Synchronization of Different Chaotic Systems Using Generalized Active Control

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Abstract

Synchronization of completely different chaotic systems via active controllers is investigated. Chaos synchronization using active controllers is generalized for different chaotic systems. Two completely different Lorenz and Rössler systems are considered for synchronization and numerical simulations are presented graphically to confirm the validity of the proposed method.

1. Introduction

Chaos is one of the most significant topics in nonlinear science, and has been intensively studying since the Lorenz system [1] was introduced. After many chaotic systems has been discovered and developed, scientists have focused on chaos control and chaos synchronization since 1990s. Chaos synchronization was discovered by Pecora and Carroll [2] and there has been great interest in it, and its applications, such as secure communication, and system identification. Given a chaotic system considered as a master system, and another identical system considered as a slave system, the dynamical behaviors of them may be identical after a transient when the slave system is driven by a control input. By using this synchronization principle, some chaotic circuits were developed and applied to secure communication systems [2-5].

Many methods have been developed to synchronize chaotic systems, including nonlinear feedback method [6], adaptive control method [7], anti-synchronization method [8] and sliding mode control method [9]. Chaos synchronization using active control which is introduced in [10] is one of the these methods. Unified chaotic systems [11], the energy resource chaotic system [12] and some other systems have been synchronized with this method. Two identical systems are usually synchronized by using of the methods mentioned above; however, it is not always possible to assume that all components are identical in engineering. Therefore, achieving synchronization of two different chaotic systems is more attractive and significant from a practical viewpoint.

The paper investigates the mathematical and practical possibilities of synchronization of completely different chaotic systems using active control. To this end, a mathematical model is provided to solve synchronization problem of completely different chaotic systems using active control in Section 2. In Section 3, numerical simulations are provided to illustrate our findings using the Lorenz system (which can be encountered in atmospheric sciences, laser devices, and some other systems related to convective heat transfer) as the master system, and the Rössler system [13] (which can be encountered in chemical reactions) as the slave system. Main conclusions to be drawn from this study are given in Section 4.

2. Active Control and Chaos Synchronization

The active control method proposed in [10] is considered to synchronize two different chaotic systems. For this purpose, consider a master system

$$\dot{x} = Ax + g(x) \tag{1}$$

where $x \in R^n$ is the state vector, $A \in R^{n \times n}$ is a constant system matrix, and $g(x)$ is a nonlinear sequence function. A slave system is defined as

$$\dot{y} = By + f(y) + \phi(t) \tag{2}$$

where $y \in R^n$ is the state vector $B \in R^{n \times n}$ is a constant system matrix, $f(y)$ is a nonlinear sequence function, and $\phi(t) \in R^n$ is an active control function. A master-slave synchronization scheme is illustrated in Figure 1. The feedback from the controller $\phi(t)$ is designed to get the error e to decay to zero.

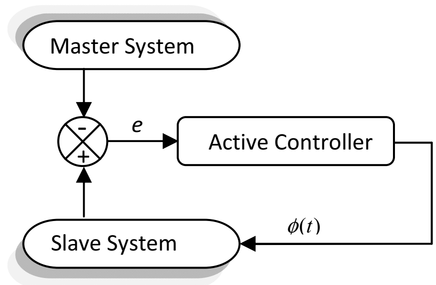

Figure 1: The master-slave synchronization scheme.

The error state is defined as $e = y - x$; therefore, the error dynamics are written as follows.

$$\dot{e} = \dot{y} - \dot{x} = Ce + G(x, y) + \phi(t) \tag{3}$$

where $C = \overline{B} - \overline{A}$ is the common parts of the system matrices in the master and slave systems; the non-common parts and nonlinear functions are gathered in $G(x, y)$ as

$$G(x,y) = f(y) - g(x) + (B-\overline{B})y - (A-\overline{A})x \qquad (4)$$

and $\phi(t)$ is the controller matrix. Error vectors with an appropriate controller $\phi(t)$ satisfying $\forall x,y,e \in R^n$ converge to zero. Hence, an appropriate controller should eliminate nonlinear terms and non-common parts, and contain another part to achieve stability, such as

$$\phi(t) = -G(x,y) + u(t) \qquad (5)$$

where $u(t) = -Ke$ is a linear controller and $K \in R^{nxn}$ is a linear gain matrix. Substitution of equation (5) into (3) leads to

$$\dot{e} = Ce + u(t) \qquad (6)$$

With replacing $u(t) = -Ke$ in the equation (6), the error dynamic is defined by

$$\dot{e} = (C-K)e \qquad (9)$$

Synchronization of chaos by using active control can be realized when master and slave systems are completely different. If the eigenvalues $\lambda_{i\,(i=1,2,\ldots,n)}$ of the matrix $C-K$ are negative ($\lambda_i < 0$), then the error state vectors exponentially converge to zero. That is, the master and slave systems exponentially synchronize.

3. Numerical Simulations

To realize and verify the chaos synchronization, two completely different chaotic systems, the Lorenz system as a master system and the Rössler system as a slave system, are selected.

The master Lorenz system [1] is defined by

$$\begin{cases} \dot{x}_1 = \sigma(y_1 - x_1) \\ \dot{y}_1 = \alpha x_1 - y_1 - x_1 z_1 \\ \dot{z}_1 = x_1 y_1 - \beta z_1 \end{cases} \qquad (10)$$

where σ, α and β are the system parameters. The slave Rössler system [13] is defined by

$$\begin{cases} \dot{x}_2 = -y_2 - z_2 + \phi_x \\ \dot{y}_2 = x_2 + a y_2 + \phi_y \\ \dot{z}_2 = b x_2 - c z_2 + x_2 z_2 + \phi_z \end{cases} \qquad (11)$$

where a, b and c are the system parameters, and ϕ_x, ϕ_y, and ϕ_z are the control signals. Figure 2 exemplifies the Lorenz and Rössler attractors in the state space.

The primary goal of the control signals defined as ϕ_x, ϕ_y, and ϕ_z is providing the slave system to pursue the master system, which is the requirement to achieve synchronization. Error vectors are defined as $e_x = x_2 - x_1$, $e_y = y_2 - y_1$, and $e_z = z_2 - z_1$. The error dynamics are defined by

$$\begin{cases} \dot{e}_x = -y_2 - z_2 - \sigma(y_1 - x_1) + \phi_x \\ \dot{e}_y = x_2 + a y_2 - \alpha x_1 + y_1 + x_1 z_1 + \phi_y \\ \dot{e}_z = b x_2 - c z_2 + x_2 z_2 - x_1 y_1 + \beta z_1 + \phi_z \end{cases} \qquad (12)$$

where the control vectors are defined as

$$\begin{cases} \phi_x = u_1 + y_2 + z_2 + \sigma(y_1 - x_1) \\ \phi_y = u_2 - x_2 - a y_2 + \alpha x_1 - y_1 - x_1 z_1 \\ \phi_z = u_3 - b x_2 + c z_2 - x_2 z_2 + x_1 y_1 - \beta z_1 \end{cases} \qquad (13)$$

By writing equation (12) into equation (13), the error dynamics occur from only linear control vectors,

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_y \\ \dot{e}_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad (14)$$

The linear controller $u(t) = -Ke$ can be defined as

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = - \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ e_z \end{bmatrix} \qquad (15)$$

There are a number of choices to obtain controller coefficients k_{ij}'s to obtain a stable closed loop system. For a particular choice of feedback gains

$$K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad (16)$$

the error dynamics have eigenvalues that are found to be -2, -2 and -2, which leads to a stable closed loop system. Hence, synchronization of the Lorenz and Rössler systems are achieved as we will observe in numerical results.

Numerical simulations are conducted by using Matlab / Simulink, and numerical results are given graphically to verify the proposed method. The simulation results are provided in Figures 2–7. The system parameters of the Lorenz and Rössler systems are selected as $\sigma = 10$, $\alpha = 28$, $\beta = 8/3$, and $a = 0.32$, $b = 0.3$, $c = 4.8$, respectively. The initial conditions of the master Lorenz and slave Rössler systems are taken as $x_1(0) = 10$, $y_1(0) = 10$, $z_1(0) = 10$ and $x_2(0) = 2$, $y_2(0) = 2$, $z_2(0) = 1$, respectively.

Figure 2 illustrates the phase portrait of the Lorenz and Rössler systems. Figure 2(a) shows the phase portrait of the Lorenz System, and Figure 2(b) shows the phase portrait of the Rössler system. In Figure 3, the time responses of the state variables of the Lorenz and Rössler systems are presented. Figure 3(a) displays x_1 and x_2, Figure 3(b) displays y_1 and y_2, and Figure 3(c) displays z_1 and z_2.

In Figures 4 and 5, control signals are activated at the time $t = 0$. In Figure 4, the time responses of the state variables of the Lorenz and Rössler systems are presented. Figure 4(a) displays x_1 and x_2, Figure 4(b) displays y_1 and y_2, and Figure 4(c) displays z_1 and z_2. It is seen that the active controller has synchronized the Lorenz and Rössler systems. Figure 5 illustrates the time responses of the error vectors. Figure 5(a) shows e_x, Figure 5(b) shows e_y, and Figure 5(c) shows e_z. After control signals are activated, the error vectors converge to zero quickly.

In Figures 6 and 7, control signals are activated at the time $t = 20$. The time responses of the state variables of the master and slave systems are given in Figure 6. Figure 6(a) displays x_1 and x_2, Figure 6(b) displays y_1 and y_2, and Figure 6(c) displays z_1 and z_2. In Figure 7, the time responses of the error vectors are presented. Figure 7(a) shows e_x, Figure 7(b) shows e_y, and Figure 7(c) shows e_z. It is clear that after control signals are activated, the error vectors converge to zero quickly.

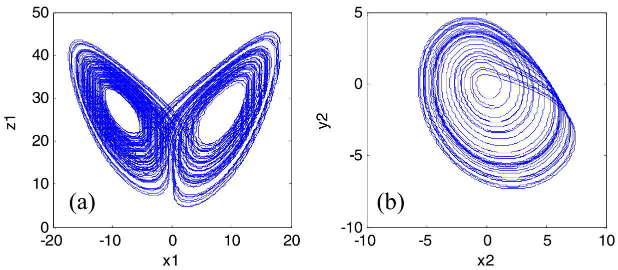

Figure 2: The phase portrait of (a) the Lorenz system, (b) the Rössler system.

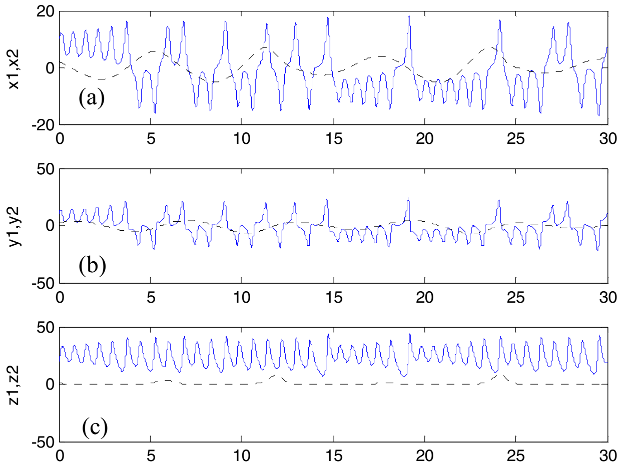

Figure 3: The time responses of the state variables of the master (x_1, y_1, z_1) and slave (x_2, y_2, z_2) (dot lines) systems.

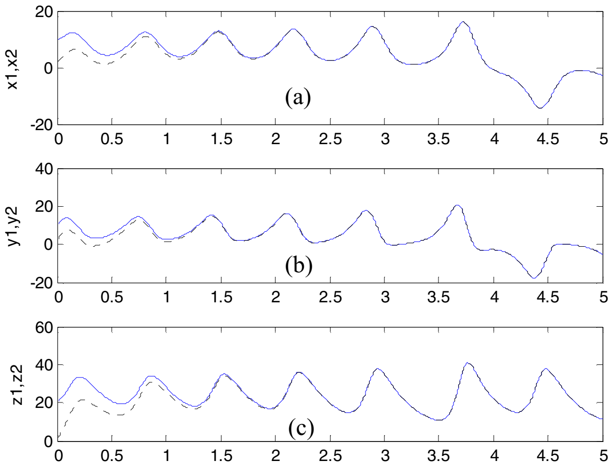

Figure 4: The time responses of the state variables of the master (x_1, y_1, z_1) (dot lines) and slave (x_2, y_2, z_2) systems with the controller activated at the time $t = 0$.

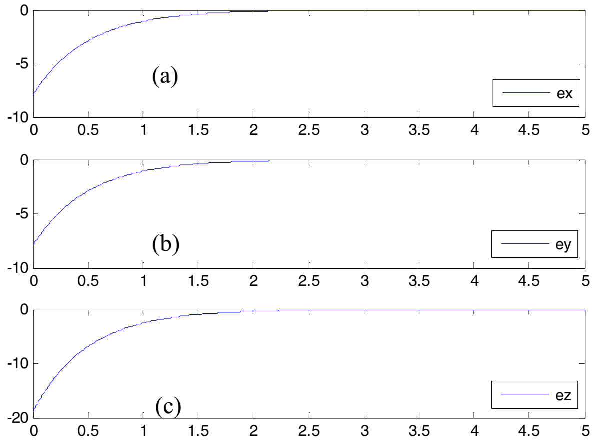

Figure 5: Dynamics of the synchronization errors (e_x, e_y, e_z) with the controller activated at the time $t = 0$.

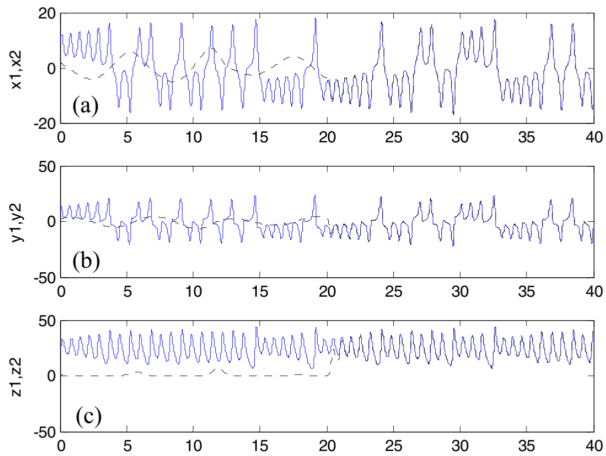

Figure 6: The time responses of the state variables of the master (x_1, y_1, z_1) and slave (x_2, y_2, z_2) (dot lines) systems with the controller activated at the time $t = 20$.

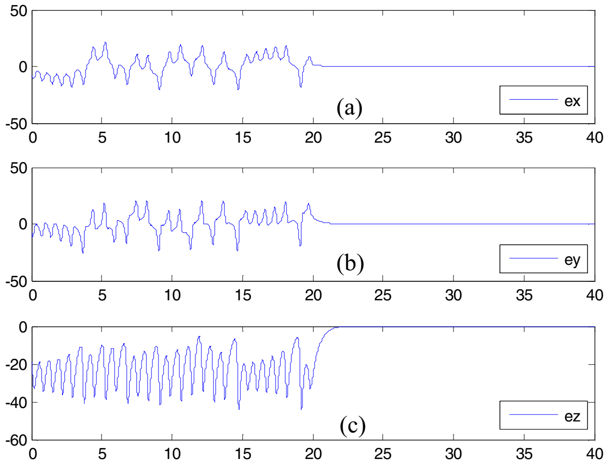

Figure 7: The time responses of the error vectors (e_x, e_y, e_z) with the controller activated at the time $t = 20$.

4. Conclusions

This study demonstrates that synchronization of completely different chaotic systems by means of active control is generalized. Numerical results verify the validity and effectiveness of the generalized active control method. Many chaotic systems in practical applications have different structures, thus, we believe that this generalized study will be a useful tool for synchronization of different chaotic systems via active control.

5. References

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