Circuit Analysis Strategy and Algorithm Based on the Diakoptic Segregation Procedure

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Abstract

The proposed analysis strategy is based on the "segregation procedure", meaning the separation between the circuit linear part and the nonlinear one. All types of nonlinearities are accepted, excepting hysteresis phenomena. We accept the general case when the mutual inductors are distributed into different subcircuits. The diakoptic way is assumed for splitting the given circuit into subcircuits, the state variables approach to build the symbolic circuit equations for each subcircuit and the waveform relaxation method for finding the solution. Compared with common approaches of state equations, the main advantage of this procedure is related to the numerical computation quality, thanks especially to the lower order of the equation systems in the mathematical model.

1. Introduction

An electric circuit can contain both linear and nonlinear elements that are connected into specified topological structure. Diakoptic way [1,2] uses a division approach by partitioning a network into multipoles to achieve a high efficiency in the solution of large-scale networks. One of the published algorithms for partitioning is "central node tearing" [2,3]. In this paper, the basic idea for circuit partitioning is the "segregation procedure" [2,4], meaning the separation between the circuit linear part and the nonlinear one.

For mutual inductors, one possible way is to assign them, by partitioning, to the same subcircuit. Unfortunately, this way is not always adequate for latency exploitation or parallel implementation. A good choice of the partitioning strategy is essential for solving circuit equations by waveform relaxation (WR) method [4-6], a natural method which is well suited for parallel simulation and fully exploitation of latency and multirate behavior. In this paper, we accept the general case when the mutual coupling branches are distributed, by partitioning, into different subcircuits.

The analyzed circuit may contain excess elements of any types. The nonlinear elements – dynamic or resistive – are treated by local linearization, using dynamic parameters and incremental sources.

We shall assume the diakoptic strategies for splitting the given network into subnetworks, the state variables approach for formulating the subcircuits equations, and the WR method for finding, step by step, the correct solution.

A circuit example illustrating the proposed strategy is finally presented.

2. Diakoptic segregation procedure

The analyzed nonlinear circuit can be viewed as interconnection between many linear and nonlinear multipoles. The segregation procedure assumes the multipoles grouping [2,4], thus the linear and nonlinear partitions (l, respectively n) on the initial circuit can be separated by a cross section S [2,7].

The nets between subcircuits are cut while partitioning, and the connections are replaced by a set of voltage or/and current sources called "virtual sources" (see figure 1).

Concerning the computational time reduction, the best results are obtained using the same or complementary type virtual sources [2,8], without changing the subcircuits order of complexity. Therefore, any virtual voltage source forms $C-E$ loop, respectively any virtual current source forms $L-J$ cutset.

Let $\boldsymbol{v}(t)$ be the vector of virtual sources inputs, the same for the two subcircuits, and let $\boldsymbol{y}(t)$ be the vector of virtual sources outputs, the same for the two subcircuits.

The output variables vector $\boldsymbol{z}_n(t)$ is necessary only for the nonlinear subcircuit and contains the voltages of the tree resistors and the currents of the cotree resistors.

Concerning the two subcircuits, (l) and (n), the inputs of the controlled sources used for modeling the cross induction voltages [9] can be expressed as:

$$\begin{cases} \boldsymbol{u}_l^m = \boldsymbol{L}_{ln} \cdot \dot{\boldsymbol{x}}_n , \\ \boldsymbol{u}_n^m = \boldsymbol{L}_{nl} \cdot \dot{\boldsymbol{x}}_l , \end{cases} \tag{1}$$

where $\boldsymbol{L}_{ln}$, $\boldsymbol{L}_{nl}$ are matrices containing mutual inductances. Using the equivalent sources method [10], we express the state equations of the two subcircuits as:

$$\begin{cases} \dot{\boldsymbol{x}}_l = \boldsymbol{A}_l \boldsymbol{x}_l + \boldsymbol{B}_l \boldsymbol{u}_l + \boldsymbol{B}_{1l} \boldsymbol{v} + \boldsymbol{B}_l^m \boldsymbol{u}_l^m + \boldsymbol{B}_{2l} \dot{\boldsymbol{u}}_l , & (2) \\ \dot{\boldsymbol{x}}_n = \boldsymbol{A}_n \boldsymbol{x}_n + \boldsymbol{B}_n \boldsymbol{u}_n + \boldsymbol{B}_{1n} \boldsymbol{v} + \boldsymbol{B}_n^m \boldsymbol{u}_n^m + \boldsymbol{B}_{2n} \dot{\boldsymbol{u}}_n + \boldsymbol{B}_{3n} \boldsymbol{z}_n . & (3) \end{cases}$$

In connection with notations

$$\boldsymbol{A}_{l0} = \boldsymbol{1} - \boldsymbol{B}_l^m \boldsymbol{L}_{ln} \boldsymbol{B}_n^m \boldsymbol{L}_{nl} , \quad \boldsymbol{A}_{l1} = \left[\boldsymbol{A}_l \;\vdots\; \boldsymbol{B}_l^m \boldsymbol{L}_{ln} \boldsymbol{A}_n \right] ,$$

$$\boldsymbol{A}_{l2} = \left[\boldsymbol{B}_l \;\vdots\; \boldsymbol{B}_l^m \boldsymbol{L}_{ln} \boldsymbol{B}_n \right] , \quad \boldsymbol{A}_{l3} = \boldsymbol{B}_{1l} + \boldsymbol{B}_l^m \boldsymbol{L}_{ln} \boldsymbol{B}_{1n} , \tag{4}$$

$$\boldsymbol{A}_{l4} = \left[\boldsymbol{B}_{2l} \;\vdots\; \boldsymbol{B}_l^m \boldsymbol{L}_{ln} \boldsymbol{B}_{2n} \right] , \quad \boldsymbol{A}_{l5} = \boldsymbol{B}_l^m \boldsymbol{L}_{ln} \boldsymbol{B}_{3n} ,$$

$$A_{n0} = \mathbf{1} - B_n^m L_{nl} B_l^m L_{ln}\,,\quad A_{n1} = \left[B_n^m L_{nl} A_l \;\vdots\; A_n \right],$$

$$A_{n2} = \left[B_n^m L_{nl} B_l \;\vdots\; B_n \right],\quad A_{n3} = B_{1n} + B_n^m L_{nl} B_{1l}\,, \tag{5}$$

$$A_{n4} = \left[B_n^m L_{nl} B_{2l} \;\vdots\; B_{2n} \right],\quad A_{n5} = B_{3n}\,,$$

where A_{l1}, A_{l2}, A_{l4}, A_{n1}, A_{n2}, A_{n4} are partitioned matrices, the state equations (2) and (3) can be expressed as:

$$\begin{cases} A_{l0}\dot{x}_l = A_{l1}x + A_{l2}u + A_{l3}v + A_{l4}\dot{u} + A_{l5}z_n\,, & (6) \\ A_{n0}\dot{x}_n = A_{n1}x + A_{n2}u + A_{n3}v + A_{n4}\dot{u} + A_{n5}z_n\,. & (7) \end{cases}$$

The initial condition vectors are the following:

$$\begin{cases} x_l(t_0) = x_l^0 \\ x_n(t_0) = x_n^0\,, \end{cases} \qquad x(t_0) = x^0 = \begin{bmatrix} x_l^0 \\ x_n^0 \end{bmatrix}. \tag{8}$$

Concerning the whole given circuit, the state variables vector x and the independent inputs vector u can be expressed as:

$$x(t) = \begin{bmatrix} x_l(t) \\ x_n(t) \end{bmatrix},\quad u(t) = \begin{bmatrix} u_l(t) \\ u_n(t) \end{bmatrix}, \tag{9}$$

where each vector is structured in connection with the segregation procedure.

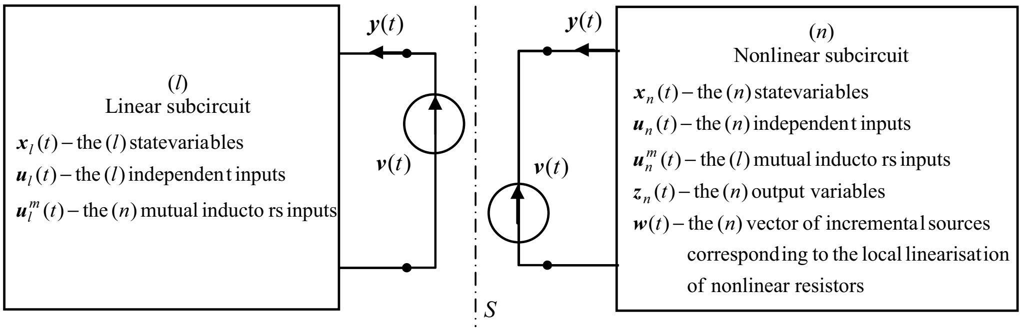

Fig. 1. Diakoptic segregation procedure

3. Liaison equations

The mathematical models of the two subcircuits, the linear and the nonlinear one, are coupled by liaison equations.

Using the partitioned matrix:

$$A_0 = \begin{bmatrix} A_{l0} & \mathbf{0} \\ \mathbf{0} & A_{n0} \end{bmatrix},\quad A_1 = \begin{bmatrix} A_{l1} \\ A_{n1} \end{bmatrix},\quad A_2 = \begin{bmatrix} A_{l2} \\ A_{n2} \end{bmatrix},$$

$$A_3 = \begin{bmatrix} A_{l3} \\ A_{n3} \end{bmatrix},\quad A_4 = \begin{bmatrix} A_{l4} \\ A_{n4} \end{bmatrix},\quad A_5 = \begin{bmatrix} A_{l5} \\ A_{n5} \end{bmatrix}, \tag{10}$$

the state equations (6) and (7) can be compacted into the form:

$$\begin{cases} A_0\dot{x} = A_1x + A_2u + A_3v + A_4\dot{u} + A_5z_n\,, \\ x(t_0) = x^0 - \text{initial conditions}\,. \end{cases} \tag{11}$$

The vector of virtual sources outputs for the linear subcircuit can be expressed as:

$$y = C_{l1}x_l + C_{l2}u_l + C_{l3}v\,. \tag{12}$$

The nonlinear subcircuit output equations can be expressed as:

$$B_{n0}z_n = B_{n1}x_n + B_{n2}u_n + B_{n3}v + w\,. \tag{13}$$

The vector of virtual sources outputs for the nonlinear subcircuit can be expressed as:

$$y = C_{n1}x_n + C_{n2}u_n + C_{n3}v + C_{n4}z_n\,. \tag{14}$$

We used the notations x_l for the linear subcircuit state variables vector, x_n for the nonlinear subcircuit state variables vector, u_l for the linear subcircuit independent inputs, respectively u_n for the nonlinear subcircuit independent inputs. The matrices A_l, B_l, B_{1l}, B_{2l}, B_l^m are always constant, but A_n, B_n, B_{1n}, B_{2n}, B_{3n}, B_{n0}, $B_{n1} \div B_{n3}$, w are generally state dependent matrices [2,8].

From equations (12) and (13), using the notations

$$C_0 = C_{l3} - C_{n3}\,,\quad C_1 = \left[-C_{l1} \;\vdots\; C_{n1} \right],$$

$$C_2 = \left[-C_{l2} \;\vdots\; C_{n2} \right],\quad C_3 = C_{n4}\,, \tag{15}$$

we obtain the liaison linear algebraic equation

$$C_0v = C_1x + C_2u + C_3z_n\,. \tag{16}$$

This equation assures the liaison between the two subcircuits and, mathematically, the liaison between proposed algorithm stages.

4. The solving algorithm

The proposed solving algorithm includes the following steps:

a) Perform an analysis time division in discrete values: $t_0, t_1, \dots, t_k, t_{k+1}, \dots, t_{final}$.

b) Starting from initial conditions $\boldsymbol{x}^0$, compute the output variables $\boldsymbol{z}_n^0$ for the moment $t = t_0$, using a classic method [2,11].

c) Solve the linear algebraic equation (16) for $t = t_0$:

$$\boldsymbol{C}_0\boldsymbol{v} = \boldsymbol{C}_1\boldsymbol{x}^0 + \boldsymbol{C}_2\boldsymbol{u}(t_0) + \boldsymbol{C}_3\boldsymbol{z}_n^0, \tag{17}$$

obtaining the virtual sources inputs for this moment, $\boldsymbol{v}^0 = \boldsymbol{v}(t_0)$.

d) Solve the nonlinear algebraic equation (13) for $t = t_1$, using a specific iterative method [2,11]:

$$\boldsymbol{B}_{n0}\boldsymbol{z}_n = \boldsymbol{B}_{n1}\boldsymbol{x}^0 + \boldsymbol{B}_{n2}\boldsymbol{u}_n(t_1) + \boldsymbol{B}_{n3}\boldsymbol{v}^0 + \boldsymbol{w}. \tag{18}$$

Using the previous calculated values $\boldsymbol{z}_n^0$ as start point, we obtain the vector $\boldsymbol{z}_n^1 = \boldsymbol{z}_n(t_1)$.

e) Solve the state equation (11) in the form:

$$\begin{cases} \boldsymbol{A}_0\dot{\boldsymbol{x}} = \boldsymbol{A}_1\boldsymbol{x} + \boldsymbol{A}_2\boldsymbol{u}(t_1) + \boldsymbol{A}_3\boldsymbol{v}^0 + \boldsymbol{A}_4\dot{\boldsymbol{u}}(t_1) + \boldsymbol{A}_5\boldsymbol{z}_n^1 \\ \boldsymbol{x}(t_0) = \boldsymbol{x}^0 \end{cases} \tag{19}$$

finding the solution

$$\boldsymbol{x}^1 = \boldsymbol{x}(t_1) = \begin{bmatrix} \boldsymbol{x}_l(t_1) \\ \boldsymbol{x}_n(t_1) \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_l^1 \\ \boldsymbol{x}_n^1 \end{bmatrix}.$$

f) For $t = t_{k+1}$ solve as follows:

- The algebraic nonlinear equation (13), starting from $\boldsymbol{z}_n^k$ and using the previous calculated values $\boldsymbol{x}_n^k$ and $\boldsymbol{v}^k$:

$$\boldsymbol{B}_{n0}\boldsymbol{z}_n = \boldsymbol{B}_{n1}\boldsymbol{x}^k + \boldsymbol{B}_{n2}\boldsymbol{u}_n(t_{k+1}) + \boldsymbol{B}_{n3}\boldsymbol{v}^k + \boldsymbol{w}. \tag{20}$$

It results $\boldsymbol{z}_n^{k+1}$.

- The state equation (11) in the particular form

$$\begin{cases} \boldsymbol{A}_0\dot{\boldsymbol{x}} = \boldsymbol{A}_1\boldsymbol{x} + \boldsymbol{A}_2\boldsymbol{u}(t_{k+1}) + \boldsymbol{A}_3\boldsymbol{v}^k + \boldsymbol{A}_4\dot{\boldsymbol{u}}(t_{k+1}) + \boldsymbol{A}_5\boldsymbol{z}_n^{k+1} \\ \boldsymbol{x}(t_k) = \boldsymbol{x}^k \end{cases} \tag{21}$$

finding

$$\boldsymbol{x}^{k+1} = \boldsymbol{x}(t_{k+1}) = \begin{bmatrix} \boldsymbol{x}_l^{k+1} \\ \boldsymbol{x}_n^{k+1} \end{bmatrix}.$$

- The liaison linear algebraic equation (16) in the particular form

$$\boldsymbol{C}_0\boldsymbol{v} = \boldsymbol{C}_1\boldsymbol{x}^{k+1} + \boldsymbol{C}_2\boldsymbol{u}(t_{k+1}) + \boldsymbol{C}_3\boldsymbol{z}_n^{k+1}, \tag{22}$$

finding $\boldsymbol{v}^{k+1}$.

g) Repeat the step f) for all the discrete moments of analysis time, finding the solutions $\boldsymbol{x}(t)$, $\boldsymbol{z}_n(t)$.

5. Example

Let us analyze the transient behavior of the nonlinear circuit shown in fig. 2. It is a common half wave rectifier with fitting transformer, *RL* load and power factor correction. Besides adjusting the level of the load voltage, the fitting transformer works like an excellent filter of the current drawn from the power system.

Thus, the given circuit contains the short-circuit impedance of the power system, the *RC* snubber circuit of the switching device and parasitic parameters of the fitting transformer. The switching device is modeled by a nonlinear voltage controlled resistance. The linear subcircuit and the nonlinear one obtained by splitting the given circuit with the cross section *S*, are shown in fig. 3. Neither circuit contains excess elements.

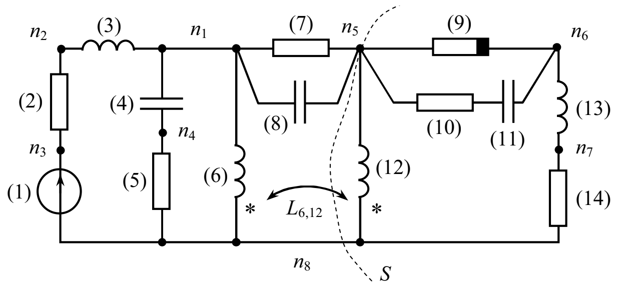

Fig. 2. Example circuit

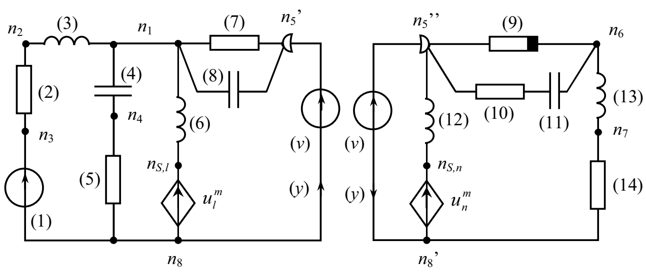

Fig. 3. Linear and nonlinear subcircuit

The state and output variables are:

$$\begin{cases} \boldsymbol{x}_l = [v_4 \quad v_8 \quad i_3 \quad i_6]^{\mathrm{t}} \\ \boldsymbol{x}_n = [v_{11} \quad i_{12} \quad i_{13}]^{\mathrm{t}} \end{cases}$$

$$\boldsymbol{z}_n = v_9.$$

Thus, we have the controlled voltage sources:

$$\begin{cases} u_l^m = L_{6,12} \cdot \dot{i}_{12} \\ u_n^m = L_{6,12} \cdot \dot{i}_6 . \end{cases}$$

The state equations of type (2) and (3), as well as the virtual sources outputs of type (12) and (14), built automatically by our dedicated program, are:

$$\begin{cases} C_4 \dot{v}_4 = -G_5 v_4 + G_5 v_8 + G_5 v \\ C_8 \dot{v}_8 = G_5 v_4 - (G_5 + G_7) v_8 + i_3 - i_6 - G_5 v \\ L_3 \dot{i}_3 = -v_8 - R_2 i_3 + v_1 - v \\ L_6 \dot{i}_6 = v_8 - u_l^m + v \end{cases}$$

from where

$$y = -G_5 v_4 + G_5 v_8 - i_3 + i_6 + G_5 v ,$$

respectively

$$\begin{cases} C_{11} \dot{v}_{11} = -G_{10} v_{11} + G_{10} v_9 \\ L_{12} \dot{i}_{12} = -u_n^l + v \\ L_{13} \dot{i}_{13} = -R_8 i_{13} - v_9 + v , \end{cases}$$

from where

$$y = -i_{12} - i_{13} .$$

The liaison equation of type (16) is now obvious:

$$G_5 v = -G_5 v_8 + i_3 - i_6 - i_{12} - i_{13} + G_5 v_4 .$$

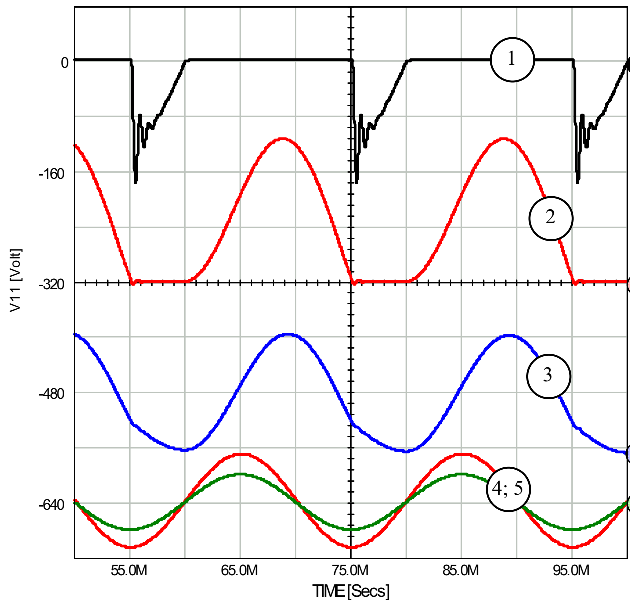

Fig. 4. Some results of the numerical computation

The output equation of type (13) is:

$$(G_{d9} + G_{10}) v_9 + G_{10} v_{11} - i_{13} + w_9 = 0 ,$$

where G_{d9} is the dynamic conductance of the nonlinear element and w_9 is the incremental independent current source introduced by the local linearization.

The above prepared symbolic equations allow using the solving algorithm described in section 4. Some results of the numerical computation are shown here, in qualitative manner only.

Also, in fig. 4 there are given: the state variable v_{11} (curve 1); the state variable i_{13} (curve 2); the state variable i_6 (curve 3); the network voltage and the drawn current (as synchronized curves 4 and 5, showing the effect of the power factor correction).

6. Conclusions

A new circuit analysis strategy is presented and its associated robust algorithm too. Using the "segregation procedure", the diakoptic way is assumed for splitting the given large-scale circuit into subcircuits and the state variables approach to build the subcircuit equations. An algebraic linear equation assures the liaison between the linear subcircuit and the nonlinear one.

The major advantages of the proposed analysis strategy and algorithm are the following:

- does not require topologic restrictions excepting the consistency assumption; the analyzed circuit may contain any type of excess elements, mutual inductors distributed between the two subcircuits and any type of nonlinear elements;
- does not require matrices inversion if the topological formulation of subcircuits state and output equations is based on a suitable method;
- offers a convenient way for equations matrices construction;
- assures a good computational stability and convergence, also a remarkable accuracy guaranteed by the convenient choice of computing algorithms;
- the computational effort and analysis time are reduced as compared to the previous studied diakoptic methods.

7. References

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