AN ANALYSIS OF THE LOWEST ORDER MODE IS PROPAGATING IN THICK-WALLED PARALLEL PLATE WAVEGUIDE

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ABSTRACT

In the radiation problem, because of the lowest order mode don’t propagate under whatever condition, to determinate the lowest order mode is propagating, which is very imported for engineering and mathematics applications. Sometimes, it is very complicate to obtain the necessary condition to ensure that the lowest order mode is propagating. In this case, graphically solution is applied.
In this study, the lowest order mode is propagating in dielectric loaded thick-walled parallel plate waveguide, which was investigated through graphically solution.

I. INTRODUCTION

Radiation from open-ended parallel-plate waveguide has been subjected to numerous investigations. In the present work, we consider the lowest order mode is propagating in dielectric filled impedance waveguide junction with a thick dielectric half-plane. In the case, where thick half-plane and inside region of waveguide has a different nonmagnetic dielectric material.

In this context, the problem of radiation from two parallel semi-infinite plates of zero thickness has been considered [1-3]. In other hands one was investigated the radiation from an open waveguide with reactive walls, which is a canonical model simulating an impedance loaded horn and horn type surface wave launchers [4]. Later, this work was generalized [5]. In all of these works, the lowest order mode is propagating, which was investigated with different methods under different conditions.

II. MATHEMATICAL ANALYSIS

We consider the radiation of the dominant TE mode wave which is incident from the left in the parallel plate region formed by two semi-infinite impedance plates defined by the impedance half plane

$S_1 = \{(x,y,z); x \in (-\infty,0), y \in (a,b), z \in (-\infty,\infty)\}$ and

$S_2 = \{(x,y,z); x \in (-\infty,0), y \in (-b,-a), z \in (-\infty,\infty)\}$

respectively, as depicted in Fig. 1.

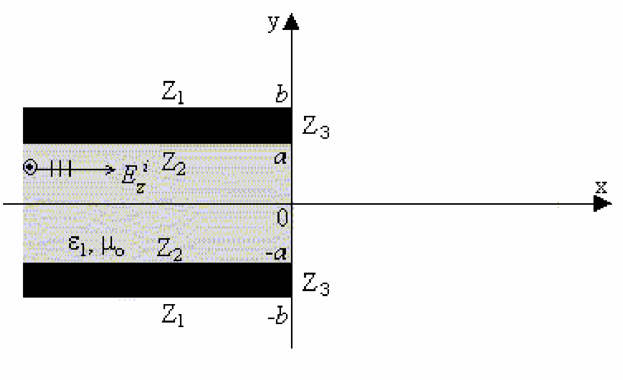

Figure 1. Geometry of radiation problem

The surface impedances of the horizontal walls $y = \pm b$, $x < 0$, and $y = \pm a$, $x < 0$ are denoted by $Z_1 = \eta_1 Z_0$ and $Z_2 = \eta_2 Z_0$ respectively, while the impedance of the vertical walls $x=0$, $y \in (a,b)$ and $x = 0$, $y \in (-b,-a)$ is $Z_3 = \eta_3 Z_0$, with Z_0 being the characteristic impedance of the free space.

Here, k_0 is the free space wave number which is assumed to have a small positive imaginary part and denoted by $k_0 = \omega\sqrt{\varepsilon_0 \mu_0}$ with ε_0 and μ_0 being the dielectric permittivity and the magnetic permeability of the free space. The lossless case can then be obtained by making $\Im m(k_0) \to 0$ at the end of the analysis. k_1 is the wave numbers denoted by $k_1 = \omega\sqrt{\varepsilon_1 \mu_0}$ with ε_1 being the dielectric permittivity in the regions $x < 0$, $y \in (-a,a)$.

Incident wave that satisfy the boundary conditions:

$$E_z^i = u^i(x,y) = [A_0 \sin[\xi_0 y] + B_0 \cos[\xi_0 y]e^{i\beta_0 x} \qquad (1a)$$

With

$$A_0 = \left(\cos\xi_0 a + \frac{\eta_2}{ik_1}\xi_0 \sin\xi_0 a\right), \quad B_0 = \left(\sin\xi_0 a - \frac{\eta_2}{ik_1}\xi_0 \cos\xi_0 a\right) \qquad (1b)$$

The configuration is two dimensional and the assumed incident field, only three field components namely,

$$E_z^i = u^i(x,y), \quad (1c)$$

$$H_x = \frac{-i}{\omega\mu_0}\frac{\partial}{\partial y}u^i(x,y), \quad (1d)$$

and

$$H_y = \frac{i}{\omega\mu_0}\frac{\partial}{\partial x}u^i(x,y) \quad (1e)$$

are nonzero.

β_0 in the incident wave equation is the lowest real root of the characteristic equation

$$L(\alpha)=\left(\cos[K_1 a]+\frac{\eta_2}{ik_1}K_1\sin[K_1 a]\right)\left(\sin[K_1 a]-\frac{\eta_2}{ik_1}K_1\cos[K_1 a]\right)=0 \quad (2a)$$

where

$$K_1(\alpha)=\sqrt{k_1^2-\alpha^2} \quad (2b)$$

and ξ_0 is defined by

$$\xi_0 = K_1(\beta_0) \quad (2c)$$

The square-root function is defined in the complex α-plane cut along $\alpha = k_1$ to $\alpha = k_1 + i\infty$ and $\alpha = -k_1$ to $\alpha = -k_1 - i\infty$ such that $K_1(0) = k_1$. Note that equation (2a) admits an infinite of symmetric roots denoted by $\pm\beta_n$ satisfying

$$L(\beta_n) = 0,\ \Im m(\beta_n) > \Im m(k_1) \quad (2d)$$

with

$$\xi_n = K_1(\beta_n)\ n = 0,1,2,.. \quad (2e)$$

Equation (2a) has two multiplier which we suppose β_n^e satisfying

$$\left(\cos[K_1 a]+\frac{\eta_2}{ik_1}K_1\sin[K_1 a]\right)=0 \quad (2f)$$

and β_n^o satisfying

$$\left(\sin[K_1 a]-\frac{\eta_2}{ik_1}K_1\cos[K_1 a]\right)=0 \quad (2g)$$

In these two equation, we can write $K_1 a$ as x, η_2 as $i\tilde{\eta}_2$ and $\frac{\tilde{\eta}_2}{k_1 a}$ as A for mathematical easiness. Thus the equations (2f) and (2g) are converted to

$$\cot x = -Ax \quad (2h)$$

and

$$\tan x = Ax \quad (2i)$$

respectively. β_n can be written as $\beta_n = \sqrt{\frac{(k_1 a)^2 - x_n^2}{a^2}}$, $n = 0,1,2,..$ The necessary condition to ensure that the lowest order mode is propagating in the waveguide one can take into account the slopes, -*A* and *A*, of the equations (2h) and (2i) respectively.

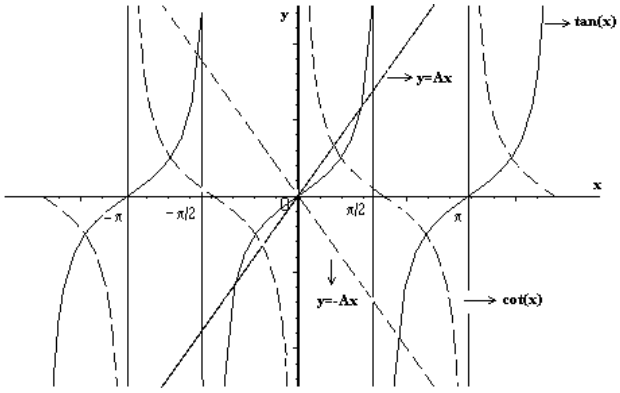

Figure 2. Graphically solution of tanx=Ax, cotx=-Ax equations.

There are three states for A.

I) If $A > 1$: First root of $\tan x = Ax$ is in the interval $x \in (0, \frac{\pi}{2})$. First root of $\cot x = -Ax$ is in the interval $x \in (\frac{\pi}{2}, \pi)$. If $k_1 a < \frac{\pi}{2}$, there is not real root for $\cot x = -Ax$. There is only one real root is root for $\tan x = Ax$ in the interval $x \in (0, \frac{\pi}{2})$ and $x_n^o < k_1 a < \frac{\pi}{2}$.

II) If $0 < A < 1$: There is not any root for $\tan x = Ax$ in the interval $x \in (0, \frac{\pi}{2})$. its first root is belong to the interval $x \in (\pi, \frac{3\pi}{2})$. $\cot x = -Ax$ has first root in the interval $x \in (\frac{\pi}{2}, \pi)$. If $x_n^o < k_1 a < \pi$, first and one real root is β_0^e; else if $x_n^o < k_1 a < \frac{3\pi}{2}$, there are two real root which satisfy $\beta_0^o < \beta_0^e$.

III) If $A < 0$: First root of the equations $\tan x = Ax$ and $\cot x = -Ax$ is in the intervals $x \in (\frac{\pi}{2}, \pi)$ and $x \in (0, \frac{\pi}{2})$, respectively. If $x_n^o < k_1 a < \frac{\pi}{2}$, there is only one real root named β_0^e else If $x_n^o < k_1 a < \pi$, there are two real roots satisfy $\beta_0^o < \beta_0^e$.

To satisfy dominant mode wave propagation in the waveguide one must take into account both the conditions $\tilde{\eta}_2 > k_1 a$ and $x_n^o < k_1 a < \frac{\pi}{2}$. In these conditions one can obtain dominant mode propagation constant β_0^o which belongs to equation (2g). Coefficient B_0 in equation (1b) is zero.

After all, consequently, the incident field expression $u^i(x,y)$ can be written as follows:

$$E_z^i = u^i(x,y) = \left(\cos\xi_0 a + \frac{\eta_2}{ik_1}\xi_0\sin\xi_0 a\right)\sin[\xi_0 y]e^{i\beta_0 x} \quad (2j)$$

III. CONCLUSION

In this study, the lowest order mode is propagating in dielectric loaded thick-walled parallel plate waveguide, which was investigated through graphically solution. Since Verification of the truthness of this solution, radiation of the end of waveguide was investigated. Existing of the mode is propagating in dielectric loaded thick-walled parallel plate waveguide, which was observed, for different values of the η_2. For the values except for the necessary condition to ensure that the lowest order mode is propagating in the waveguide, one was observed that any mode is propagating. This is guarantied that only one and the lowest order mode is exist.

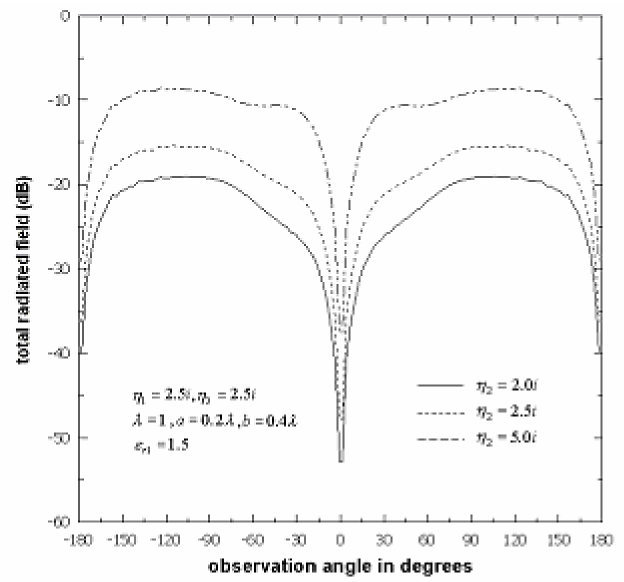

Figure 10. Radiated field amplitude versus the observation angle for different values of the η_2

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