CFOA-BASED WIEN-BRIDGE TYPE RC CHAOS OSCILLATOR

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Abstract: Wien-bridge sinusoidal oscillator, based on current-feedback opamp is modified to obtain chaotic oscillator. This modification is done following the simple linear domain issues. Experimental results, SPICE and numerical simulations are given.

1. Introduction

In recent years, there has been growing interest in the nonlinear dynamics of electronic systems, especially in chaotic oscillators. This is due to fact that complicated behaviour of the chaos is expected to be of commercial value for the applications as secure communication and chaotic synchronisation. Eventhough chaotic oscillators have been studied since early 80's, Chua's chaotic oscillator has been a paradigm for studying chaos in nonlinear circuits [1]. Different implementations of Chua's circuit are given [2,3].

On the other hand, the current-feedback opamps (CFOA's) are receiving considerable attention recently (see [4-6] and the references cited therein) as these elements offer the following advantages over the conventional opamps: i) constant bandwidth almost independent of the gain, ii) practically no slew rate limitation (typically 2000V/μsec). Also, it has been shown that sinusoidal oscillators implemented from a CFOA, e.g. Wien-bridge oscillator, exhibit superior features with respect to their classical opamp-based counterparts, such as higher frequency of operation and lower distortion levels [7].

In the literature, there is a large amount of chaos oscillators obtained by modifying Wien-bridge type sinusoidal oscillator [8-13]. In this work, a Wien-bridge type oscillator based chaos generator will be introduced.

The proposed one here is an RC chaos oscillator based on CFOA and the involved active nonlinear resistor is provided by a general purpose signal diode. The advantages of the proposed circuit over the mentioned ones can be summarized by the advantages of RC over LC, advantages of diode over JFET [8] and above mentioned advantages of CFOA's over OA's. Moreover, it will be shown that this chaos generator obeys most of the design rules proposed in [14] which aim to make use of simple linear domain design issues in nonlinear domain.

First, dynamic equations of CFOA based Wien-bridge oscillator will be given and modification done to obtain chaos will be explained. Also SPICE simulations will be introduced. Secondly, mathematical model will be given, numerical simulations done by MATLAB will be discussed and experimental results, which verify the numerical simulations will be presented.

2. The Proposed Chaotic Oscillator

In order to obtain chaotic oscillator, the Wien-bridge sinusoidal oscillator based on CFOA given in Figure 1 will be modified.

Here the amplifier block of Wien-bridge oscillator is implemented around a CFOA which is used as a non-inverting voltage controlled voltage source with gain $k = \frac{R_{a2}}{R_{a1}}$.

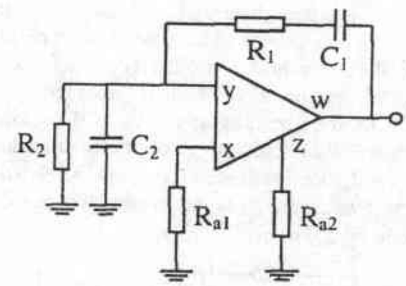


Figure 1: Wien-bridge oscillator based on CFOA

Feedback block of Wien-bridge oscillator is composed of R_1, C_1 in series and R_2, C_2 in parallel. State-space equations of this Wien-type sinusoidal oscillator are as following:

$$\begin{bmatrix} \dot{v}_{c1} \\ \dot{v}_{c2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1C_1} & \frac{k-1}{R_1C_1} \\ -\frac{1}{R_1C_2} & \frac{k-1-\frac{R_1}{R_2}}{R_1C_2} \end{bmatrix} \begin{bmatrix} v_{c1} \\ v_{c2} \end{bmatrix} \quad (1)$$

The oscillation condition can be easily obtained from these equations as:

$$\frac{1}{R_1C_1} = \frac{1}{R_1C_2}(k-1-\frac{R_1}{R_2}) \quad (2)$$

and oscillation frequency is ω_0:

$$\omega_0 = \frac{1}{\sqrt{R_1C_1R_2C_2}} \quad (3)$$

Before introducing the chaos oscillator which is the modified version of this Wien type sinusoidal oscillator, the parasitic effects of CFOA will be considered. Following the design rules in [14], the aim is to reproduce the chaotic dynamics that does not depend on any device-specific parasitic effect. So the parasitic capacitance of the compensating terminal, appearing in parallel with R_{a2} will be neglected by properly choosing the values of C_1, C_2 and k. Another parasitic effect of CFOA is the small input resistance R_x ($R_x \cong 65\Omega$) of the inverting terminal. Its effect will be neglected by choosing R_{a1} large enough. Thus oscillation condition will not be effected by R_x.

It is well-known from Poincare-Bendixson theorem [15] that two dimensional autonomous systems do not have chaotic solutions, so for chaos generation the order of the circuit has to be at least three. To obtain chaos oscillator, an additional capacitor C_3 is connected in parallel with a nonlinear resistor as shown in Figure 2.

This chaotic oscillator follows another design rule of [14] which states that "the nonlinear element should be separated from the linear blocks so that the functionality of these blocks are clear, ideal and independent of the parameters of any nonlinear element". As explained above, the linear building blocks of the Wien-bridge oscillator are not used in obtaining the nonlinearity which is provided by the second CFOA and general-purpose diode. This second CFOA plays the role of a negative impedance converter and due to diode it is only activated at voltages $v_{c3}>v_0$, where v_0 is the forward voltage drop of the diode ($v_0 \cong 0.6V$).

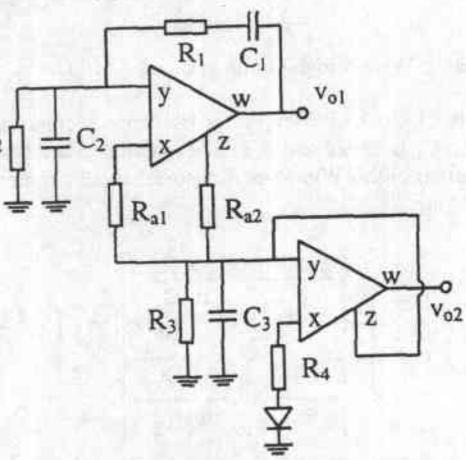


Figure 2: Modified Wien-bridge oscillator for chaos

Thus obtained nonlinear active resistor with antisymmetric characteristic can be modelled by two segment piecewise nonlinear characteristic.

By using general-purpose diode in order to obtain active nonlinear resistor, another design rule of [14] is fulfilled.

From this modified chaos generating Wien-bridge oscillator, it is easy to go back to sinusoidal Wien-bridge oscillator by just short-circuiting the capacitor C_3.

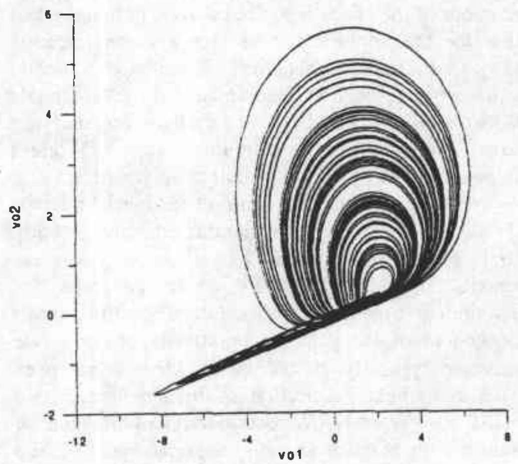
This circuit also provides two buffered output voltages, which avoids loading effects during measurements.

The equations describing this chaotic oscillator are as follows:

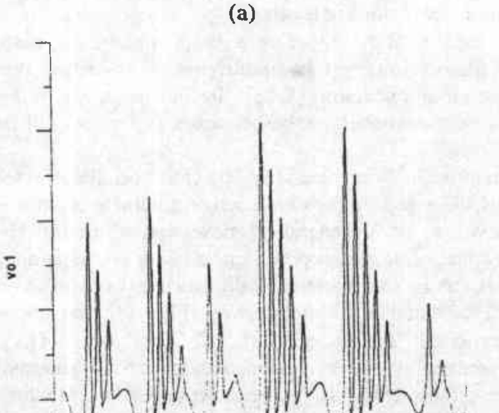
$$\begin{bmatrix} \dot v_{c1} \\ \dot v_{c2} \\ \dot v_{c3} \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1C_1} & \frac{k-1}{R_1C_1} & -\frac{k-1}{R_1C_1} \\ -\frac{1}{R_1C_2} & \frac{k-1-\frac{R_1}{R_2}}{R_1C_2} & -\frac{k-1}{R_1C_2} \\ 0 & \frac{2k}{R_{a2}C_3} & -(\frac{2k}{R_{a2}C_3}+\frac{1}{R_3C3}) \end{bmatrix} \begin{bmatrix} v_{c1} \\ v_{c2} \\ v_{c3} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{v_{c3}-v_0}{R_4C_3}H(v_{c3}-v_0) \end{bmatrix} \quad (4)$$

where $k=R_{a2}/R_{a1}$ and H(x) is the Heaviside function, that is $H(x)=\begin{cases} 0 & x<0 \\ 1 & x \geq 0 \end{cases}$.

Figure 3 shows the phase portrait and the output waveform for v_{o1} obtained by SPICE simulation with the following element values: $R_1=10k\Omega$, $R_2=40k\Omega$, $R_3=2k$, $R_4=1430\Omega$, $C_1=1nF$, $C_2=0.25nF$, $C_3=0.5nF$,



(a)



(b)

Figure 3: a) The phase portrait of v_{o2} versus v_{o1}. b)Typical waveform of $v_{o1}(t)$

Ra_1=8kΩ, R_{a2}=17kΩ; and using a macromodel of a commercially available CFOA, AD844 [4] and 1N4148 diode.

It can be easily observed that with the given element values, the chaotic oscillator exhibits chaotic behaviour after the Wien-bridge oscillator operates in the oscillatory mode.

3. Mathematical Model and Experimental Results

The state-space equations given above can be transformed into the following form:

$$\begin{bmatrix}\gamma_1\dot{x}\\ \gamma_2\dot{y}\\ \gamma_3\dot{z}\end{bmatrix}=\begin{bmatrix}-1 & k-1 & -(k-1)\\ -1 & k-1-\eta & -(k-1)\\ 0 & 2k & -(2k+\alpha)\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}+\begin{bmatrix}0\\ 0\\ \beta\end{bmatrix}(z-1)H(z-1) \quad (5)$$

where $x=\frac{v_{c1}}{v_0}, y=\frac{v_{c2}}{v_0}, z=\frac{v_{c3}}{v_0}, \alpha=\frac{R_{a2}}{R_3}, \eta=\frac{R_1}{R_2}$,

$\beta=\frac{R_{a2}}{R_4}$ $\gamma_1=R_1C_1$, $\gamma_2=R_1C_2$, $\gamma_3=R_{a2}C_3$.

The phase portrait obtained by solving these equations using MATLAB with k=2, η=0.25, γ_1=1, γ_2=0.25, γ_3=0.8, α=8, β=11.2 is given in Figure 4. In this figure, new state variables are defined corresponding to the outputs of the CFOA's in Fig. 2. In this figure, phase portrait of 2y+z versus z is shown. This dynamical system has two equilibrium points and eigenvalues calculated at these equilibrium points are as following:

$$\begin{cases}-13.7348, & 0.3674\pm j0.978, \quad z<1\\ -0.0525, & 0.5262\pm j4.33, \quad z\geq 1\end{cases}$$

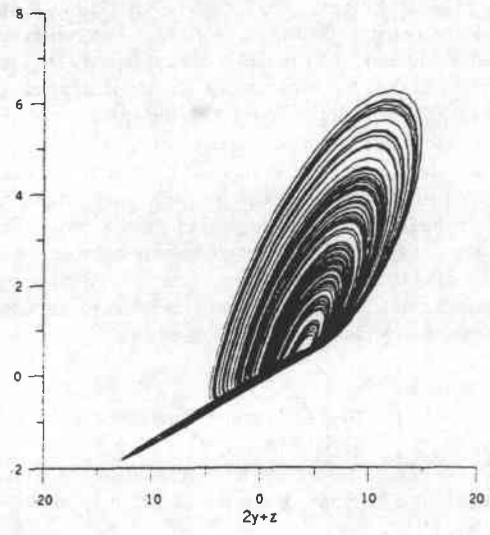


Figure 4 The phase portrait of the new variables corresponding to the CFOA's outputs in Figure 2.

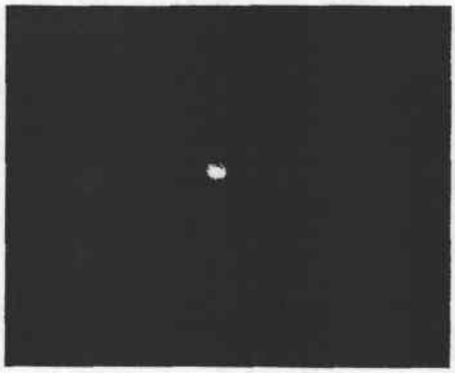
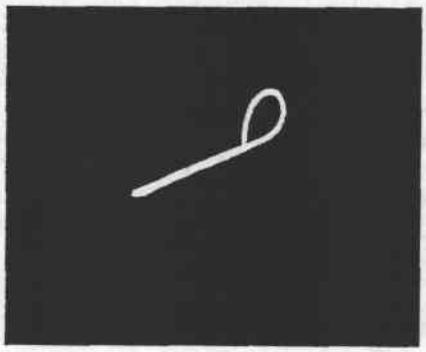
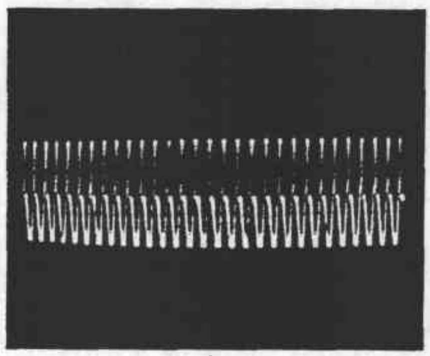


Figure 5 Stable equilibrium point for R_{a2}=14kΩ.

These equilibrium points are of hyperbolic saddle focus type, since they both have real γ, and complex σ+jω, eigenvalues satisfying γσ<0 [16]. When z<1, the hyperbolic saddle focus type equilibrium point is at the origin, but in the case of z≥1, the other same type equilibrium point is virtual and it is at (14,0,-14). Eventhough there exist two equilibrium points of hyperbolic saddle focus type, since $\frac{\sigma_1\sigma_2}{\gamma_1\gamma_2}>1$, the attractor is not a Shilnikov type. Instead, there must exist a heteroclinic cycle in which there is a chaotic attractor [16]. This phenomena is observed in experimental results.



(a)



(b)

Figure 6 a) Phase portrait b) Waveform of heteroclinic orbit.

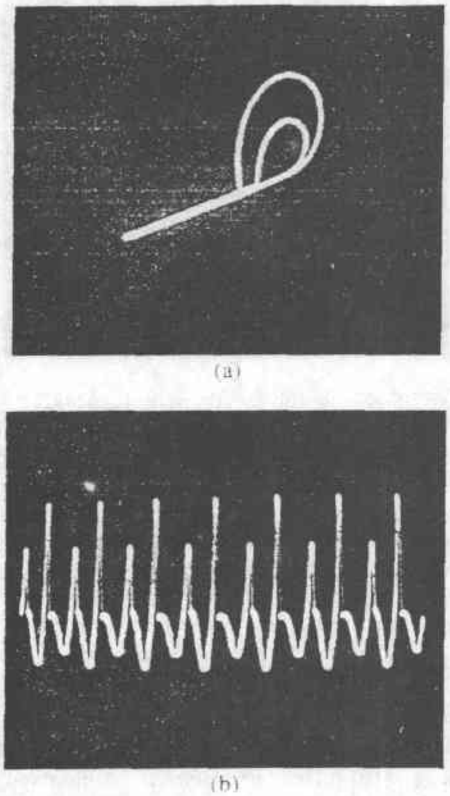


Figure 7 a) Phase portrait b) Waveform of heteroclinic orbit with two loops around each equilibrium point.

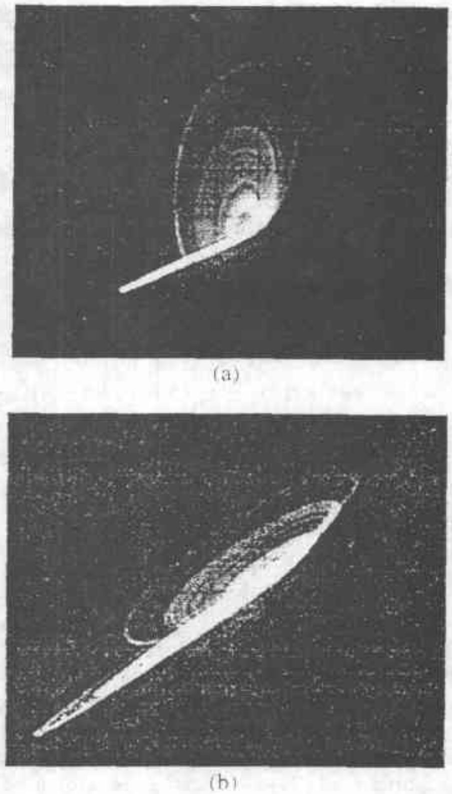


Figure 8 a) The chaotic attractor b) The phase portrait of v_{c2} against v_{o1}.

The circuit is constructed with R_1=10kΩ, R_2=40kΩ, R_3=2kΩ, R_4=1428Ω, C_1=1nF, C_2=0.25nF, C_3=0.5nF, R_{a1}=8kΩ. R_{a2} is used for tuning and its value is changed from 14kΩ to 20kΩ. The CFOA's are realised by using AD844 from Analog Devices [4] supplied with ±15 V. As shown in Figure 5 when R_{a2}=14kΩ, stable equilibrium point is observed. When R_{a2} is tuned to 14.35kΩ period-1 is obtained. Then R_{a2} is tuned to 15kΩ and heteroclinic orbit connecting two hyperbolic saddle foci is obtained. The phase portrait of v_{o2} against v_{o1} in this case is given in Figure 6a and the waveform of v_{o2} is given in Figure 6b. Heteroclinic orbit with two loops around each equilibrium point is obtained for R_{a2}=16kΩ and the corresponding phase portrait and the waveform of v_{o2} are given in Figures 7a and 7b, respectively. In Figure 8a, the chaotic attractor is given for R_{a2}=16.9kΩ. The heteroclinic orbits between two equilibrium points are observed in Figure 8b where the phase portrait of v_{c2} against v_{o1} is given. The oscilloscope horizontal and vertical scale values in figure 8a are twice as large as those used in figures 6a and 7a.

During the route from steady-state to chaos solutions, the chaotic attractor is observed without continuous evolution through quasi-periodic solutions. This is due to the heteroclinic orbit between two hyperbolic saddle focus type equilibrium points [15].

4. Conclusion

In this work, Wien-bridge type sinusoidal oscillator is modified to obtain chaotic attractor. The modification is carried out by following the design rules proposed in [14]. SPICE and MATLAB simulations andexperimental results are given. The observed chaotic attractor and route to it are explained using the observations in the experiments and the mathematical results. The equations obtained for the chaos attractor resemble that of the given in [8], but the implementation here is simpler as 6 resistors are used instead of 8. Also, here CFOA's are used instead of OA's, therefore a better high frequency performance is expected owing to the well-known advantages of the CFOA's over classical opamps. Since the equations resemble each other, the chaotic attractor observed is the same as the one given in [8].

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