Solution of Economic Dispatch Problem using Gravitational Search Algorithm

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Abstract

An efficient and optimum economic operation of electric power generation systems has always played an important role in the operation of the electric power industry. Low fuel cost and reliable operation of power systems is desired the available electricity generation resources to supply the load demand on the power system. Economic dispatch problem is one of the nonlinear optimization problems of electrical power systems. This study presents a new heuristic approach to solve the well-known power system economic dispatch problem using Gravitational Search Algorithm (GSA). In order to test the effectiveness of proposed approach it was applied to different test systems. To show that effectiveness of proposed algorithm the results obtained are compared to those reported in the literature. The simulation results demonstrate that this proposed approach is a very useful and efficient approach to solve economic dispatch problem.

1. Introduction

In recent years, developments in technology cause more energy demand in power systems and make these systems more complicated. Operation of power systems on optimal conditions and planning of it are required due to the increased energy demand. The Economic Dispatch (ED) problem, one of the non-linear optimization problems in electrical power systems, has an important place in the economical operation of the power system. In solving the ED problem, the objective is to minimize the total fuel cost, while satisfying the various physical and operational constraints. In the traditional ED problem, the fuel cost function of a generator is performed as quadratic function [1, 2].

So far, too many optimization methods have been used in the solution of ED problem. These methods can be classified as classical optimization and heuristic algorithms. In the solution of ED problem the conventional methods are mainly classical methods, which include gradient method, Lagrange relaxation method and linear programming method. Because of the nonlinear characteristics of ED problem with many local optimum solutions and a large number of constraints, the classical methods cannot execute well in solving ED problem [2, 3]. In recent years, many approaches have been presented in the literature to formulate and to solve ED with heuristic techniques. Generally, heuristic search techniques are used as tools for the solution of complex nonlinear optimization problems because of their robustness to overcome the deficiencies of the traditional methods [4].

Abido proposed multiobjective evolutionary algorithm for environmental/economic power dispatch problem [5]. Wang and Singh used particle swarm optimization with local search for multiarea environmental/economic dispatch problem [6]. Sharma et al. presented differential evolution algorithm with time-varying mutation for multiarea economic dispatch problem [7]. Lu et al. utilized chaotic differential evolution algorithm for dynamic economic dispatch problem with valve-point effects [8]. Abido proposed multiobjective evolutionary algorithms for environmental/economic power dispatch problem [9].

One of the recently improved heuristic algorithms is the gravitational search algorithm (GSA) based on the Newton's law of gravity and mass interactions [10]. GSA has been verified high quality performance in solving different optimization problems in the literature [11-14]. In this paper, a newly developed heuristic optimization called GSA method is proposed to solve the ED problem which is formulated as a non-linear optimization problem with equality and inequality in power systems. The proposed algorithm is tested in different test systems to show its effectiveness. Results obtained from GSA are compared to results reported in the literature. Results show that proposed approach is more effective and powerful than other algorithms in solution of ED problem.

2. Problem Formulation

The objective ED problem is to find the optimal combination of generators' output powers so as to minimize the total incremental fuel cost while all generating units satisfying the operating constraints and meeting the total load demand of a power system. The ED problem is to minimize the total fuel cost which can be defined mathematically as the sum of the cost function of each generator. The problem is formulated as follows:

$$\min F_t = \sum_{i=1}^{N} F_i\left(P_i\right) \tag{1}$$

Where F_i is the fuel cost for the generator unit i (\$/hr), N is the number of generators, P_i is the generation output of i^{th} generator. The fuel cost function is usually approximated by a quadratic function as follows:

$$F_i = a_i \times P_i^2 + b_i \times P_i + c_i \tag{2}$$

Where a_i, b_i and c_i represent the fuel cost coefficients of the i^{th} generator. Equation (3) is the inequality constraint between its minimum and maximum values for the electrical output power of each generator in the system.

$$P_i^{\min} \le P_i \le P_i^{\max} \qquad i=1,2,\ldots,N \tag{3}$$

Where P_i^{min} is the minimum operating limit of i^{th} generator, P_i^{max} is the maximum operating limit of i^{th} generator. Equation (4) is power equilibrium. P_L is total transmission line power losses. The ED problem considered does not take into account the transmission line.

$$D = \sum_{i=1}^{N} P_i - P_D - P_L = 0 \tag{4}$$

P_D represents the total active power demand in the system.

3. Gravitational Search Algorithm

Rashedi et al. proposed a new meta-heuristic searching algorithm called Gravitational Search Algorithm (GSA) in 2009. GSA is a stochastic optimization algorithm population based search algorithm motivated by the Newton's laws of gravity and mass interaction. According to the proposed algorithm, agents are assumed to be objects that their performances are measured by means of masses. The whole agents pull each other by the gravitational attraction force and this force induces the movement of all agents globally towards the agents with heavier masses. In GSA, each mass has four particulars: its position, its inertial mass, its active gravitational mass and passive gravitational mass. The position of the mass equaled to a solution of the problem and its gravitational and inertial masses are specified using a fitness function [10]. GSA algorithm can be summarized following steps:

3.1. Step 1: Initialization

When it is assumed that there is a system with N (dimension of the search space) masses, position of the i^{th} mass is described as follows. At first, the positions of masses are fixed randomly.

$$X_i = \left(x_i^1, \ldots, x_i^d, \ldots x_i^n\right) \quad \text{for, } i=1,2,\ldots,N \tag{5}$$

where, x_i^d is the position of the i^{th} mass in d^{th} dimension.

3.2. Step 2: Fitness Evaluation of All Agents

In this step, to execute for all agents at each iteration and *best* and *worst* fitness are computed at each iteration described as follows.

$$best(t) = \min_{j\in\{1,..,N\}} fit_j(t) \tag{6}$$

$$worst(t) = \max_{j\in\{1,..,N\}} fit_j(t) \tag{7}$$

where $fit_j(t)$ is the fitness of the j^{th} agent of iteration t, $best(t)$ and $worst(t)$ are best (minimum) and worst (maximum) fitness of all agents.

3.3. Step 3: Compute the Gravitational Constant (G(t))

In this step, the gravitational constant at iteration t ($G(t)$) is computed as follows.

$$G(t) = G_0 \exp(-\alpha \frac{t}{T}) \tag{8}$$

where G_0 is the initial value of the gravitational constant chosen randomly, α is a constant, t is the current iteration and T is the total iteration number.

3.4. Step 4: Update the Gravitational and Inertial Masses

In this step, the gravitational and inertial masses are updated for each agent at iteration as follows.

$$M_{ai} = M_{pi} = M_{ii} = M_i, \qquad i = 1,2,\ldots,N \tag{9}$$

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \tag{10}$$

where $fit_i(t)$ is the fitness of the i^{th} agent at iteration t.

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)} \tag{11}$$

where M_{ai} is the active gravitational mass of the i^{th} agent, M_{pi} is the passive gravitational mass of the i^{th} agent, M_{ii} is the inertia mass of the i^{th} agent, $Mi(t)$ is the mass of the i^{th} agent at iteration t.

3.5. Step 5: Calculate the Total Force

In this step, the total force acting on the i^{th} agent $(F_i^d(t))$ is calculated as follows.

$$F_i^d(t) = \sum_{j\in kbest j\neq i} rand_j F_{ij}^d(t) \tag{12}$$

where $rand_j$ is a random number between interval [0,1] and *kbest* is the set of first K agents with the best fitness value and biggest mass.

The force acting on the i^{th} mass ($M_i(t)$) from the j^{th} mass ($M_j(t)$) at the specific iteration t is described according to the gravitational theory as follows.

$$F_{ij}^d(t) = G(t)\frac{M_{pi}(t)\times M_{aj}(t)}{R_{ij}(t)+\varepsilon}(x_j^d(t) - x_i^d(t)) \tag{13}$$

where $R_{ij}(t)$ is the Euclidian distance between i^{th} and j^{th} agents $\left(\left\|X_i(t), X_j(t)\right\|_2\right)$ and ε is the small constant.

3.6. Step 6: Calculate the Acceleration and Velocity

In this step, the acceleration $(a_i^d(t))$ and velocity $(v_i^d(t))$ of the i^{th} agent at iteration t in d^{th} dimension are calculated through law of gravity and law of motion as follows.

$$a_i^d(t) = \frac{F_i^d(t)}{M_{ii}(t)} \quad (14)$$

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t) \quad (15)$$

where $rand_i$ is the random number between interval [0,1].

3.7. Step 7: Update the Position of Agents

In this step the next position of the i^{th} agents in d^{th} $(x_i^d(t+1))$ dimension are updated as follows.

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \quad (16)$$

3.8. Step 8: Repeat

In this step, steps from 2 to 7 are repeated until the iterations reach the criteria. In the final iteration, the algorithm returns the value of positions of the corresponding agent at specified dimensions. This value is the global solution of the optimization problem also.

All these steps explained above describes how the GSA works. Besides, the principle diagram of the GSA is illustrated in Fig. 1.

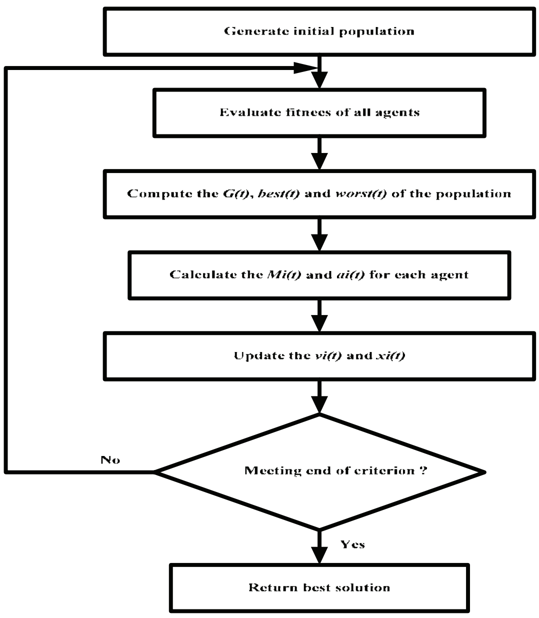

Fig. 1. The principle diagram of the GSA [10]

4. Simulation Results

Proposed approach has been applied to solve ED problem. In order to demonstrate the efficiency and robustness of proposed GSA approach based on Newtonian physical law of gravity and law of motion which is tested for different test systems. These are 3, 10 and 18 units systems ignored transmission line losses.

The setup parameters for the proposed algorithm are given in Table 1.

Table 1. Setting parameters of proposed approach for test systems

Parameters	Test System I	Test System II	Test System III
N	200	150	50
G_0	100	100	100
α	10	10	10
T	200	250	300

4.1. Test System I

This test system considered of 3 thermal units as given in Table 2. In this case, the total load demand expected to be determined was P_D = 850 MW and other parameters of these generators is taken from ref. [15].

Table 2. Generator data for test system I

Units	P_i^{min}	P_i^{max}	a_i	b_i	c_i
1	150	600	0.001142	7.2	510
2	100	400	0.001942	7.85	310
3	50	200	0.00482	7.97	78

The results obtained for this test system study are given Table 3, which shows that the GSA has good solution for power demand of 850 MW.

Table 3. Results obtained by the proposed approach for test system I

Units	Proposed GSA
1	438.8519
2	301.9486
3	109.1995
Total Power Output (MW)	850
Total Cost ($/h)	**8141.790495**

The best total fuel cost result obtained from proposed GSA and other optimization algorithms such as Artificial Immune System (AIS), Meta Evolutionary Programming, and Genetic Algorithm (GA) are compared in Table 4. From the Table 4 it is clear that result obtained from sum of generation units does not meet the load demand for AIS approach.

Table 4. Comparison of proposed approach for test system I

Units	Proposed GSA	AIS* [15]	Meta EP [15]	GA [15]
1	438.8519	438.8570	-	-
2	301.9486	301.9090	-	-
3	109.1995	109.2339	-	-
Total Power Output (MW)	850	850.00	-	-
Total Cost ($/h)	**8141.790495**	8141.7905	8141.7905	8194.3600

*Sum of the generation units are 849.9999 MW according to result obtained from AIS approach.

Figure 2 shows convergence of the best total fuel cost result obtained from GSA for test system I.

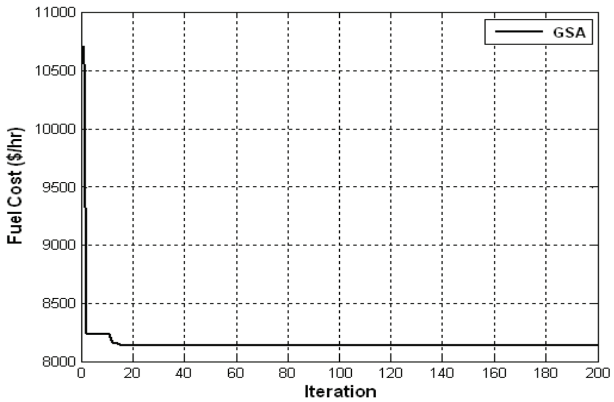

Fig. 2. Convergence of the GSA for test system I

4.2. Test System II

This test system considered of 10 thermal units as given Table 5. In this case, the load demand expected to be determined was P_D = 600 MW other parameters of these generators is taken from ref. [16].

The results obtained from proposed approach for this test system study are given in Table 6, which shows that the simulation results obtained by GSA for the best solution for power demand of 600 MW.

Table 5. Generator data for test system II

Units	P_i^{min}	P_i^{max}	a_i	b_i	c_i
1	0	72	0.0036	1.6358	22.734
2	0	70	0.0042	1.5519	23.150
3	0	64	0.0025	1.6728	21.680
4	0	61	0.0089	1.3690	17.797
5	0	72	0.0065	1.4409	21.150
6	0	71	0.0033	1.6290	22.490
7	0	73	0.0022	1.5615	24.670
8	0	73	0.0022	1.5615	24.670
9	0	143	0.0023	1.5499	42.384
10	0	143	0.0026	1.3807	46.881

Table 6. Results obtained by the proposed approach for test system II

Units	Proposed GSA
1	42.5352
2	46.2620
3	53.5925
4	32.2544
5	38.6003
6	47.2835
7	73.0000
8	73.0000
9	85.5011
10	107.9710
Total Power Output	600.0000
Total Cost ($/h)	**1304.577587**

Proposed approach has successfully specified the optimal value of generated power by each generating unit to minimize the total cost in the system. The best total fuel cost result obtained from proposed GSA and other optimization algorithms are compared in Table 7. From the Table 7 it is clear that EIM and AIACO approaches did not meet the load demand, its' result obtained from sum of generation units. Figure 3 shows convergence of the best total fuel cost result obtained from GSA for test system II.

Table 7. Comparison of proposed approach for test system II

Units	Method				
	EIM* [16]	IPSO [16]	AIACO* [16]	HPSO [17]	Proposed GSA
1	42.51	41.93	42.69	42.51	42.5352
2	46.42	48.03	46.95	46.425	46.2620
3	53.81	51.50	53.76	53.814	53.5925
4	32.18	32.10	32.37	32.184	32.2544
5	38.54	37.05	38.57	38.536	38.6003
6	47.40	49.23	47.29	47.404	47.2835
7	73.00	73.00	73.00	73.00	73.0000
8	73.00	73.00	73.00	73.00	73.0000
9	85.21	86.01	85.09	85.211	85.5011
10	107.92	108.36	107.27	107.93	107.9710
Total Power Output	600.00	599.99	600.00	600.00	600.0000
Total Cost ($/h)	1304.56	1304.58	1304.58	1304.577	**1304.577587**

*Sum of the generation units are 599,99 and 599,99 MW according to result obtained from EIM and AIACO approaches, respectively.

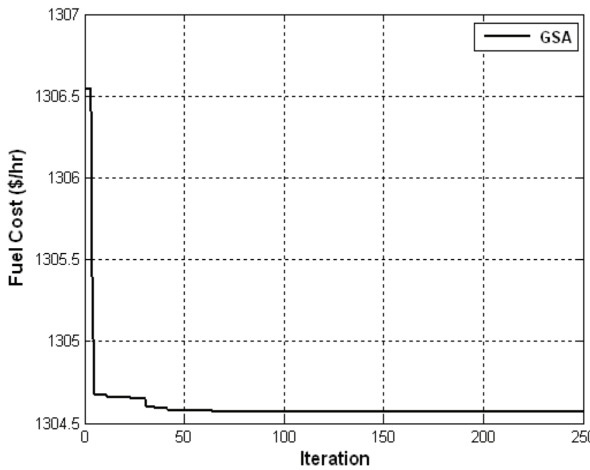

Fig. 3. Convergence of the GSA for test system II

4.3. Test System III

This test system considered of 18 thermal units as given Table 8. In this case, the load demand expected to be determined was P_D = 365 and 433.22 MW respectively; and other parameters of these generators is taken from ref. [18].

The results obtained from proposed approach for this test system study are given Table 9, which shows that the simulation results obtained by GSA for the best solution for power demand of 365, 80%P and 70%P MW respectively. The best total fuel cost result obtained from proposed GSA and other optimization algorithms are compared in Table 10. Figure 4 shows convergence characteristic curve of the best total fuel cost result obtained from GSA for 365 MW.

Table 8. Generator data for test system III

Units	P_i^{min}	P_i^{max}	a_i	b_i	c_i
1	7	15.00	0.602842	22.45526	85.74158
2	7	45.00	0.602842	22.45526	85.74158
3	13	25.00	0.214263	22.52789	108.9837
4	16	25.00	0.077837	26.75263	49.06263
5	16	25.00	0.077837	26.75263	49.06263
6	3	14.75	0.734763	80.39345	677.73
7	3	14.75	0.734763	80.39345	677.73
8	3	12.28	0.514474	13.19474	44.39
9	3	12.28	0.514474	13.19474	44.39
10	3	12.28	0.514474	13.19474	44.39
11	3	12.28	0.514474	13.19474	44.39
12	3	24.00	0.657079	56.70947	574.9603
13	3	16.20	1.236474	84.67579	820.3776
14	3	36.20	0.394571	59.59026	603.0237
15	3	45.00	0.420789	56.70947	567.9363
16	3	37.00	0.420789	55.965	567.9363
17	3	45.00	0.420789	55.965	567.9363
18	3	16.20	1.236474	84.67579	820.3776

Table 9. Results obtained by the proposed approach for test system III

Units	365 MW	80% P	70% P
1	15.0000	15.0000	15.0000
2	45.0000	45.0000	44.6580
3	25.0000	25.0000	25.0000
4	25.0000	25.0000	25.0000
5	25.0000	25.0000	25.0000
6	5.5700	3.0000	3.0000
7	6.2689	3.0000	3.0000
8	12.2800	12.2800	12.2800
9	12.2800	12.2800	12.2800
10	12.2800	12.2800	12.2800
11	12.2800	12.2800	12.2800
12	22.9371	18.9000	15.3936
13	3.0619	3.0000	3.0000
14	31.0380	31.8000	20.9806
15	35.9837	31.7000	22.9961
16	35.9272	34.0452	24.1027
17	37.0933	34.0108	24.0030
18	3.0000	3.0000	3.0000
Total Power Output	365.0000	346.5760	303.2540
Total Cost ($/h)	**25438.1**	**23858.5**	**20386.4**

Table 10. Comparison of proposed approach for test system III

Method	Demand Load (MW)		
	80%P	70%P	365 MW
y-iteration [18]	23861.58	20393.43	-
Binary GA[18]	23980.24	20444.68	-
Real GA[18]	23861.58	20396.39	25768.57
Hybrid GA-PSO [19]	23859.0012	20390.2625	25451.5661
Proposed GSA	**23858.5**	**20386.4**	**25438.1**

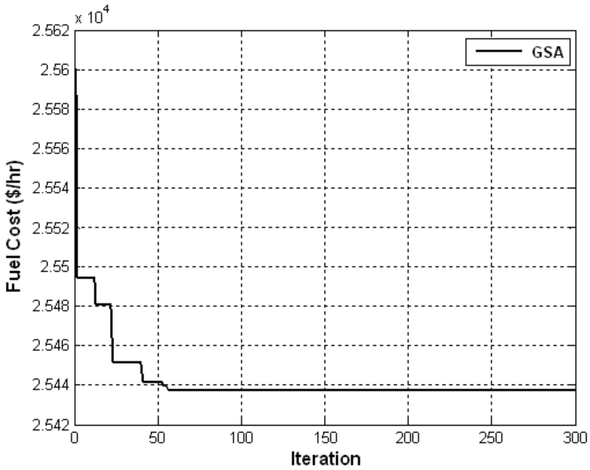

Fig. 4. Convergence of the GSA for test system III

5. Conclusion

In this paper, a novel approach based on the Newton's laws of gravity and mass interaction GSA has been presented and applied to economic power dispatch optimization problem. . The ED problem is formulated as a nonlinear optimization problem with equality and inequality constraint in power systems. The proposed approach has been tested for different test systems and results obtained are compared to other previously reported in the literature. The simulation results demonstrate the effectiveness and robustness of the proposed algorithm to solve ED problem in test systems. From the outcome of the results, it is obvious that the proposed approach can acquire satisfactory solution.

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