, W_R can be calculated as:

$$W_R = \frac{3s^2}{(0.1s+1)^2} \quad (16)$$

So the structure of W_R will be:

$$W_R = \frac{a_1 s^2}{(a_2 s+1)^2} \quad (17)$$

The initial value of a_1 and a_2 will be chosen as (16) accordingly. Nominal performance weight W_P is like structure 1 and W_T is taken to be constant starting from 1.

C. SATISFACTORY PERFORMANCE INDEX

A performance index should be determined to evaluate different controller designs. Satisfactory performance index is defined as the average of satisfactory ratio computed for each specification. Computing of satisfactory ratio may vary depending on problem and desired specifications. The satisfactory ratio which is used for flexible transmission problem is defined 100% if a condition is achieved by a controller and is defined 0 if the corresponding characteristic is two times greater (or less) than the limited value. The intermediate values are computed by linear interpolation. This definition can be used for a large variety of problems. In some cases, it is possible that a number of specifications are more important than others. Since all the specifications might not be satisfied by mixed sensitivity controller design, the importance of the specifications should be considered in optimizing the parameters. This can be done by defining the satisfactory performance index as the weighted mean of the satisfactory ratio. Here weights indicate the importance of the specifications. It is obvious that if a controller destabilizes the system, its satisfactory performance index will be zero. In mixed sensitivity problem satisfactory performance index (η) depends on selecting weights, thus it is a function of weights' parameter:

$$\eta = F(\bar{a}), \quad \bar{a} = (a_1, a_2, ..., a_m) \quad (18)$$

where F is a scalar function and m is the number of parameters used in structure of weight functions. The goal is to maximize the performance index. However calculating an analytic form for F is very difficult (if not impossible), it is possible to find its maximum using software packages like MATLAB.

D. USING GENETIC ALGORITHM TO MAXIMIZE THE PERFORMANCE

As said before, the main goal is to maximize the performance index. Thus using Matlab robust control toolbox an m-file function is defined whose input is vector $\bar{a}$ and output is satisfactory performance index η. Various optimization methods can be used to maximize η including direct search (which needs the initial vales) and genetic algorithm.

E. SUMMARY

From the above analysis, a design procedure for improved performance mixed sensitivity problem is proposed as follows:

1- Defining the problem and desired specification.
2- Determining a structure for weight functions with some free parameters.
3- Determining a satisfactory performance index according to the problem and desired specification as a function of weight functions parameters.
4- Using optimization algorithms to tune weight function parameter in order to maximize the satisfactory performance index.

IV. SIMULATION RESULTS

The usefulness of the above procedure is demonstrated by designing an H_∞ controller for the benchmark problem illustrated in section II. Weights structure and satisfactory performance index used are the ones determined in previous section. The result for each structure is mentioned separately.

STRUCTURE 1: MULTIPLICATIVE UNCERTAINTY

Using weight functions (7) and (11) the resulting controller performance is not acceptable as specification for t_r and t_s are not achieved and performance index is about 55%. The proposed approach in [3], has tuned the weight functions parameters manually and increased the performance index to 84%. Using the procedure presented in the previous section, the resulting controller performance index improved to 92 %. Figure 4 shows the magnitude Bode plot of tuned weighting functions. Almost all specifications are indeed satisfied with a highly acceptable performance ratio except settling time for full loaded model. The simulation results for the three load cases are given in Figures 5-7 and Table 1.

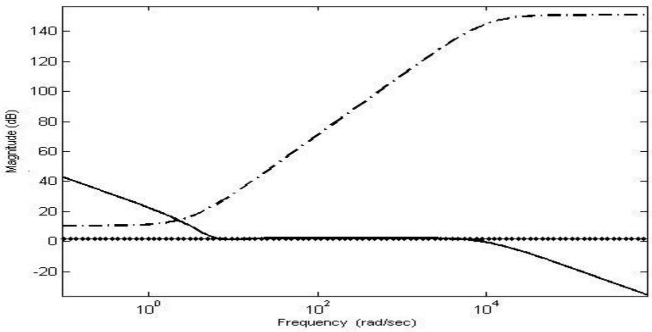

Figure 4. The Bode magnitude plot of tuned weighting functions (multiplicative perturbation), W_T (dash-dot), W_P (solid), W_R (dot).

Table 1: Resulting specifications for multiplicative uncertainty.

	No load	Half load	Full load	Performance ratio
tr (sec)	0.8	0.8	0.6	100 %
Mp (%)	1.5	2	5.8	100 %
ts (sec)	1.5	1.5	6	50 %
$\|\|S\|\|_\infty$ (dB)	3.3	3.3	5.8	100 %
$\|\|KS\|\|_\infty$ (dB)	0	0	0	100 %
$\|S(jw)\|$ (dB) for f < 0.2 Hz	-1.1	-1.2	-1.3	100%
Satisfactory Performance Index = 92 %				

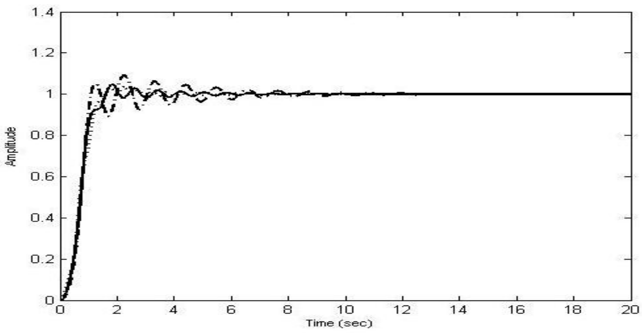

Figure 5. Closed-loop step response (multiplicative perturbation), full load (dash-dot), half load (solid), no load (dot).

STRUCTURE 2: ADDITIVE UNCERTAINTY

Again, using weight functions primarily computed will not satisfy specifications. The controller designed, using genetic algorithm for optimizing the parameters, satisfies specifications with about 90% performance indexes. The Bode magnitude plot of the tuned weighting functions has been shown in Figure 8. The simulation results are given in Figures 9-11 and Table 2.

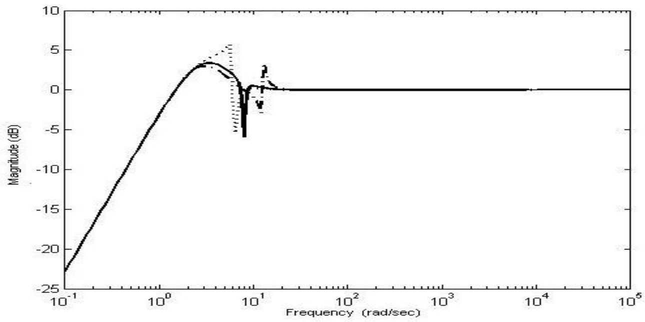

Figure 6. Output sensitivity function (multiplicative perturbation), full load (dot), half load (solid), no load (dash-dot).

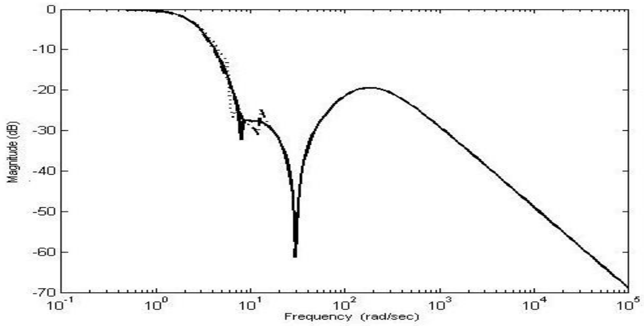

Figure 7. Input sensitivity function (multiplicative perturbation), full load (dot), half load (solid), no load (dash-dot).

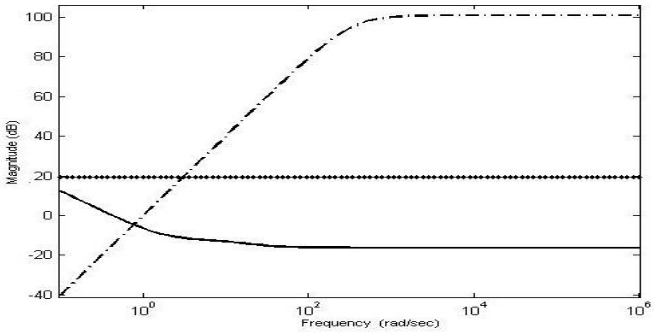

Figure 8. The Bode magnitude plot of tuned weighting functions (multiplicative perturbation), W_R (dash-dot), W_P (solid), W_T (dot).

Table 2: Resulting specifications for additive uncertainty.

	No load	Half load	Full load	Performance ratio
tr (sec)	1	1	0.7	100 %
Mp (%)	1.3	1.4	6	100 %
ts (sec)	1.8	1.7	7	35 %
$\|\|S\|\|_\infty$ (dB)	2.9	3.2	4.7	100 %
$\|\|KS\|\|_\infty$ (dB)	0	0	0	100 %
$\|S(jw)\|$ (dB) for f < 0.2 Hz	0	0	0	100%
Satisfactory Performance Index = 90 %				

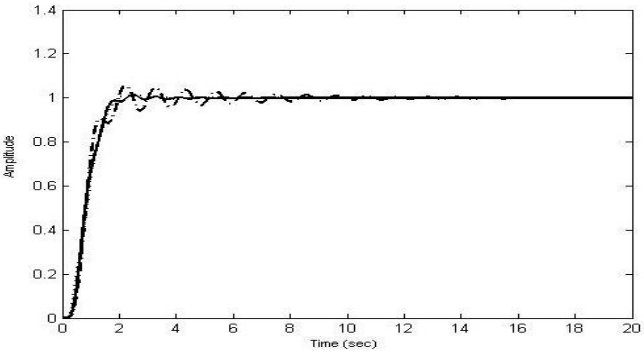

Figure 9. Closed-loop step response (additive perturbation), full load (dash-dot), half load (solid), no load (dot).

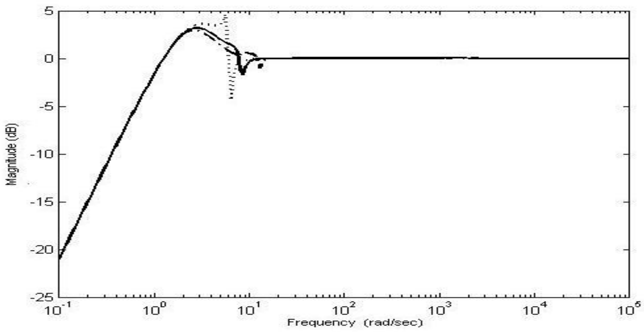

Figure 10. Output sensitivity function (additive perturbation), full load (dot), half load (solid), no load (dash-dot).

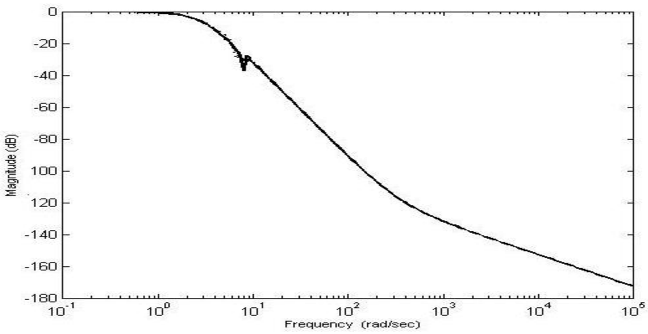

Figure 11. Input sensitivity function (additive perturbation), full load (dot), half load (solid), no load (dash-dot).

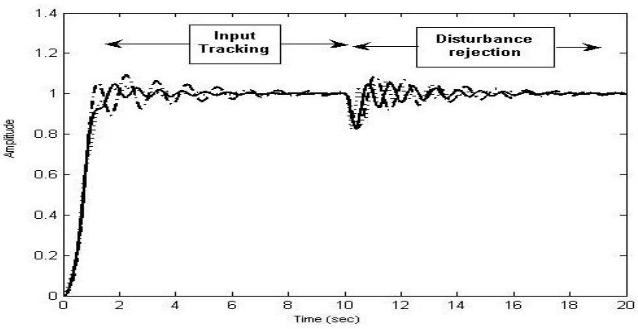

Figure 12. Output disturbance rejection response (multiplicative perturbation), full load (dash-dot), half load (solid), no load (dot).

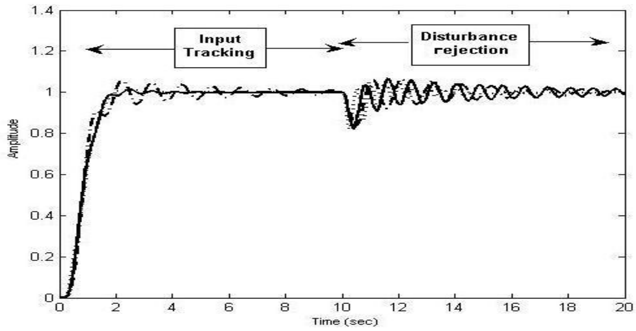

Figure 13. Output disturbance rejection response (additive perturbation), full load(dash-dot), half load (solid), no load (dot).

There is no condition on disturbance rejection time in problem specifications in section II (i.e. the time necessary to reject to 10 % of measured peak value a step output disturbance filtered by 1/A(s), where A(s) is the denominator of plant model). This objective have not used during optimization; however, the designed controllers have acceptable disturbance rejection as showed in Fig. 12 add Fig. 13. This and any other objective specifications can easily be considered in performance index and be optimized with genetic algorithm consequently.

V. CONCLUSION

In this paper, the problem of selecting weighting functions in mixed sensitivity problem has been studied. A structure for weighting functions has obtained and relating parameters is optimized using genetic algorithm to improve the robust performance. Application of results in improving the performance of flexible system has been demonstrated using two different types of perturbation. The final solutions achieve almost all specification with highly performance index.

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