PHYSICAL OPTICS (PO) APPROACH OF THE PLANE WAVE DIFFRACTION BY THE JUNCTION OF A THICK IMPEDANCE HALF-PLANE AND A THICK DIELECTRIC SLAB

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ABSTRACT

Plane wave diffraction at the junction of thick impedance half-plane and a thick dielectric slab is investigated by using Physical Optics approach. In this approach, by using infinite length plane which is a canonical problem, equivalent sources on the scattering object are obtained approximately. Then by writing this approximately result in the Kirchoff-Huygens equation, scattering field is obtained. The effects of the material properties of the dielectric slab, thickness and the impedance of the junction on the diffraction phenomenon are presented graphically. At the end of the analysis the results are compared with the Wiener-Hopf technique given in [4]. The result of the comparison shows that the PO approximation is effective for small thickness of the junction.

I. INTRODUCTION

As known, half planes and two-part planes with non vanishing thickness are important canonical problems, which have been subject to numerous past investigations. By using a modified type of Wiener-Hopf technique, diffraction from thick perfectly conducting half plane was first considered by Jones [1]. In this sense, Aoki and Uchida studied diffraction from two thick dielectric half planes [2]. The thick metal-dielectric join problem was investigated by Volakis and Ricoy [3]. Later, Tayyar and Büyükaksoy reconsidered the geometry treated in [3] in more general case where the thick metallic half-plane is replaced by an impedance one [4-5]. The aim of this study is, by using Physical Optics approach, to solve the diffraction problem of the geometry was mentioned in [4] and to compare these solutions. The comparison shows us that the approach using in this work gives a good result for small joint thickness.

II. ANALYSIS

In reference [4] the diffraction problem was solved with the Wiener-Hopf method. Using the image bisection principle the original impedance-dielectric join problem was split divided into two types of step discontinuities with dielectric slab loading, corresponding to even and odd excitation cases. By means of the Fourier transform technique, each dielectric loaded step discontinuity problem was formulated as a modified Wiener-Hopf equation involving infinitely many unknown expansion coefficients satisfying an infinite system of linear algebraic equations. Solving diffraction problem by using Wiener-Hopf technique takes lots of computational time. For small joint thickness values using PO approach gets the solution easier. The scope of this work is to improve an approximate solution for the diffraction problem (Figure 1).

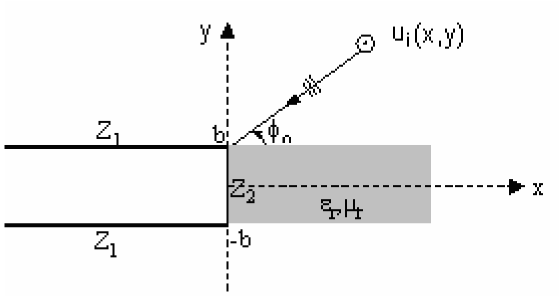

Figure1. Geometry of the problem

Let an E_z-polarized time harmonic plane wave illuminate a junction formed by a thick impedance half-plane and a semi-infinite dielectric slab of thickness *2b*. The lateral walls of the impedance half-plane defined by $\mathbf{S}_1 = \{(x, y, z);\ x \in (-\infty, 0),\ y = b,\ z \in (-\infty, \infty)\}$ and $\mathbf{S}_2 = \{(x, y, z);\ x \in (-\infty, 0),\ y = -b,\ z \in (-\infty, \infty)\}$ are characterized by the same surface impedance $Z_1 = \eta_1 Z_0$ while the surface impedance of the end face $\mathbf{S}_3 = \{(x, y, z);\ y \in (-b, b),\ x = 0,\ z \in (-\infty, \infty)\}$ is $Z_2 = \eta_2 Z_0$ with Z_0 being the characteristic impedance of the free space. The relative parameters of the dielectric slab are ε_r , μ_r (see Figure 1).

The incident wave defined as:

$$E_z^i = \exp[-ik_0(x\cos\phi_0 + y\sin\phi_0)] \quad (1)$$

with k_0 and ϕ_0 being the free space wave number and the angle of incidence, respectively.

III. PHYSICAL OPTICS (PO) APPROACH

Physical Optics approach is a semi-analytical method. In this approach, by using infinite length plane which is a canonical problem, equivalent sources on the scattering object are obtained approximately. Then by writing this approximate result in the Kirchoff-Huygens equation, scattering field is obtained. The Kirchoff-Huygens principle uses Green function to express wave function $\phi(\mathbf{p})$:

$$\phi(\mathbf{p}) = \int_v G(\mathbf{p},\mathbf{p}')\rho(\mathbf{p}')dv' + \oint_{\partial S}\left\{G(\mathbf{p},\mathbf{p}')\nabla'\phi(\mathbf{p}') - \nabla' G(\mathbf{p},\mathbf{p}')\phi(\mathbf{p}')\right\}d\mathbf{n}' \quad (2)$$

Here, $G(\mathbf{p},\mathbf{p}')$ is the Green function which provides scalar Helmholtz equation, $\rho(\mathbf{p}')$ is source distribution which generates field, $\mathbf{p}$ and $\mathbf{p}'$ illustrate observation and source points, respectively. The first expression which is on the right side of equation (2) can be calculated easily while the sources are known. In order to calculate second expression, wave function on the surface $(\nabla'\ (\mathbf{p}'),\ (\mathbf{p}'))$ should be known. By using Physical Optics method one can obtain these unknown equivalent sources on the surface and then the scattering wave is calculated.

When equivalent sources are obtained on the scattering object for illuminated region, they are assumed to be zero on the shadow regions. Thus, the error related to scattering wave will increase in the shadow region.

One can write the scattering field in the Physical Optics approach as follow:

$$u_s(x,y) = u_r(x,y) + u_g(x,y) \quad (3)$$

where $u_r(x,y)$ denotes the field reflected from the plane y=b, namely

$$u_r(x,y) = \Re_1 e^{-ik_0[x\cos\phi_0 - (y-2b)\sin\phi_0]} \quad (4)$$

In (4) $\Re_1$ is reflection coefficient defined as:

$$\Re_1 = \frac{\eta_1\sin\phi_0 - 1}{\eta_1\sin\phi_0 + 1} \quad (5)$$

In (3) u_g denotes the contribution of the dielectric half plane to the scattering field

Assume that the integral representation of the scattering field is as follows:

$$u_s(x,y) = \frac{1}{2\pi}\int_C A(\alpha)e^{iK_0(\alpha)(y-b)-i\alpha x}d\alpha. \quad (6)$$

After putting $y=b$ in equation (6), $A(\alpha)$ can be obtained by taking the inverse Fourier transform of $u_s(x, y)$ as:

$$A(\alpha) = \int_{-\infty}^{\infty} u_s(x,b)e^{i\alpha x}dx \quad (7)$$

By taking into account to the reflected field from the dielectric half-plane

$$u_{r2}(x,y) = \Re_2 e^{-ik_0[x\cos\phi_0 - (y-2b)\sin\phi_0]}, \qquad x\in(0,\infty) \quad (8)$$

scattering field expression at $y=b$ is written as follows:

$$u_s(x,b) = e^{-ik_0(x\cos\phi_0 + b\sin\phi_0)}\left[(\Re_2 - \Re_1)\mathbf{H}(x) + \Re_1\right] \quad (9)$$

Here,

$$\mathbf{H}(x) = \begin{cases} 1 & 0<x \\ 0 & x<0 \end{cases} \quad (10)$$

Z_0, Z_1 are being the characteristic impedance of the free space and slab; k_1 is wave number in the slab, ϕ_1 is the propagation direction of the wave in the slab. In this case reflection coefficient $\Re_2$ from the dielectric slab with the thickness $2b$ is obtained as follow:

$$\Re_2 = \frac{-2i(Z_0^2 - Z_1^2)\sin(k_1 b\sin\phi_1)}{(Z_0+Z_1)^2 e^{-2ik_1 b\sin\phi_1} - (Z_0-Z_1)^2 e^{2ik_1 b\sin\phi_1}} \quad (11)$$

Replacing (9) into (7) one gets:

$$A(\alpha) = (\Re_2 - \Re_1)e^{-ikb\sin\phi_0}\int_0^{\infty}\mathbf{H}(x)e^{ix(\alpha - k_0\cos\phi_0)}dx + \Re_1 e^{-ikb\sin\phi_0}\int_{-\infty}^{\infty}e^{ix(\alpha - k\cos\phi_0)}dx \quad (12)$$

Thus, A (α) gets:

$$A(\alpha) = (\Re_2 - \Re_1)e^{-ikb\sin\phi_0}\frac{-1}{i(\alpha - k_0\cos\phi_0)} + 2\pi\Re_1 e^{-ik_0 b\sin\phi_0}\delta(\alpha - k_0\cos\phi_0) \quad (13)$$

where $\delta(\alpha)$ illustrates Dirac distribution. Replacing (13) into (6) and the asymptotic evaluation of the integral through the saddle-point technique enables us to obtain the diffracted field related to the geometry as follows:

$$u_g(\rho,\phi) = \frac{(\Re_1 - \Re_2)\sin\phi e^{-ik_0 b\sin\phi_0}}{\cos\phi_0 + \cos\phi}\frac{e^{-i\pi/4}}{\sqrt{2\pi}}\frac{e^{ik\rho}}{\sqrt{k\rho}} \quad (14)$$

where change of variables defined as,

$$x = \rho\cos\phi, \qquad y - b = \rho\sin\phi \quad (17)$$

IV. CONCLUSIONS

In this section some graphical results showing the effects of the thickness, wall impedance and the material properties of the dielectric slab on the diffraction phenomenon are presented.

Figure 2 shows the dependence of the diffracted field on the junction thickness b. Figure 3 displays the variation of the diffracted field versus the observation angle for different values of dielectric permittivity of the slab.

Comparison between the Physical Optics approximation and Wiener-Hopf solution of the diffraction problem is shown in Figure 4. When the observation angle is in the interval $\phi \in \left(0^\circ, 90^\circ\right)$ the differance between the PO approximation and Wiener-Hopf solution is sensible. The reason is, in the Physical Optics approximation, wall impedance η_2 of the junction does not appear in the diffracted field expression. The contrubiton of this wall impedance to the diffraction field loses importance for small thickness of junction and our approximation gives successful results.

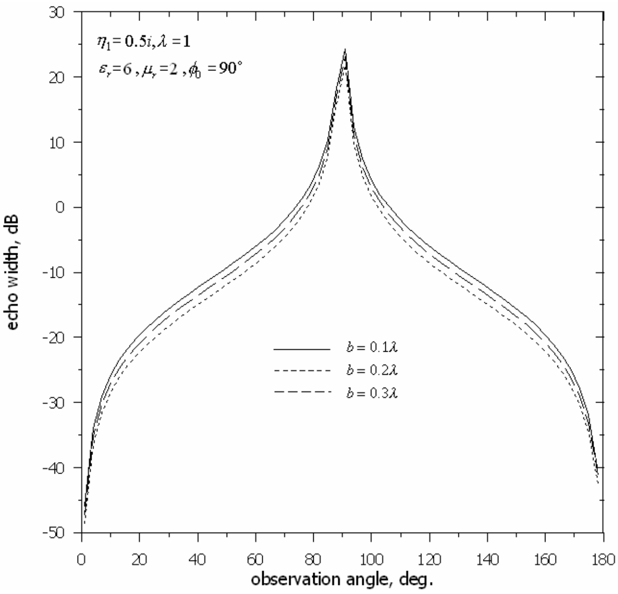

Figure 2. Bistatic echowidth family curves for different values of the junction thickness *b*.

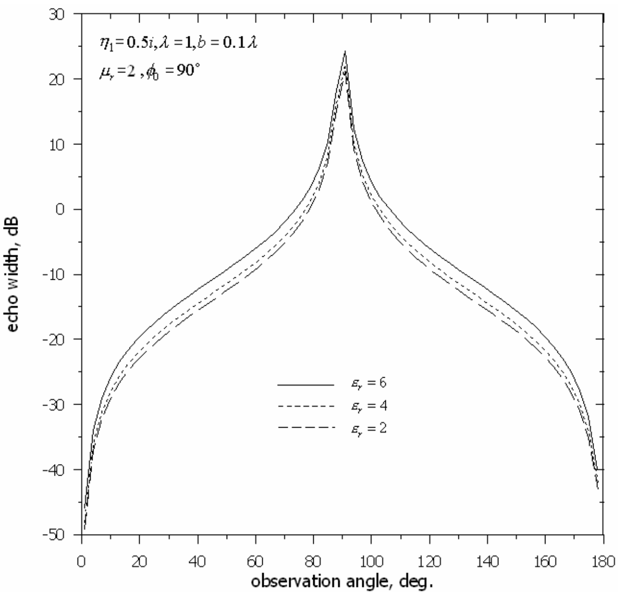

Figure 3. Bistatic echowidth family curves for different values of the dielectric permittivity ε_r.

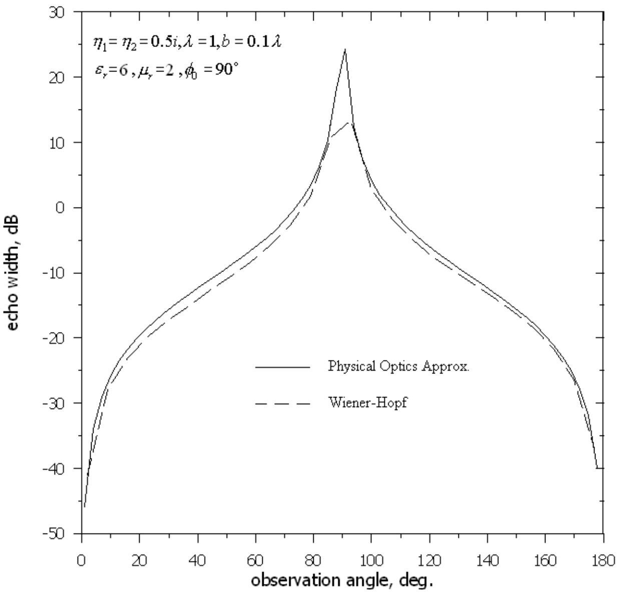

Figure 4. Comparison with the Physical Optics approximation and Wiener-Hopf solution

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