# Identification Of Low Frequency Oscillations In Power System

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## Abstract

**In power system, stability problem and problems with electromechanical oscillations or generators swinging are consistent in power system. Electromechanical oscillations are noticeable in characteristic variables of synchronous generators. Consequences of synchronous generator connecting on grid are physical nature, apropos more generators connecting on one power system. The responses of power system on any system disturbance are electromechanical oscillations. Oscillations can be low damped or undamped with constant or increasing amplitude, so they can achieve value which can disrupt the system operation. The monitoring of power system electromechanical oscillations is very important in the frame of modern power system management and control. This paper presents techniques for identification and analysis of low-frequency oscillations. Simulations and analysis shall be performed on Two-Area Test system.**

## 1. Introduction

Like operations limits, the monitoring of power system oscillating modes is a relevant aspect of power system operation and control. Non-prevented low-frequency power swings can be cause of cascading outages that can rapidly extend effect on wide region. On this regard, a WAMPC systems help in detecting such phenomena and assess power system dynamics security. Oscillations in power systems are classified by the system components that they affect. Some of the major system collapses attributed to oscillations are described [1]. Electromechanical oscillations are of the following types: interplant mode oscillations, local plant mode oscillations, inter-area mode oscillations, control mode oscillations, tensional modes between rotating plant. Machines on the same power generation site oscillate against each other at 2.0 to 3.0 Hz depending on the unit ratings and the reactance connecting them. This oscillation is termed as interplant because the oscillations manifest themselves within the generation plant complex. The rest of the system is unaffected. In local mode, one generator swings against the rest of the system at 1.0 to 2.0 Hz. Inter-area mode oscillations is observed over a large part of the network. It involves two coherent group groups of generators swinging against each other at 1 Hz or less.

This paper introduces the technique for identification and analysis low frequency oscillations in power system with special focus on multi-resolution wavelet analysis. After multi-resolution decomposition of characteristic signals, in signal components physical characteristics of system oscillations are identified and presented on the map using the Fast-Fourier Transform (FFT) in time-frequency domain representation. The results of the Eigenvalue analysis are compared with the results coming from the Prony and wavelet analysis.

The remainder of this paper is organized as follows. In Section 2, basic theory of small signal stability of multi-machine systems and also Prony and wavelet theory basics are presented. Practical application results identification and analysis low frequency oscillations on test system are given in Section 3. Section 4 contains the main conclusions.

## 2. The techniques of identification low frequency oscillations in power system

The analysis and monitoring of transient oscillations can be accomplished by means of several methodological approaches. Each approach has its own advantages and feasible applications, providing a different view of the system dynamic behavior. Eigenvalue analysis technique is based on the linearization of the nonlinear equations that represent the power system around an operating point which is the result of electromechanical modal characteristics: frequency, damping and shape. Direct spectral analysis of power response signals use the Fourier Transforms (or Short Time Fourier Transform (STFT), Prony or Wavelet analysis technique.

### 2.1. Modal analysis - small signal stability of multi-machine systems

Analysis of practical power systems involves the simultaneous solution of equations representing the following: (i) synchronous machines, and the associated excitation systems and prime movers, (ii) interconnecting transmission network (iii) static and dynamic (motor) loads and (iv) other devices such, as HVDC converters, static VAR compensators [2]. Low frequency electromechanical oscillations range from less than 1 Hz to 3 Hz other than those with sub-synchronous resonance. Multi-machine power system dynamic behavior in this frequency range is usually expressed as a set of non-linear differential and algebraic equations. The algebraic equations result from the network power balance and generator stator current equations. The high frequency network and stator transients are usually ignored when the analysis is focused on low frequency electromechanical oscillations. The initial operating state of the algebraic variables such as bus voltages and angles are obtained through a standard power flow solution. The initial values of the dynamic variables are obtained by solving the differential equations through simple substitution of algebraic variables into the set of differential equations. The set of differential and algebraic equations is then linearized around the equilibrium point and a set of linear differential and algebraic equations is obtained:

$$\dot{x} = f(x, z, u) \quad (1)$$
$$0 = g(x, z, u) \quad (2)$$
$$y = h(x, z, u) \quad (3)$$

where $f$ and $g$ are vectors of differential and algebraic equations and h is a vector of output equations. The inputs are normally reference values such as speed and voltage at individual units and can be voltage, reactance and power flow asset in FACTS devices. The output can be unit power output, bus frequency, bus voltage, line power or current etc. By linearization (1) to (3) around the equilibrium point following equations (4) to (6) are given:

$$\Delta\dot{x} = \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial z}\Delta z + \frac{\partial f}{\partial u}\Delta u \quad (4)$$
$$0 = \frac{\partial g}{\partial x}\Delta x + \frac{\partial g}{\partial z}\Delta z + \frac{\partial g}{\partial u}\Delta u \quad (5)$$
$$y = \frac{\partial h}{\partial x}\Delta x + \frac{\partial h}{\partial z}\Delta z + \frac{\partial h}{\partial u}\Delta u \quad (6)$$

Elimination of the vector algebraic variable $\Delta z$ from (4) and (6), gives:

$$\Delta\dot{x} = A\Delta x + B\Delta u \quad (7)$$
$$\Delta y = C\Delta x + D\Delta u \quad (8)$$

where $A, B, C, D$ are the matrix of partial derivatives in (4) to (6) evaluated at equilibrium. Power system state space representation is normally linearized around an operating point. The symbol $A$ from (7) and (8) is omitted so as to follow the standard state space making $x$ and $u$ into the incremental values. This is the representation of a linearized differential and algebraic equations model of a power system on which standard linear analysis tools.

### 2.2. Basis of Prony analysis

Prony analysis is a signal processing method that extends Fourier analysis. It is a technique of analyzing signals to determine model, damping, phase, frequency and magnitude information contain within the signal. Prony method is a technique for sample data modeling as a linear combination of exponentials, it has a close relationship to least squares linear prediction algorithm used for AR (Autoregressive) and ARMA (Autoregressive moving average) parameter estimation. Prony analysis is a method of fitting a linear combination of exponential terms to a signal. Each term in (9) has four elements: the magnitude $A_n$, the damping factor $\sigma_n$, the frequency $f_n$, and the phase angle $\theta_n$. Each exponential component with a different frequency is viewed as a unique mode of the original signal $y(t)$ [3]. The four elements of each mode can be identified from the state space representation of an equally sampled data record. The time interval between each sample is $T$:

$$y(t) = \sum_{n=1}^{N} A_n e^{\sigma_n t} \cos(2\pi f_n t + \theta_n), \quad n = 1,2,3,....,N. \quad (9)$$

Using Euler's theorem and letting $t=MT$, the samples of $y(t)$ are:

$$y_M = \sum_{n=1}^{N} B_n \lambda_n^M \quad (10)$$
$$B_n = \frac{A_n}{2} e^{j\theta_n} \quad (11)$$
$$\lambda_n = e^{(\sigma_n + j2\pi f_n)T} \quad (12)$$

Prony analysis consists of three steps. In the first step, the coefficients of a linear predication model are calculated. The linear predication model (LPM) of order $N$, shown in (13), is built to fit the equally sampled data record $y(t)$ with length $M$. Normally, the length $M$ should be at least three times larger than the order $N$:

$$y_M = a_1 y_{M-1} + a_2 y_{M-2} + ... + a_N y_{M-N} \quad (13)$$

Estimation of the LPM coefficients $a_n$ is crucial for the derivation of the frequency, damping, magnitude, and phase angle of a signal. To estimate these coefficients accurately, many algorithms can be used. A matrix representation of the signal at various sample times can be formed by sequentially writing the linear prediction of $y_M$ repetitively.

In the second step, the roots $\lambda_n$ of the characteristic polynomial shown as (14) associated with the LPM from the first step are derived. The damping factor $\sigma_n$ and frequency $f_n$ are calculated from the root $\lambda_n$ according to (12):

$$\begin{aligned}\lambda^N - a_1\lambda^{N-1} - ... - a_{N-1}\lambda - a_N \\ = (\lambda-\lambda_1)(\lambda-\lambda_2)\cdots(\lambda-\lambda_n)\cdots(\lambda-\lambda_N)\end{aligned} \quad (14)$$

In the last step, the magnitudes and the phase angles of the signal are solved in the least square sense. According to (10), (15) is built using the solved roots $\lambda_n$:

$$\mathrm{Y} = \phi \mathrm{B} \quad (15)$$
$$\phi = \begin{bmatrix} 1 & 1 & ... & 1 \\ \lambda_1 & \lambda_2 & ... & \lambda_N \\ . & . & . & . \\ \lambda_1^{M-1} & \lambda_2^{M-1} & ... & \lambda_N^{M-1} \end{bmatrix} \quad (16)$$
$$\mathrm{B} = [B_1 \quad B_2 \quad ... \quad B_N]^T \quad (17)$$

The magnitude $A_n$ and phase angle $\theta_n$ are thus calculated from the variables $B_n$ according to (11).

### 2.3. Wavelet transform

Wavelet analysis is a relatively new signal processing tool and is applied recently by many researchers in power systems due to its strong capability of time and frequency domain analysis [4][5][6]. The Wavelet transform is a mathematical tool, like the Fourier transform for signal analysis. A wavelet is an oscillatory waveform of effectively limited duration that as average value of zero [7][8]. Similarly, wavelet analysis is the breaking up of a signal into shifted and scaled versions of the original (or mother) wavelet. As an example, Daubechies (db4) wavelet is presented on the Fig. 1.

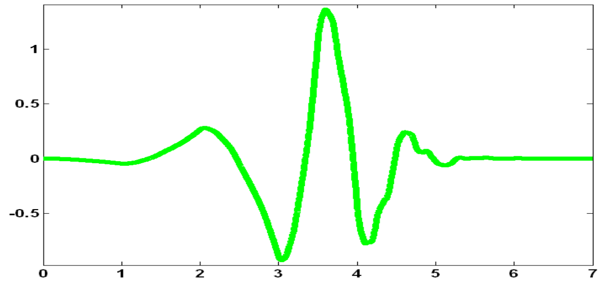

**Fig. 1.** Wavelet (db4)

The wavelet transform of a time dependent signal $f(t)$ consists of a set coefficients $W_s(a,b)$. These coefficients measure the similarity between the signal $f(t)$ and a set of functions $\psi_{a,b}(t)$. All the functions $\psi_{a,b}(t)$ are derived from a 'mother wavelet' as follow:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right), \quad a>0 \qquad (18)$$

Where $a$ represent a time dilatation and $b$ a time translation. The Continuous Wavelet Transformation (CWT) of a time domain signal is defined by:

$$CWT_f(a,b) = (f,\psi_{a,b}) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} f(t)\overline{\psi}\left(\frac{t-b}{a}\right)dt \qquad (19)$$

where: $\psi(t)$ is the basis wavelet function (or mother wavelet), that can be real or complex, $a$ is the dilatation scale parameter, $b$ is the time scale parameter, $\psi\left(\frac{t-b}{a}\right)$ are the daughter wavelet function. The application of wavelet transform in engineering areas usually requires a discrete wavelet transform Discrete Wavelet Transformation (DWT). A square integrable signal $f(t)$ is decomposable into different time-frequency scales. In wavelet analysis, such a signal can be represented by a linear combination of two parameter wavelet functions:

$$f(t) = \sum_{k=-\infty}^{\infty} a_{j_0}(k)\varphi(t-k) + \sum_{j=j_0}^{\infty}\sum_{k=-\infty}^{\infty} d_j(k)2^{j/2}\psi(2^j t-k) \qquad (20)$$

The wavelet functions $\varphi(t)$ and $\psi(t)$ are localized in time. Parameters $k$ and $j$ perform translation and time scaling of the original functions. The functions $\varphi(t)$ and $\psi(t)$ are usually chosen so that the functions on the right side of (19) form an orthonormal basis. Then decomposition and reconstruction are efficient using orthogonal projection. The $a_j(k)$ and $d_j(k)$ terms are referred to as approximation and detail coefficients, respectively (coefficients of low-pass and high-pass filters). These coefficients can be order according fallowing relations:

$$a(k) = \sqrt{2}\int_{-\infty}^{+\infty}\varphi(t)\cdot\varphi(2t-k)dt$$
$$d(k) = \sqrt{2}\int_{-\infty}^{+\infty}\psi(t)\cdot\varphi(2t-k)dt \qquad (21)$$

They reflect a range from local to global characteristics of the original signal $f(t)$ because their associated functions have different time-frequency scales. A very useful implementation of DWT, called multi-resolution analysis, is demonstrated in Fig. 2.

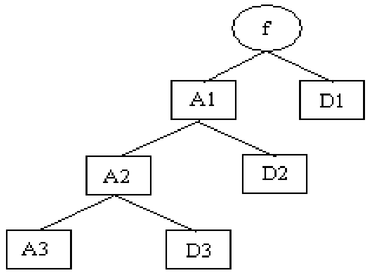


**Fig. 2.** Wavelet multi-resolution analysis

The original sampled signal $f$ is passed through a highpass filter (D) and a lowpass filter (A). Then the outputs from both filters are decimated by 2 to obtain the detail coefficients and the approximation coefficients at level 1 (A1 and D1). The approximation coefficients are then sent to the second stage to repeat the procedure. Finally, the signal is decomposed at the expected level.

## 3. An identification low frequency oscillations in power system- Test results

Simulation and analysis was done by using Two-Area System present on Fig. 3 and using software Power system analysis Toolbox (PSAT), Wavelet toolbox and Prony toolbox. The Eigenvalue analysis of the system for a specific operating point led to the identification of several modes of oscillation.

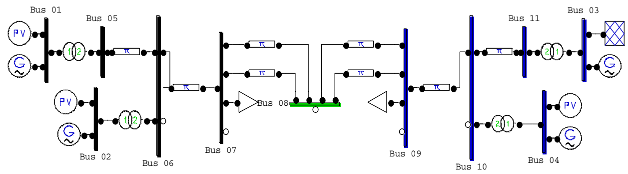


**Fig. 3.** Test system

In Table 1, selected modes of oscillation and their characteristics are reported.

After simulation small disturbance (the system has been perturbed by applying small active power load increase at bus 8) observed oscillation throughout the system.

In this case, Fig. 4 shows the voltage oscillation at bus 9. Fig. 5 presents the Prony approximation in time domain and Fig. 6 present Prony analysis in frequency domain. The Prony analysis leeds to the identification of dampings of all the identified modes indicated in Fig. 6 and reported in Table 2. The value of the dominant oscillations modes of 0.59 Hz and 1,2 Hz are correctly identified in agreement with the Eigenvalue analysis.

**Table 1.** Modes of oscillations

| mode | eigenvalue 1/s | damping ratio | frequency Hz |
|---|---|---|---|
| 1 | -0.102 ± j0 | 1 | 0 |
| 2 | -0.181 ± j0 | 1 | 0 |

| | | | |
|---|---|---|---|
| 3 | -0,433 ± j3,863 | 0,111 | 0,615 |
| 4 | -0,498 ± j0,114 | 0,975 | 0,018 |
| 5 | -1,548 ± j7,932 | 0,192 | 1,262 |
| 6 | -1,640 ± j8,219 | 0,196 | 1,308 |
| 7 | -5,364 ± j0,062 | 1,000 | 0,010 |
| 8 | -19,018 ± j20,714 | 0,676 | 3,297 |
| 9 | -19,686 ± j14,411 | 0,807 | 2,294 |

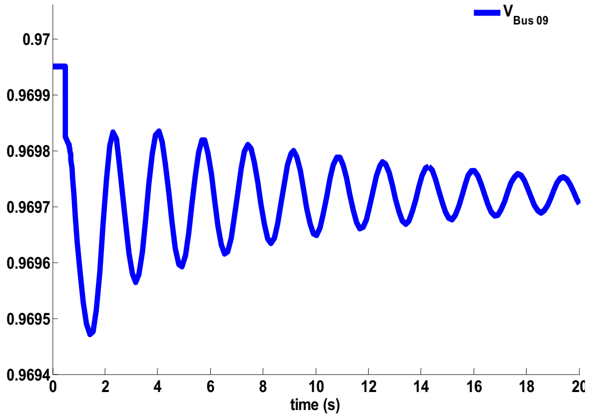


**Fig. 4.** Voltage oscillation

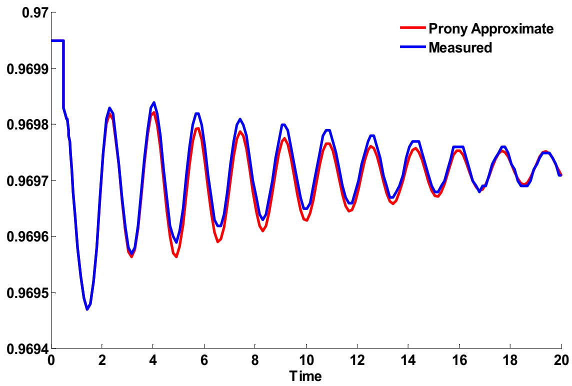


**Fig. 5.** Prony analysis in time domain

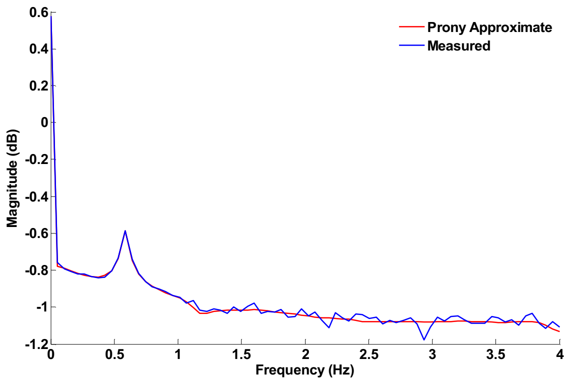


**Fig. 6.** Prony analysis in frequency domain

**Table 2.** Prony analysis: Frequency and damping estimation

| damping ratio | frequency Hz |
|---|---|
| 0.091 | 0.59 |
| 0.2 | 1.2 |
| 0.81 | 4.0 |
| 0.58 | 1.8 |
| 1.2 | 3.5 |
| 0.91 | 2.4 |
| 0.64 | 2.9 |

In wavelet analysis, we often speak of approximations (A) and details (D). The approximations are the high scale, low-frequency components of the signal. The details are low-scale, high-frequency components. DWT comprises two functions. The first one is the scaling function, related to the low-pass filters and the second one is the wavelet function, related to the high-pass filters. Signal with low-pass and high-pass filters dispense on approximations (A) and details (D). For many signals, the low-frequency content is the most important part, giving to the signal's identity. The high- frequency content, on the other hand, gives the flavour or nuance.

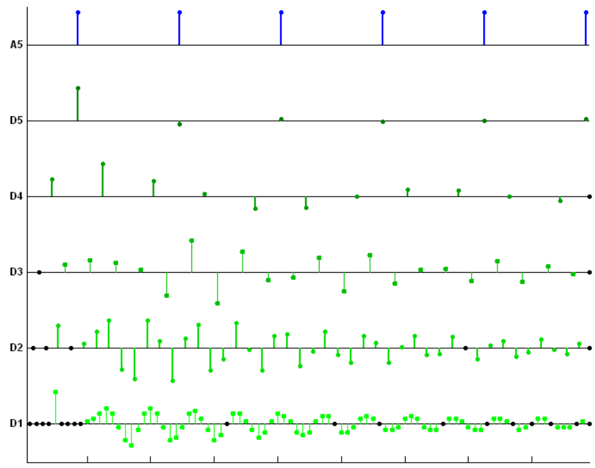


**Fig. 7.** Wavelet multi-resolution analysis signal of voltage oscillation

The Haar's mother wavelet is selected to analysis voltage signal on Fig. 4. Using DWT multi-resolution analysis, this signal is decomposed on approximations and details coefficients at the five levels (Fig. 7). Identification of onset of the system disturbance is possible with the usage of the first inner decomposition level of the signal (D1) which is normally adequate to detect any disturbance in the signal [9]. However other coarser resolution levels are used to extract more features which can help in the estimation process. As it shown previously, wavelet transform is applied to extract the signal containing the dominant mode from the voltage signal. And afterward, by using the Fast-Fourier Transform (FFT) in time-frequency domain representation, as shown on Fig. 8., the frequency characteristic and power spectrum of the dominant oscillation mode in the frequency domain only of D3 component is done.

After time frequency analysis of the component signal, it is possible to detect character of low frequency oscillations in the signal. The value of the dominant oscillations modes of 0.6 Hz as the dominant mode is correctly identified in agreement with the Eigenvalue analysis. The dominant mode of the D1 component is 3.8 Hz and 0.61 Hz, while in D2 and D3 four dominant modes are identified: 0.6 Hz, 1.6 Hz, 2.8 Hz and 3.8 Hz.

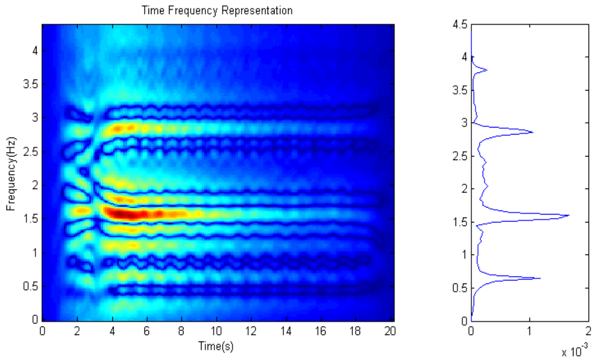


**Fig. 8.** FFT time frequency representation and power spectrum of dominant low frequency mode of the D3 component

The chart on the Fig 8. shows the time-frequency behavior of the oscillation modes hidden into the signal and gives rise to a qualitative approach for estimation the damping of the oscillation modes [10][11]. Like the Eigenvalue and Prony analysis, wavelet analysis identifies dominant modes and characters of these oscillations with obvious identification of onset of system disturbance.

## 6. Conclusions

In this paper, the technique for identification and analysis low frequency oscillations in power system is presented. The results of the Eigenvalue analysis, applied to the test system, are compared with the results coming from Prony and wavelet analysis. The test results show that with the support of this technique is possible to: (i) identify the onset of events/disturbance on the power system, (ii) efficiency in fast identification oscillations in power systems, (iii) detection dominant electromechanical oscillatory mode in power system and (iv) oscillatory mode character. Real-time monitoring of power system dynamics can help in identifying poorly damped modes of oscillations and possible threats to the security of the system. Regarding its own advantages over the standard mathematical signal processing tools, it is expected for wavelet transform to be used in modern wide area monitoring systems.

## 7. References


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