# Composite Power System Reliability Modeling and Evaluation Considering Aging Components

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# Abstract

As the components grow older, most power systems may enter a wear out stage indicating an aging trend. This paper is focused on aging effects in composite power system reliability evaluation. Quantitative reliability indices such as Loss of Load Expectation (LOLE) are calculated. The IEEE Reliability Test Systems is used to illustrate this technique.

#### 1. Introduction

Effective reliability evaluation is important to planners and operators of power systems. However, high level of reliability may also cause high costs. So it is important to study the trade off between these two variables in ensuring their acceptable levels using multi-objective optimization technique [1], [22].

A major part of the power system consists of three divisions [2]: generation, transmission and distribution, shown by Fig.1. Reliability indices can be evaluated in each hierarchical level and provide planners or operators with alternate planning or operating strategies [3]. In this paper, reliability modeling and analysis is based on the composite system [4-6] which is hierarchical level 2. Composite power system reliability assessment deals with transmission constraints as well as generation capacity [7]. AC load flow or DC load flow [8-9] may be used depending upon the needs of the study. DC load flow method is assumed in this paper.

In this paper Sequential Monte Carlo [10-14] simulation is used to build reliability models and carry out assessment.



Fig. 1. Three zones of power systems

#### 2. Background

Sequential Monte Carlo simulation is based on sampling the probability distribution function of each component. Time to next transition is sampled using the inverse transform method [18], illustrated by (1).

$$x = F^{-1}(Z) \tag{1}$$

Where F ( $\bullet$ ) is a probability distribution function of random variable x, Z is uniform random generating number on interval (0, 1).

In general, electromechanical equipment of power systems develops aging tendency [19-21] as the age of component increases. As quantitative indices, Loss of Load Expectation (LOLE), Loss of Load Probability (LOLP), Loss of Load Duration (LOLD), Loss of Load Frequency (LOLF), and Expected Energy Not Supplied (EENS) [23-25] are used to examine aging effects on composite system reliability. The methodology is applied to the IEEE Reliability Test Systems (RTS) [26-27] illustrated by Section 7.

#### 3. Problem Formulation

## 3.1. Stochastic Process [10], [15]

As a mathematical model of power system, Stochastic Process is introduced. Power systems consist of generators, transmission lines, transformers, and so on. In this paper, for transmission lines and transformers, it is assumed that they never fail. The history of all the generators is modeled as a stochastic process.

## 3.2. DC Power Flow and Linear Programming [28-29]

For composite system reliability analysis, a power flow method is needed to determine the system status. DC power flow is used. Following assumptions are made.

- 1. Each bus voltage magnitude is one per unit
- 2. No line losses. Only imaginary part of Y matrix is con sidered.

So that,

$$P_i = \sum_{j} -B_{ij} \theta_{ij}, \text{ for bus } i$$
(2)

Matrix form follows,

$$P = -B\theta \tag{3}$$

Where  $P_i$  is real power flow at bus i, matrix B is an imaginary part of Y matrix,  $\theta_{ij}$  is the difference between angles from bus i to j.

Following equations are described in (4)-(8).

Load curtailment = 
$$Min \sum_{i}^{N} C_{i}$$
 (4)

Subject to

$$B\theta = P_G - P_D + C \tag{5}$$

$$P_G \leq P_G^{\max} \tag{6}$$

$$0 \leq C \leq P_D \tag{7}$$

$$|P_{line}| \leq P_{line}^{\max} \tag{8}$$

Where N is the bus number

C is the vector of load curtailments

- $P_G$  is the vector of generation
- $P_G^{\text{max}}$  is the vector of upper limits of generation

 $P_D$  is the vector of load

 $P_{line}$  is the vector of line real power flows

 $P_{line}^{\text{max}}$  is the vector of upper limits of flows

In the above equations,  $B, P_G^{\max}, P_{line}^{\max}, and P_D$  are known, and  $\theta, P_G, and, C$  are unknowns. And,  $P_{line}$  is the function about  $\theta$  by (2). So above equations are based on standard linear programming model.

## 4. Sequential Monte Carlo Technique

To model system failure or success state, Sequential Monte Carlo is applied. The chronological system state transition is performed by distribution functions of each component.

## 5. System State Sampling

# 5.1. Sampling Time for the Renewal Model

There are several renewal distribution functions: Exponential, Weibull, Normal, Log-normal, etc. In this paper, exponential distribution function is used for modeling of non-aging component, shown by (9). Then, sampled time x is obtained by (1).

$$F(x) = 1 - e^{-\rho x} \tag{9}$$

$$x = \frac{-\ln(Z)}{\rho} \tag{10}$$

Expected value of N (t):

$$E[N(t)] = \Lambda(t) = \int_{0}^{t} \int \rho(u) du = \rho t$$
(11)

Probability that the number of events during delta t will be k:

$$\Pr(N(\Delta t) = k) = \frac{\left(\int_{0}^{\Delta t} \rho(t)dt\right)^{k} e^{-\int_{0}^{\Delta t} \rho(t)dt}}{k!}$$
$$= \frac{\left(\rho\Delta t\right)^{k} e^{-\rho\Delta t}}{k!}$$
(12)

Where N(t) is the number of events during time t. Intensity rate  $\rho$  is failure rate or repair rate, depending on the state of a component is up or down.

# 5.2. Sampling Time for the Aging Model

Non-homogeneous Poisson Process (NHPP) [16-17] is commonly used for modeling of aging components. In this paper, Power Law Process (PLP) [16], [30-31], one of NHPP, is introduced. This model is applied to only up times, since repair time is not affected by aging. Basically, in a PLP, the failure rate function is the same as that of Weibull distribution, shown by (13). However, they are different from each other. For Weibull distribution, failure rate of each cycle is repeated with same value, which is called perfect repair action [35]. Expected value of N (t) and probability of N (t)=k are given in (14), (15).

$$\lambda(t) = \lambda\beta t^{\beta-1} \tag{13}$$

$$\Lambda(t) = E[N(t)] = \int_{0}^{t} \int \lambda(u) du = \lambda t^{\beta}$$
(14)

$$\Pr(N(\Delta t) = k) = \frac{\lambda^k (\Delta t)^{\beta k} e^{-\lambda (\Delta t)^{\beta}}}{k!}$$
(15)

There are a number of techniques [32-34] to sample a NHPP. Interval by Interval method is based on time distribution between event arrivals. At arrival time  $t_k$ , the interval time to next arrival, x has following distribution function:

$$F_{t_{k}}(x) = 1 - \exp[-\int_{0}^{x} \int \lambda(t_{k} + u) du]$$
  
= 1 - exp[-\lambda{(t\_{k} + x)^{\beta} - t\_{k}^{\beta}}] (16)

To get time to sample, inverse transform method is used.

$$Z = F_{t_k}(x) = 1 - \exp[-\lambda\{(t_k + x)^{\beta} - t_k^{\beta}\}]$$
(17)

which gives:

$$t_{k+1} = (t_k^{\beta} - \frac{\ln Z}{\lambda})^{\frac{1}{\beta}}, \text{ for } k \ge 0, t_0 = 0$$
 (18)

General form follows.

$$x_{k} = \left(\frac{-\ln Z}{\lambda}\right)^{\frac{1}{\beta}}, \text{ for } k = 1$$
(19)

$$\mathbf{x}_{k} = \{ (\sum_{i=1}^{k-1} \mathbf{x}_{i})^{\beta} - \frac{\ln Z}{\lambda} \}^{\frac{1}{\beta}} - \{ \sum_{i=1}^{k-1} \mathbf{x}_{i} \}, \text{ for } k \ge 1$$
 (20)

Interval time to next event is calculated by (19) and (20) which have recursive form.

Here is a consideration related to aging. Basically it is expected that system may have the same reliability at the beginning regardless of aging effects. Equations (21) and (22) are described from (15).

$$MTTFF = \int_{0}^{\infty} \int r(N(\Delta t) = 0) = \frac{\frac{1}{\beta}\Gamma(\frac{1}{\beta})}{\lambda^{\frac{1}{\beta}}} = \frac{1}{\lambda_{exp}} \quad (21)$$
$$\lambda = \lambda_{exp}^{\beta}(\frac{1}{\beta})^{\beta}\Gamma(\frac{1}{\beta})^{\beta} \quad (22)$$

## 6. System Reliability Indices

Fig. 2 shows the flowchart of composite system reliability assessment. Expected load curtailments value during one year is given by the expected energy not supplied, EENS [MWh/year] according.



#### 7. Case Studies

The layout of the 24 bus IEEE RTS is shown by Fig. 3. MATLAB is used for system modeling and simulation.

To study the aging effects of system components on composite system reliability evaluation in detail, cases proposed are shown in table 1. For case 1, transmission constraints are not considered. Reliability assessment is performed only by generation capacity. On the other hand, case 2 includes consideration of transmission system. In this case, linear optimization technique based on DC power flow, described in Section 3, is used.



Fig. 3. Single Area IEEE RTS

Table 2, 3, and 4 describe location of generating units and their reliability data.

 Table 1. Description of cases

Case	Description			
1	HL 1 (generation system)			
2	HL 2 (composite system)			

Table 2. Reliability data of generating units

Generating units	Capacity [MW]	Failure Rate [1/h]	Repair Rate [1/h]
G1-G5	12	1/2940	1/60
G6-G9	20	1/450	1/50
G10-G15	50	1/1980	1/20
G16-G19	76	1/1960	1/40
G20-G22	100	1/1200	1/50
G23-G26	155	1/960	1/40
G27-G29	197	1/950	1/50
G30	350	1/1150	1/100
G31-G32	400	1/1100	1/150

Table 3. Generator bus data

Bus	Units [MW]	Capacity [MW]
1	G6/G7/G16/G17	192
2	G8/G9/G18/G19	192
7	G20/G21/G22	300
13	G27/G28/G29	591

15	G1/G2/G3/G4/G5/G23	215
16	G24	155
18	G31	400
21	G32	400
22	G10/G/11/G12/G13/G14/G15	300
23	G25/G26/G30	660

Bus	Load percent	Bus	Load percent	Bus	Load percent
1	3.8	7	4.4	15	11.1
2	3.4	8	6.0	16	3.5
3	6.3	9	6.1	18	11.7
4	2.6	10	6.8	19	6.4
5	2.5	13	9.3	20	4.5
6	4.8	14	6.8		

#### Table 4. Bus load percent of system

# 7.1. Case 1

Reliability indices are shown in Table 5 at HL 1 level. For non-aging model, all generators are modeled by exponential distribution.

Table 5. Reliability indices in HL 1

Non-Aging Model		LOLE [h]	EENS [MWh/y]	LOLD [h]	LOLF [#/h]
		9.42	1095.76	2.37	4.55×10 <sup>-4</sup>
Aging (β)	1.0	9.35	1113.95	2.22	4.81×10 <sup>-4</sup>
	1.2	54.08	8018.73	5.40	$11.45 \times 10^{-4}$
	1.4	185.05	33829.67	6.13	$3454 \times 10^{-4}$
	1.6	455.07	95821.07	6.78	76.76×10 <sup>-4</sup>
	1.8	723.81	174535.44	7.31	113.28×10 <sup>-4</sup>

# 7.2. Case 2

For linear programming, additional line flow limit data are given in references [26-27]. Table 6 shows the results of composite system reliability evaluation. Similarly, as parameter  $\beta$  is increased, reliability indices tend to grow. To visualize aging effects on composite system reliability, LOLP is compared with different  $\beta$  in HL 1 and HL 2, shown in Fig. 4.

Table 6. Reliability indices in HL 1

Non-Aging Model		LOLE [h]	EENS [MWh/y]	LOLD [h]	LOLF [#/h]
		31.19	3978.09	3.47	$1.02 \times 10^{-3}$
Aging (β)	1.0	31.25	4101.52	3.84	0.93×10 <sup>-3</sup>
	1.2	140.54	24686.03	6.93	2.32×10 <sup>-3</sup>

1.4	529.38	96500.03	8.08	7.49×10 <sup>-3</sup>
1.6	796.65	219923.57	9.44	9.66×10 <sup>-3</sup>
1.8	995.94	285900.64	9.79	11.64×10 <sup>-3</sup>



Fig. 4. Load curtailment versus  $\beta$ 

#### 8. Conclusions

Aging of components is an important fact in power systems. So it is necessary to examine aging characteristics in system reliability or in economic evaluation.

Sequential Monte Carlo based on Stochastic Process is applied to Single Area IEEE RTS which is used to test and analyze reliability assessment. As aging parameter  $\beta$  representing aging level is increased, probability and frequency of system failure become higher.

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