SIMULATION OF SYNTHETIC APERTURE RADAR IMAGING USING CAPON SPECTRUM ESTIMATION METHOD

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ABSTRACT

The matched-filter bank (MAFI) spectral estimators have received considerable attention in a number of applications. In this paper we present an adaptive finite impulse response (FIR) filtering approach which is Capon algorithm. We compare the Capon method with fast Fourier transform (FFT) and also we compare the forward only Capon with the forward-backward Capon.

I. INTRODUCTION

The matched filter bank estimators have received considerable attention in a variety of applications including target range signature estimation and synthetic aperture radar (SAR) imaging. The classical approaches to spectral estimation include the discrete Fourier transform (DFT), and its variants which are typically based on smoothing the spectral estimate or windowing the data [1], [2]. An important matched-filter bank (MAFI) spectral estimation method is CAPON [3]. It is well known that Capon can yield more accurate spectral estimates with much lower sidelobes and narrower spectral peaks than the fast Fourier transform (FFT) method, which is also a special case of the FIR filtering approaches [4]. It was a controversial subject that whether or not the forward-backward Capon method is better than the forward-only Capon spectral estimation. It was showed that the forward-backward MAFI estimator usually provides better estimation results than the forward-only approaches [5].

In this paper, we compare the Capon method with fast Fourier transform (FFT).We show by means of experimental examples the Capon method can provide more accurate spectral estimates , narrower spectral peaks and lower sidelobe levels than the fast Fourier transform method (FFT) and we demonstrate the forward-backward Capon estimator yields better estimation than the forward-only Capon approach.

In section 2, we formulate the forward-backward filtering and the Capon spectral estimation approach and we explain how to apply FIR filtering approaches to SAR imaging. In section 3, we present experimental results, we compare the Capon spectral estimation to FFT and we discuss the forward-backward Capon estimators. And finally in section 4, we conclude the paper.

II. PROBLEM FORMULATION

We consider firstly 1-D data sequences after we extend them to 2-D cases Capon spectral estimation.
Let y (n) shows a discrete time 1-D sequences and n = 0,1,…,N

$$y(n) = \alpha(w)\, e^{jnw} + e_n(w) \qquad (1)$$

where α (w) denotes the complex amplitude of a 1-D sinusoid with frequency (w) and $e_n(w)$ denotes unmodeled noise term at frequency (w) with zero mean. The problem of interest is to estimate α (w) for any given (w).

One of the traditional method to obtain α (w) is fast Fourier transform (FFT), which is computationally very efficient algorithm. However it is well known that FFT methods suffer from high sidelobe effects, wider spectral peaks and poor accuracy. Although windowed FFT algorithms reduce the sidelobs, it decreases the resolution. We present 1-D Capon algorithm which is an adaptive FIR filtering approach for estimation of α (w). We begin describing the use of FIR filters for estimation of α (w_1)

and after we present extend them to the 2-D CAPON algorithm. Capon is a non-parametric adaptive match filter bank (MAFI) approach [5] and follows two main step:

1-)Pass the data thorough a band pass filter with varying centre frequency (w)

2-)Estimate the power at (w) for any $w \varepsilon [0, 2\pi)$ of interest from the filtered data.

The band pass filter used is usually an M-tap FIR filter with its coefficient vector given by

$$h(w)=[h_1(w)\ h_2(w)\ \ldots\ldots\ldots h_M(w)]^T \quad (2)$$

With $(.)^T$ denoting the transpose. The choice of M has been widely discussed in [5]. Let

$$\hat{y}_i = [y_i \quad y_{i+1} \ldots\ldots\ldots y_{i+M-1}]^T, \quad i = 0, 1, \ldots\ldots, N - M \quad (3)$$

be the overlapping sub vectors of the data vector

$$y = [y_0\ y_1 \ldots\ldots\ldots y_{N-1}]^T \quad (4)$$

The output of the FIR filter when the input is the raw data sequence $\hat{y}_i$, is given by

$$h^H(w)\ \hat{y}_i = \alpha\ (w)\ [h^H(w)\ a(w)]\ e^{jiw} + w_i(w), \quad i = 0, 1, \ldots\ldots, N\text{-}M \quad (5)$$

where $(.)^H$ denotes complex conjugate transpose and

$$a(w) = [1\ e^{jw} \ldots\ldots e^{j(M-1)w}]^T . \quad (6)$$

Let

$$h^H(w)\ a(w) = 1. \quad (7)$$

thus, (5) becomes

$$h^H(w)\ \hat{y}_i = \alpha\ (w)\ e^{jiw} + w_i(w). \quad (8)$$

as we refer before forward and backward approaches provide better estimation than forward only estimation [5], we now consider the filter output of the data sequence in backward order:

$$\tilde{y} = [y^*_{N-1} \quad y^*_{N-2} \ \ldots\ldots y^*_0] \quad (9)$$

Let

$$\tilde{y}_i = [y^*_{N-i-1}\ y^*_{N-i-2} \ldots\ldots\ldots y^*_{N-i-M}]^T, \ i = 0, 1,., N\text{-}M. \quad (10)$$

The output of the backward FIR filter when the input is $\tilde{y}_i$ is given by

$$h^H(w)\ \tilde{y}_i = e^{-j(N-1)w}\ \alpha^*(w)\ e^{\ jiw} + w^*_i(w) \quad (11)$$

where $w^*_i(w)$ denote the backward unmodeled noise at the filter output.

It is usually expected that the forward and backward FIR filter outputs given in (8) and (12), respectively, give better spectral estimation performance [4].

Using the forward and backward FIR filter outputs, it can be shown in [4] that the least-squares estimate of $\alpha(w)$ as

$$\acute{\alpha}\ (w) = ½[\ h^H(w)\ \hat{y}\ (w) + e^{-j(N-1)w}\ \ \tilde{y}^H(w)\ h(w)]. \quad (12)$$

It is also remarkable that when M = 1, h(w) becomes a scalar and FIR filtering approaches become the same as the FFT approach. For N >> M, the FIR filtering approach approximately reduces to FFT. Therefore, significant differences between FFT and the FIR filtering approaches occur only when M is sufficiently large as compared with N [4].

We can show that the h(w) in (11) satisfies [4]

$$Jh^*(w) = h(w)\ e^{-j(M-1)w} \quad (13)$$

Therefore, after some algebra (12) is equivalent to

$$\acute{\alpha}\ (w) = h^H(w)\ \hat{y}\ (w). \quad (14)$$

Although the forward backward estimate of α is the same form as the forward only estimate of α, the filter vector h(w) obtained with the forward backward approach is different form that related to the forward only approach.

Let R denote the covariance matrix of the data vector y(n);

$$R = E\{y(n)\ y^H(n)\} \quad (15)$$

Where {.} denotes expectation operator. To make sure $^{R-1}$ exists, we must choose M < N/2. The power of the filter output can be written as

$$E\{\left| y_F(n)^2 \right|\} = h^H R\ h \quad (16)$$

Where $y_F(n)$ is the MAFI filter output.

$$y_F(n) = \sum_{m=0}^{M} h_m\, y(n-m) = h^H \begin{vmatrix} y(n) \\ y(n-1) \\ \\ y(n-M) \end{vmatrix} \cong h^H\ y(n) \quad (17)$$

The filter frequency response is given by

$$H\ (w) = \sum_{m=0}^{M} h_m e^{-jwm} \ = h^H\ a(w) \quad (18)$$

The Capon method uses a band pass filter satisfying which means minimising of the power of the filter output;

h = arg $\min_{h(w)} h^H R h$, subject to $h^H a(w) = 1$. (19)

the solution of (17) is well known and is given by [1]:

$$h_w = \frac{R^{-1}a(w)}{a^H(w)R^{-1}a(w)} \tag{20}$$

If we insert (20) into filter output (16) then we can obtain 1-D Capon [3] Power spectral density estimator (PSD)

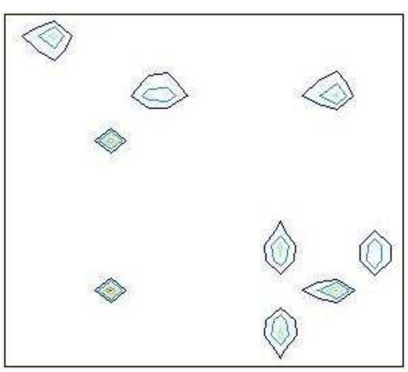
$$\acute{\alpha}_{\text{Capon 1-D}}(w) = \frac{1}{a^H(w)R^{-1}a(w)} \tag{19}$$

The 2-D Capon estimator can be achieved as a fairly straightforward extension of the 1-D version:

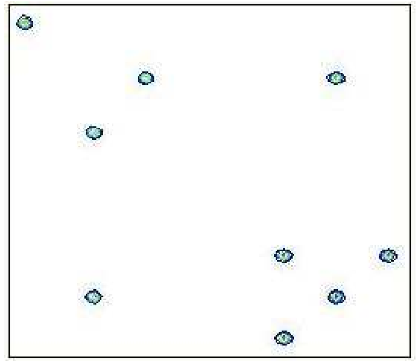
$$\acute{\alpha}_{\text{Capon 2-D}}(w_1,w_2) = \frac{a(w_1,w_2)^H R^{-1} a(w_1,w_2)}{a(w_1,w_2)^H R^{-2} a(w_1,w_2)} \tag{20}$$

III. EXPERIMENTAL RESULTS

The conventional SAR imaging method is FFT. Many parametric and non-parametric spectral estimation methods have also more recently been used for SAR imaging [1], [2]. It has been shown in [6] that the Capon method gives good SAR images. We present experimental results showing the performance of the Capon algorithm in synthetic aperture radar (SAR) imaging. We compare the performance of Capon with FFT and also we compare the forward-only Capon with the forward-backward Capon.



(a)



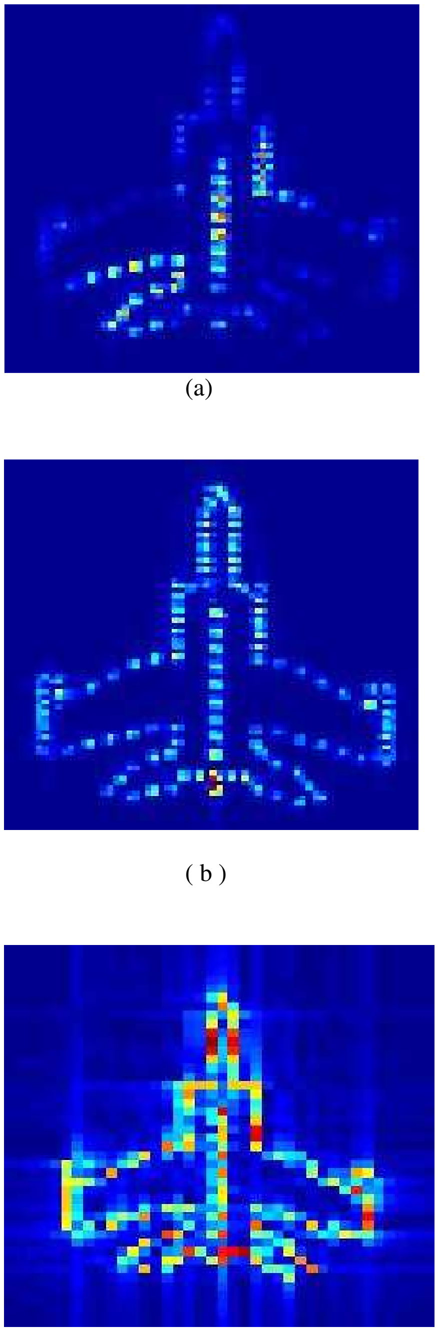
(b)

Fig. 1. SAR images by using (a) 2-D FFT, (b) 2-D Capon

Fig. 1(a) shows the FFT power spectral density estimation. It is clearly shown that FFF estimates are not very accurate and but as it is seen in Fig. 1(b), Capon method gives more accurate spectral estimation. And FFT method results in higher sidelobs than the Capon method. The spectral peaks given by the capon method are narrower than FFT method.

Here we also mention that the choice of M is so critical. For too large M, the matrix R may be singular. On the other hand for N >> M, the FIR filtering approach approximately reduces to FFT. Therefore, significant differences between FFT and the FIR filtering approaches occur only when M is sufficiently large as compared with N [4]. Hence , it is recommended that M should be between N/4 and N/2.

Now we compare FFT with the forward-only Capon and the forward-backward Capon for MIG-25 airplane SAR data. Although the choice of M for Capon is difficult to make, we choose M = N/4 which usually gives the best result [3]. Fig. 2(a) shows the result of the forward-only and fig.2 (b) shows the forward-backward Capon spectral estimation. It is obviously seen that the forward-backward Capon method gives better resolution and better estimation. The main reason of this is that while the forward-backward Capon uses both the forward and backward data vectors (12) to obtain the estimate of the covariance matrix (15), the forward-only Capon uses only the forward data vectors (8) to estimate the covariance matrix. As R is persymmetric, we can expect that the forward-backward covariance matrix is generally a better estimate of forward-only covariance matrix [5].



(a)

(b)

(c)

Fig. 2. SAR images of MIG-25 airplane obtained by using (a) Forward-only 2-D Capon, (b) Forward-backward 2-D capon and (c) 2-D FFT

Fig. 2(c) shows the result of FFT estimator. Both the forward-only in fig. 2(a) and the forward-backward in fig. 2(b) Capon estimators give better result than the FFT estimator. It is easy to see that both of the Capon methods can yield better resolution with much lower sidelobs and narrower spectral peaks than the FFT method. Because the power out of the FFT filter is the same for all frequencies and the shape of the filter frequency response does not depend on the frequency of the sequence. The filter response is only shifted with w_0 to ensure a peak response at $w=w_0$ [2]. On the other hand, the filter shape of the Capon depends on the noise background near the centre frequency $w=w_0$. For a given noise covariance w will form a different filter for each assumed value of w_0. The filter adjust itself to reject noise components with frequencies not near $w=w_0$ and pass signal components at and near $w=w_0$ [2].

IV. CONCLUSION

We have presented an adaptive FIR filtering approach which is the Capon spectral estimation method. We have compared the Capon method with FFT and also we compared the forward-only Capon with the forward-backward Capon method. We have shown by means of experimental examples that the Capon method can yield better resolution with much lower sidelobs and narrower spectral peaks than FFT which is also a special case of the FIR filtering approaches. And also we have shown that the forward-backward Capon gives more accurate spectral estimates and better resolution than the forward-only Capon method.

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