An Efficient Approach for the Solution of Inverse Scattering Problems via Neural Network Method

Burak Gürlek[1], Ali Yapar[1]

[1]Electronics and Communications Eng. Dept., Istanbul Technical University, 34469, Istanbul, Turkey
gurlekb@itu.edu.tr, yapara@itu.edu.tr

Abstract

In this paper, an efficient approach to the solution of inverse scattering problem related to circular hole buried in a wall is presented. The approach is based on neural network method in which the reconstruction domain is first restricted to a narrower one by Electromagnetic Back Propagation Method (EBPM), and then classical neural network method is applied. Comparisons between the classical and proposed neural network solution of the problem show that the present method can detect the circular crack more accurately than classical solution for the same number of education set element. Also proposed solution provides faster education than classical solution by means of restriction. The approach may also be used to accelerate the neural network based methods for more complex inverse scattering problems.

1. Introduction

Detection of small cracks or inhomogeneities within a known host medium is an important problem since it has a wide range of applications in the areas of construction engineering, aero-nautics, material science, biomedical etc. This problem can be considered as a nondestructive testing one since the main aim is to determine the location, size and the physical parameters of inhomogeneities in a known material by the remote measure-ments. The concrete structures in civil engineering applications, composite materials in airplane and related industries, and an organ of body in medical sciences are the examples of such host media. From the electromagnetic point of view the solution strategies for the aforementioned problem can be simply classified into two main approaches. In the radar based approaches pulses in time domain used in order to obtain the information about the inhomogeneity [1], while the second approach is based on frequency domain nonlinear integral equations [2]. Within this framework a neural network based method for the determination of the location of circular holes in a wall is presented in this study by integral equation based formalism. For the sake of simplicity a 2-D formulation is investigated and thus the problem is reduced to a scalar one. The entire reconstruction domain (the wall) is first restricted to a smaller rectangular region by Electromagnetic Back Propagation Method (EBPM) which substantially reduces the computational cost and increases the efficiency of the neural network method. Then the conventional neural network approach is applied for the solution of the problem. As it is shown by numerical examples, this two-step algorithm is very effective and reliable. The details of the algorithm and the numerical results are given in the following sections. Throughout the paper e^{-iwt} time convention is used.

2. Formulation of the Problem

Consider the problem whose generic configuration is given in Figure 1, where a lossy dielectric wall with parameters ε_r=4, σ=0.001 and d=0.2 m, stands for the thickness, is placed in free space. Within the wall a circular cylindrical hole D, whose cross sectional area is denoted by S models the crack. Let us denote z component of incident, scattered and total electric fields by E^i

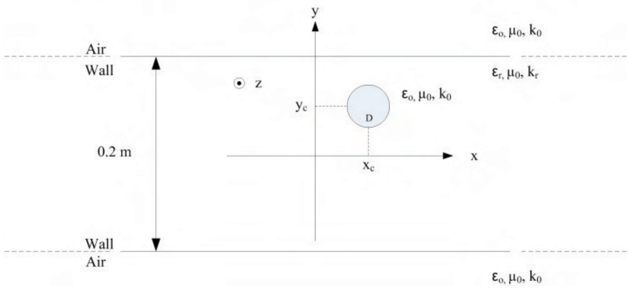

Fig. 1. Geometry of the problem

E^s, and E, respectively. From these definitions and using 2-D Green's theorem with Helmholtz equation in three part space, one can formulate the problem by the following electric field integral equations

$$E^s(x,y) = k_r^2 \iint_S G(x,y;x',y')\left(\frac{k_0^2}{k_r^2} - 1\right)E(x',y')dx'dy' \qquad (1)$$

$$E(x,y) = E^i(x,y) + k_r^2 \iint_S G(x,y;x',y')\left(\frac{k_0^2}{k_r^2} - 1\right)E(x',y')dx'dy' \qquad (2)$$

which are also known as data and object equations, respectively. In (1) and (2) G is the Green's function of there part space, $k_0^2 = w^2\varepsilon_0\mu_0$ is the square of the wave number of free space, $k_r^2 = w^2\varepsilon_0\varepsilon_r\mu_0 + iw\sigma\mu_0$ is the square of the wave number of the wall and primed coordinates is defined over the cross-section of the circle D. Equation (1) can be used for calculating the scattered field at any point in the whole space, provided that the total electric field over crack is known. On the other hand the total electric field can be easily obtained by using the object equation. In this work this type of integral equations are solved by using Method of Moments (MoM) [3] numerically.

The main aim of the inverse scattering problem considered in the present study is to find the location of the crack D by using the scattered field measurements performed on a line parallel to the wall. For this aim we suggest a two-step algorithm, details are given below.

3. Neural Network Solution of Inverse Scattering Problem

Neural network, inspired from human neural system, is widely used as a computation tool. Like human's neural system, neural network has simple computation unit called neuron and uses it to construct a network. This computation network can be used to model every system easily, since it constitutes a manifold which characterizes input/output relationship of the system.

The most critical point of neural network approach is training process. To model any system, neural network must learn input/output relation of the system in training process. After successive training step neural network can be used to solve trained system easily. Therefore accuracy of solution heavily depends on quality of training process. For modeling complex systems neural network require large training set which is obtained by sampling the investigated system densely. Therefore usually training process is time and CPU consuming for successive learning [4].

Most of the inverse scattering methods in frequency domain, equation (1) is used to invert the data. Because of its non-linearity and compactness of its kernel, inverse scattering problems are complex and cannot be solved easily and directly. According to described advantages, neural networks may be used to overcome this difficulty despite of its complex training process. This idea has applied to many problems and quite satisfactory results have been obtained [4 - 8].

In inverse scattering problems the only known quantity is the measured scattered field data. With the aid of this data, physical properties of object and indirectly its location is aimed to reconstruct. So input of neural network must be scattered field data, and outputs are determined by the nature of problem which also called a priori information. In our problem cracks are modeled with circular cylinders along z-axis and clearly its inside is air - filled, so its shape and dielectric properties are known. This a priori information provides a simpler inverse scattering problem for neural network. Thus only unknown parameter about crack is its location and therefore outputs of neural network system are coordinates of cracks in 2D Cartesian coordinate plane. The neural network model can be seen in Figure 2.

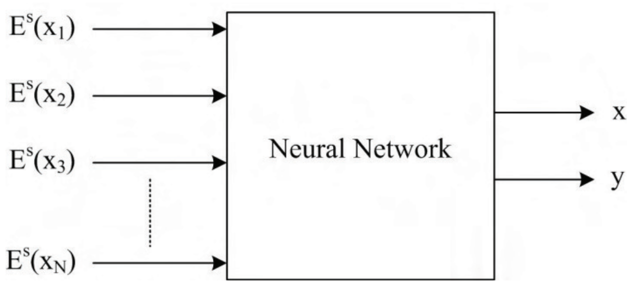

Fig. 2. Neural network model for given inverse scattering problem for N observation point

To solve the prescribed problem a multi - layer feed forward neural network with single hidden layer and sigmoid activation function are used. According to Cybenko's Universal Approximation Theorem [9] this type of neural network is sufficient to approximate every continuous function which defines input/output relationship of any system [10].

Training of phase is realized by example based learning process. To achieve this, training set which consists of scattered field values for each known crack location in observation points are necessary. Training set can be easily constituted from solution of (1) by using MoM for known crack locations numerically. Also this solution is used for construction of test set. After the construction of training set, Error Back Propagation algorithm which uses gradient descent method is used to train neural network [10].

4. Electromagnetic Back Propagation Method and Its Application to Neural Network Approach

4.1. Electromagnetic Back Propagation Method

Electromagnetic Back Propagation Method (EBPM) is one of the fundamental and basic solution approaches for inverse scattering problems. It can be used to provide initial value about dielectric properties of object needed in iterative inverse scattering algorithms. As many inverse scattering methods, its mathematical foundation is originated from data equation (1).

Let us define a function

$$F(x',y') \triangleq \left(\frac{k_0^2}{k_r^2} - 1\right) E(x',y'), \tag{3}$$

which we called density function whose support determines the cross section S. By this definition the data equation can be reduced to

$$E^s(x,y) = k_r^2 \iint_R G(x,y;x',y') F(x',y') dx'dy', \tag{4}$$

where R denotes the reconstruction domain. As we also mentioned in previous sections, in inverse scattering problems scattering fields and Green's function of corresponding space is known. MoM is used to solve (4) to obtain the density function over reconstruction domain. By definition, this function should be zero everywhere in R except the cross section S of crack. Note that the equation (4) is an ill – posed one by nature and only an approximation of F can be obtained by applying a regularization scheme. This is done here by classical Tikhonov Regularization [11].

Although the support of density function determines the cross section of crack theoretically only a rough approximation can be extracted from the regularized solution and therefore an approximate region can be determined by observing the variation of F. As a result, the reconstruction domain for neural network approach can be restricted to a narrower one which reduces the computational cost of the algorithm substantially. Furthermore it is observed that the accuracy of the results can be enhanced by this preprocess. This result of EMPM motivates us to proposed new method on neural network solution of inverse scattering problem.

4.2. Application of Electromagnetic Back Propagation Method to Neural Network Approach

As mentioned before training process is the most important step for neural network. Because of the training process, neural network establishes a relationship between input and output of system. Quality of this relationship determines the interpolation ability of neural network in output space and accuracy of nonlinear system function approximation. For working of neural network with low order error, training set samples for the given problem also sample nonlinear system functions densely. This increases the number of element in training set. For example, consider a big reconstruction domain, in order to establish successive relationship between input and output, reconstruction domain is sampled smoothly which cause more training points relative to smaller one.

To overcome complexity of training process we proposed a method which is application of EBPM to neural network approach. We know from previous section that EBPM can be used to acquire coarse information about the location of crack. In proposed method first EBPM is applied to scattered field data. After application of EBPM a region which includes the crack can be determined easily. This coarse location information is used to restrict reconstruction domain of inverse scattering problem. Neural network is trained over restricted re-construction domain, than scattered field is applied to neural network and output is determined. The Most important advantage of this method is restriction of reconstruction domain around cracks. With the aid of this preprocess, neural network learns smaller area than the original one, namely the wall itself. It is expected that this proposed method provide shorter training time and simplify the training process also for neural network solution.

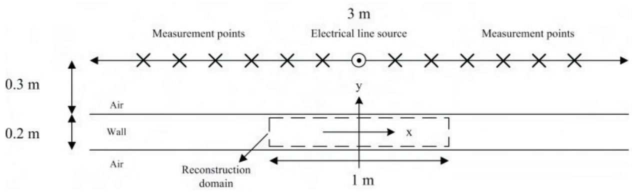

Fig. 3. Geometry of scattered field measurement

5. Numerical Results

In this section the numerical results related to wall imaging problem whose configuration is shown in Figure 3 will be given. The main aim here is to compare the efficiency of proposed and classical neural network solutions. Before this comparison, it is investigated that how EBPM gives a location information about location of crack. Within this context a measurement set up given in Figure 3 is considered where the scattered field is assumed to be measured at 60 equally spaced observation points, 0.3m far away from wall along 3m in x coordinate. Crack is searched in a reconstruction domain having the dimensions 1x0.2 m^2. In all numerical examples the illuminations are carried out by using monochromatic electrical line source whose operating frequency is 500 MHz located at y=0.3m along z axis.

Crack is modeled as a circular cylinder whose radius is $\lambda/15$ where λ is wavelength in wall. For simulating noisy case a random term $n_\ell \left|E^s\right| e^{2ir_d\pi}$ is added to each scattered field data E^s at measurement points where n_ℓ is the noise level and r_d a random number between 0 and 1.

Capability of determining approximate crack location via EBPM can be seen in Figure 4 and 5. In Figure 4, the amplitude of the density function is plotted for noise free case. As can be seen from Figure 4, the location of crack in x direction can be clearly determined in the region (0.2 – 0.3) by observing the density function in reconstruction domain. The variation of the amplitude of density function for the same configuration with 10% noisy data is shown in Figure 5. It is obvious from the figure that again a satisfactory approximation for the location of crack is determined in the interval (0.2 – 0.3) in x direction for the noisy case. It can be seen from these results that EBPM can be efficiently used to restrict reconstruction domain for neural network solution.

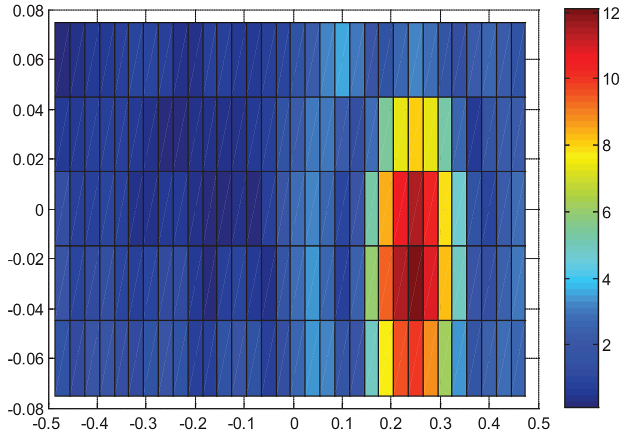

Fig. 4. Dielectric profile of reconstruction domain via EBPM; 1 cm crack located at (0.2, -0.03); noise free

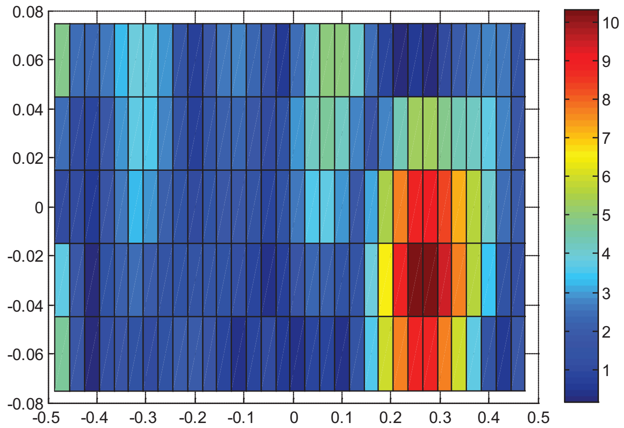

Fig. 5. Dielectric profile of reconstruction domain via EBPM; 1 cm crack located at (0.2, -0.03); 10% noise level

To compare proposed and classical neural network method, in given measurement geometry equally spaced 20 scattered fields is chosen from 60 measured scattered field data and this 20 values are used as input of neural network along x dimension of reconstruction domain. Since the measured scattered field has real and complex parts, 40x20x2 network used to solve inverse scattering problem. In order to compare the classical neural network and presented one the problem is solved by the parameters; 100 training point and 20 test point. For performance analysis, neural network run 15 times with same test set to acquire static behavior of network. All given values are mean of 15 results of neural network for both proposed and classical solution.

For comparison of proposed and classical neural network solution of inverse scattering problem, defined problem is trained for same number of element and tested for 20 random points over reconstruction domain. Output of classical neural network for test set is given in Figure 6. The results of the presented method for the given test set are shown in Figure 7

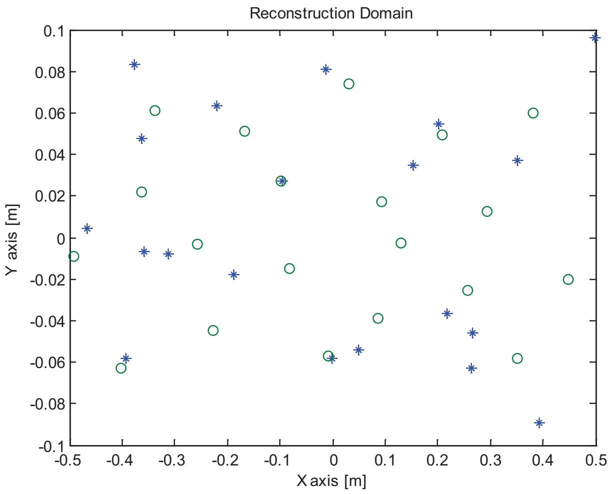

Fig. 6. Classical neural network solution of given problem for 20 test element; circles original location, stars neural network solution; noise free

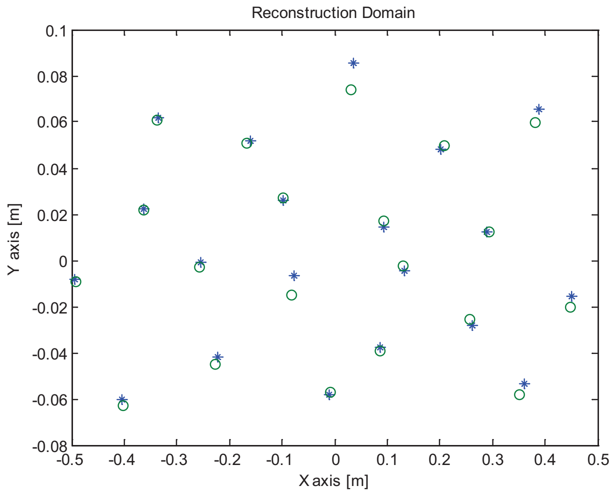

Fig. 7. Proposed neural network solution of given problem for 20 test element; circles original location, stars neural network solution; noise free

which has maximum mean error among 15 repetitions. By comparing Figure 6 and 7 we can easily state that proposed neural network gives higher accuracy than classical neural network solution for determining center of crack. More accurate result of proposed neural network solutions only originates from usage of EBPM before direct application of neural network method. Although 100 training elements are not sufficient to sample original reconstruction domain accurately, this number of training points is more sufficient for narrower reconstruction domain around each test points. It is also worth to mention that in classical neural network approach corner regions may not be learned very well, which can be seen from Figure 6, because of small number of training points in this region. Conversely in proposed neural network solution reconstruction domain is restricted around each crack, and therefore one does not need to apply a sampling strategy.

To be able to give a quantitative comparison we also give a detailed error analysis. To do this absolute error analysis can be conducted by subtracting original crack location from both

Table 1. Comparison of proposed and classical neural network solution numerically; noise free

		EBPM	Classical
Mean Absolute Error in X		0.0022 m	0.0504 m
Mean Absolute Error in Y		0.0023 m	0.0241 m
Worst Case			
	Absolute Error in X	0.0037m	0.0741 m
	Absolute Error in Y	0.0031 m	0.0314 m
Best Case			
	Absolute error in X	0.0015 m	0.0282 m
	Absolute Error in Y	0.0018 m	0.0168 m
Mean Training Time		2.36 sec	10.9 sec

neural network solutions in x and y coordinates. For noise free case this comparison can be seen in Table 1. It is clearly observed that mean absolute error of EBPM based neural network solution along 15 run of neural network with same test points give approximately 23 times smaller than classical solution in x coordinates and 10 times in y coordinates. Because of rectangular geometry of reconstruction domain, absolute error efficiency of proposed method in x and y coordinates are different. Variation interval (domain) of y coordinates is smaller than x coordinates, so y coordinates can be learned more successfully relative to x coordinates. Besides in Table 1, it can be seen that even worst absolute error situation for proposed method along 15 repetitions is approximately 5 – 7 times better than the best case of classical neural network solution. In addition to error analysis training times are compared for both methods. In Table 1, it is observed that training of proposed method is faster than classical neural network solution. Because; in proposed method training is performed in restricted reconstruction domain which provides close correlation between training points as well as narrower domain which will be learned. Therefore neural network can easily and rapidly establish a relationship between input/output.

To see behavior of proposed method and compare it with classical solution in noisy case, a similar analysis can be conducted; results are given in Table 2. In this analysis 1% noise level is used since the neural network is not robust against noise. Since it can be seen from Figure 5 that EBPM is not so sensitive to 10% noise level, the nature of neural network for this configuration is responsible for the noise sensitivity. Again similar results are observed for noisy case in the sense of error analysis of same reason explained before.

6. Conclusions

In this paper an efficient neural network method is proposed for the solution of the inverse scattering problems related to detection of circular holes in a wall. The fundamental principle of EBPM based neural network approach is the restriction of reconstruction domain to a narrower one. Proposed method is compared with classical solution for crack detection in wall problem.

Table 2. Comparison of proposed and classical neural network solution numerically; 1% noise level

		EBPM	Classical
Mean Absolute Error in X		0.0541 m	0.2364 m
Mean Absolute Error in Y		0.0464 m	0.0621 m
Worst Case			
	Absolute Error in X	0.0591 m	0.2589 m
	Absolute Error in Y	0.0571 m	0.0743 m
Best Case			
	Absolute Error in X	0.0475 m	0.1974 m
	Absolute Error in Y	0.0372 m	0.0496 m
Mean Training Time		3.07 sec	7.88 sec

It is shown by numerical examples that proposed method not only gives more accurate result but also provides fast training process with same number of training element for both noisy and noise free cases. These are main advantages of proposed method over neural network solution of inverse scattering problems. Since proposed method simplifies complex training process of neural network for inverse scattering problems, more complex inverse scattering problems can be solved easily and rapidly by the use of this two-step method. The future studies will be devoted to application of the proposed method to more complex problems including 3-D structures.

7. References

[1] S. Doi, S. Tanaka, "Non - destructive inspection of concrete structures using an electromagnetic wave radar," *SICE 2003 Annual Conference*, vol.2, no., pp.1147-1150, 4-6 Aug. 2003.

[2] G. Bozza, M. Brignone, M. Pastorino, M. Piana, A. Randazzo, "A linear sampling approach to crack detection in microwave imaging," *Imaging Systems and Techniques, 2008. IST 2008. IEEE International Workshop on*, vol., no., pp.222-226, Sept. 2008.

[3] J. H. Richmond, “Scattering by a dielectric cylinder of arbitrary cross section shape”, *IEEE Transactions on Antennas Propagation*, vol. 13 , no. 3, pp. 334–341, May 1965.

[4] E. Bermani, S. Caorsi, M. Raffetto, "A microwave objects recognition approach based on neural networks" *Instrumentation and Measurement Technology Conference, 1999. IMTC/99. Proceedings of the 16th IEEE*, vol.3, no., pp.1582-1585, 1999.

[5] E. Bermani, S. Caorsi, and M. Raffetto, "An inverse scattering approach based on a neural network technique for the detection of dielectric cylinders buried in a lossy half-space," *Progress in Electromagnetic Research*, Vol. 26, no., pp. 67-87, 2000.

[6] S. Caorsi, P. Gamba, "Electromagnetic detection of dielectric cylinders by a neural network approach," *Geoscience and Remote Sensing, IEEE Transactions on*, vol.37, no.2, pp.820-827, Mar 1999.

[7] I. Elshafiey, L. Udpa, S.S. Udpa, "Solution of inverse problems in electromagnetics using Hopfield neural networks," *Magnetics, IEEE Transactions on*, vol.31, no.1, pp.852-861, Jan 1995.

[8] I.T. Rekanos, “Neural-network-based inverse-scattering tech-nique for online microwave medical imaging”, *IEEE Trans. Mag.,* vol. 38, no. 2, pp. 1061–1064, March 2002.

[9] G. Cybenko, “Approximations by superpositions of sigmoidal functions”. *Mathematics of Control, Signals, and Systems*, vol. 2, no. 4, pp. 303-314, Dec. 1989.

[10] S. Haykin, “Neural Networks - A comprehensive Foundation”, MacMillan, New York, 1999.

[11] C.W. Groetsch, “The Theory of Tikhonov Regularization for Fredholm Equations of the First Kind”, White Plains, NY: Longman, 1984.