Optimal Coupling Medium For Brain Stroke Imaging with Microwaves

M. N. Akıncı, O. Güren, M. Cayoren, I. Akduman
Department of Electronics and Communication Engineering
Istanbul Technical University
Istanbul, Turkey
Email: akincime@itu.edu.tr

***Abstract*—In this work, we present a numerical approach to determine the electrical parameters (relative dielectric permittivity ϵ_r, conductivity σ) of the optimum coupling medium for brain stroke imaging. In contrast to previous works, which utilizes a planar model for determining the optimum coupling medium, we utilize a simple 2D circular model. Furthermore, instead of calculating the transmission coefficient, the inverse problem is solved by means of the well-known contrast source inversion (CSI) method. Optimality comparisons between coupling mediums are made with respect to mean square errors between exact models and the resultant distributions, which are obtained from CSI. Besides giving an idea about the illumination frequency that must be used, obtained results clearly reveals the set of electrical parameters of coupling medium, which makes the imaging feasible.**

***Keywords*—Microwave imaging, Biomedical imaging, Contrast source inversion method, Coupling medium**

I. Introduction

Microwave imaging continues to attract attention as it was in a past few decades. Although the applications of microwave imaging can range from subsurface imaging [1]–[3] to imaging of comets [4], an indispensable research area is certainly the biological imaging [5]–[8].

Today, one of the popular problems in the medical imaging is the non-invasive brain stroke monitoring. Scientists propose different methods for screening the brain stroke via microwave scattering [7], [9], [10]. Their results show that the electrical parameters of the coupling medium and the frequency of the illumination can have large impacts on the quality of the results. This phenemenon can account for the current research for tissue mimicking materials [11], [12].

In this regard, we present a numerical analysis for the optimal value of the electrical parameters of the coupling medium. In a previous work [7], the authors calculate the transmission coefficients for a planar model of brain. Here, we use a 2D circular model of brain and reconstruct the electrical parameters of whole phantom by using the well-known contrast source inversion (CSI) algorithm [13]. Then, we calculate the root of the mean square difference between reconstructed profile and the exact phantom. Note that, solving the inverse problem when searching for the optimal coupling medium is more suitable; since it reveals the effect of the coupling medium on the imaging results in a more precise manner. Obtained results show that the optimum parameters for the coupling medium ranges between $30 < \epsilon_r < 50$ and 0 S/m $< \sigma < 1$ S/m for 1.25 GHz and $10 < \epsilon_r < 70$ and 0 S/m $< \sigma < 1$ S/m for 0.5 GHz.

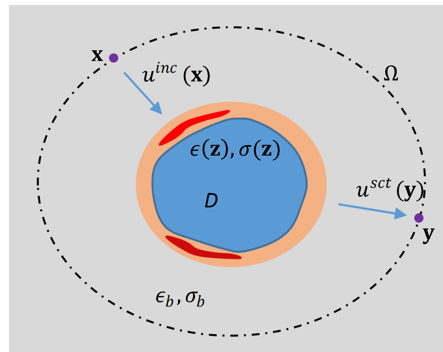

Fig. 1. Configuration of the problem

II. Contrast Source Inversion Method

We confine the analysis to 2D configuration as given in Figure 1. The domain D comprises of a transverse slice of brain. Measurement points $\mathbf{x}$ and source points $\mathbf{y}$ are located on the domain Ω, which surrounds D. It is assumed that the brain is illuminated by a set of incident fields of angular frequency ω, $u_j^{inc}(\mathbf{x})$, $j = 1, \cdots, J$. It is well known that, electric field satisfies the below integral equation [13]:

$$u_j^{tot}(\mathbf{x}) = u_j^{inc}(\mathbf{x}) + k_b^2 \int_D G(\mathbf{x}, \mathbf{z})\chi(\mathbf{z})u_j^{tot}(\mathbf{z})dv(\mathbf{z}), \quad x \in D \tag{1}$$

where

$$G(\mathbf{x}, \mathbf{z}) = \frac{i}{4} H_0^{(1)}(k_b|\mathbf{x} - \mathbf{z}|) \tag{2}$$

denotes Green function of background medium with $H_0^{(1)}$ standing for the zero-order Hankel function of the first kind. The contrast function of the brain, χ is defined as:

$$\chi(\mathbf{z}) = \frac{k^2(\mathbf{z})}{k_b^2} - 1 \tag{3}$$

where $k = \sqrt{\omega^2 \epsilon \mu_0 + i\omega\sigma\mu_0}$ is the wavenumber of the brain and k_b is wavenumber of the background medium. Reconstructing χ from 1 is a non-linear and ill-posed problem. Following the CSI method proposed in [13], contrast source term is defined as $w = \chi u$. In this manner, the scattered field

TABLE I. COLE-COLE PARAMETERS FOR DIFFERENT TISSUES

Tissues	ϵ_∞	Δ_ϵ	τ	α	σ_i
Skin	4	32	7.23e-12	0	2e-4
Fat	2.5	3.0	7.96e-12	0.2	1e-2
Bone	2.5	10	13.26e-12	0.2	2e-2
Gray Matter	4	45	7.96e-12	0.1	2e-2
White Matter	4	32	7.96e-12	0.1	2e-2

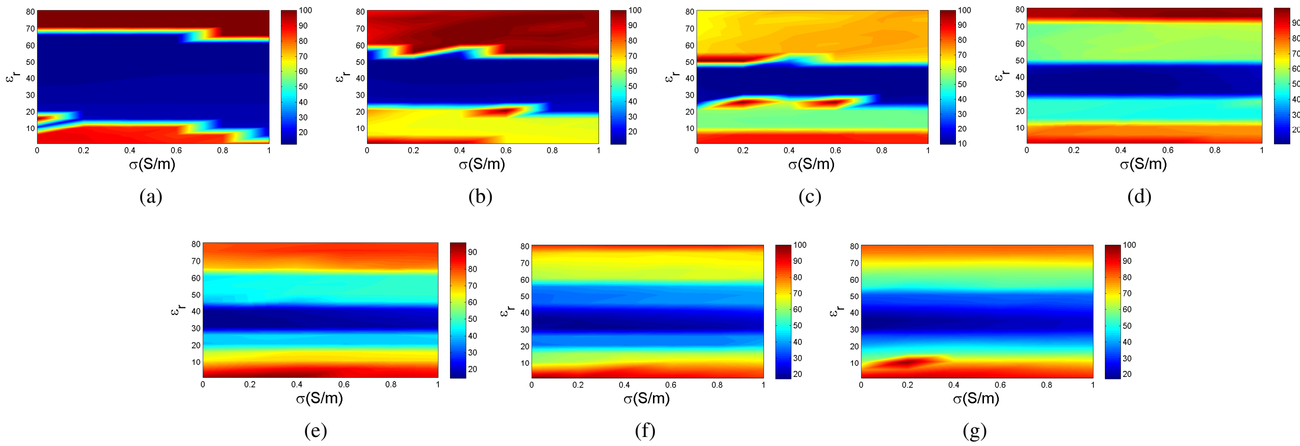

Fig. 2. The root of the mean square errors between estimated electrical parameter distribution $\hat{\chi}$ and exact distribution χ ($100 \times \frac{||\hat{\chi}-\chi||_{L^2(D)}}{||\chi||_{L^2(D)}}$) for the frequencies of: a) 0.50 GHz b) 0.75 GHz c) 1.00 GHz d) 1.25 GHz e) 1.50GHz f) 1.75GHz g) 2.00GHz

u_j^{sca} can be given as:

$$u_j^{sca}(\mathbf{x}) = k_b^2 \int_D G(\mathbf{x}, \mathbf{z}) w_j(\mathbf{z}) dv(\mathbf{z}), \quad x \in \Omega \tag{4}$$

and the object equation becomes:

$$w_j(\mathbf{z}) = \chi u_j^{inc}(\mathbf{z}) + \chi k_b^2 \int_D G(\mathbf{x}, \mathbf{z}) w_j(\mathbf{z}) dv(\mathbf{z}), \quad x \in D \tag{5}$$

The CSI method conceives the reconstruction of χ as a minimization problem where cost functional defined as:

$$F = \frac{\sum_j ||u_j^{sca} - G_\Omega w_j||_\Omega^2}{\sum_j ||u_j^{sca}||_\Omega^2} + \frac{\sum_j ||\chi u_j^{inc} + \chi G_D w_j - w_j||_D^2}{\sum_j ||\chi u_j^{inc}||_D^2} \tag{6}$$

Here $||.||_\Omega$ and $||.||_D$ denote the norms on $L_2(\Omega)$ and $L_2(D)$, respectively. The iterative minimization is done in two steps. At the initial stage of n$^{\text{th}}$ iteration, supposing $w_{j,n-1}$, $\chi_{j,n-1}$ are known, the contrast sources updated as:

$$w_{j,n} = w_{j,n-1} + \beta_{j,n} v_{j,n} \tag{7}$$

where $\beta_{j,n}$ is the parameter adjusting update amount and $v_{j,n}$ stands for the update direction. Here $v_{j,n}$ is chosen as the Polak-Ribiere conjugate gradient direction and $\beta_{j,n}$ is selected to minimize the cost function. In the second step, the contrast function is updated in a similar manner to the contrast source:

$$\chi_n = \chi_{n-1} + \alpha_n d_n \tag{8}$$

where d_n is the Polak-Ribiere conjugate gradient direction and α_n is the tuning parameter. Explicit expressions of the parameters in 7, 8 can be found in [13]. To start the iterative process, the initial value of $w_{j,0}$ is obtained by the backpropagation [13]:

$$w_{j,0} = \frac{||G_\Omega^* u_j^{sca}||_D^2}{||G_\Omega G_\Omega^* u_j^{sca}||_\Omega^2} G_\Omega^* u_j^{sca} \tag{9}$$

where G_Ω^* stands for the conjugate operator of G_Ω. As an a-priori knowledge, at each step of the CSI algorithm, the relative dielectric parameters smaller than 1 is forced to be 1 and negative conductivities are made 0. Finally, we also assume that the shape of the head is known completely.

III. RESULTS AND DISCUSSION

In this section we present the numerical results obtained for different coupling mediums. The brain that is used in the simulations is assumed to consists of five concentric regions, which are skin, fat, bone, cerebrospinal fluid (CSF) and brain (gray + white matter) from outer to the inner parts respectively. We assume that the brain is circular and the radius of the head is 8.0 cm. The thickness of the layers are taken as 4 mm for skin, 4 mm for fat, 7 mm for bone and 3 mm for CSF [7]. The dielectric parameters of these tissues is taken from [14] and they are given in the Table I. For the tissues that Cole-Cole parameters exist the following formula is used to calculate the complex relative dielectric parameter:

$$\epsilon_r(\omega) = \epsilon_\infty + \frac{\Delta\epsilon}{1 + (-i\omega\tau)^{1-\alpha}} + \frac{i\sigma_i}{\omega\epsilon_0} \tag{10}$$

For the CSF the electrical parameters are taken as $\epsilon_r = 69.3$, $\sigma = 0.4$ S/m [9] and the stroke is modelled with a circular region having radius of 1 cm, centered around $(x = 2cm, y = -1.5cm)$ and with electrical parameters of $\epsilon_r = 36$, $\sigma = 0.72$ S/m [9]. For the CSF and the stroke, the complex relative dielectric permittivity is computed as:

$$\epsilon_r(\omega) = \epsilon_r + \frac{i\sigma}{\omega\epsilon_0} \tag{11}$$

Here, it is understood from [7], [9], the optimal coupling medium can have a conductivity between 0 S/m - 1 S/m and

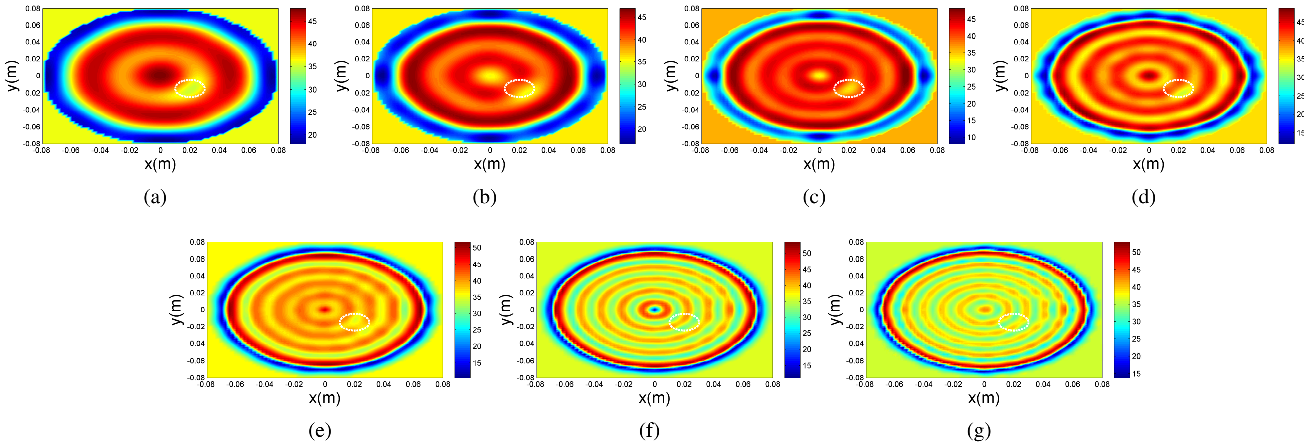

Fig. 3. Retrieved relative dielectric parameter of the brain for the frequency of: a) 0.50 GHz b) 0.75 GHz c) 1.00 GHz d) 1.25 GHz e) 1.50 GHz f) 1.75 GHz g) 2.00 GHz

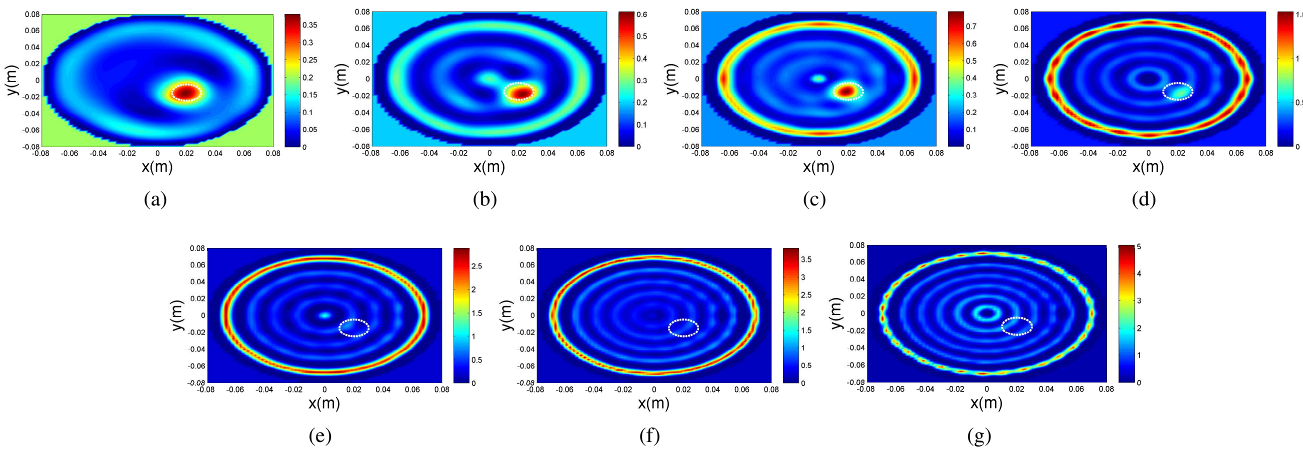

Fig. 4. Retrieved conductivity of the brain for the frequency of: a) 0.50 GHz b) 0.75 GHz c) 1.00 GHz d) 1.25 GHz e) 1.50GHz f) 1.75GHz g) 2.00GHz

a relative dielectric permittivity between 1-81. Besides, it is known that increasing frequency too much decreases the penetration into tissues whereas using too low frequencies causes resolution problems. Hence, the frequency of illumination must be restricted between 0.50-2.00 GHz. Considering these, the conductivity is swept between 0-1 S/m with an incremental step of 0.2 S/m, the relative dielectric parameter is swept between 1-81 with an incremental step of 5 and frequency of operation is swept between 0.5-2.0 GHz with an incremental step of 0.25 GHz. For each set of parameters, the forward problem is solved with a 2D method-of-moments (MoM) solver, whose accuracy is tested with analytical solutions. This MoM solver always discretize the objects so that the largest dimensions of the cells is smaller than the one-tenth of the minimum wavelength in that medium. For illimunating the brain 32 line sources, which are uniformly distributed on a circle having a radius of 15 cm, are utilized. The scattered electric fields are sampled at the points on which the line sources are located. Later, the inverse problem is solved by means of CSI method, for each set of parameters. Here, when the inverse problem is solved, it is assumed that the shape of head is known as a-priori and the domain is discretized into squares whose edge has a length of 2.5 mm. Besides using different meshes in the forward and inverse problems, we also add 50 dB additive white Gaussian noise (AWGN) to scattered field data to be able to prevent any inverse crime issue. The root of mean square errors between estimated electrical parameter distribution $\hat{\chi}$ and exact distribution χ, which can be given as $\frac{||\hat{\chi}-\chi||_{L^2(D)}}{||\chi||_{L^2(D)}}$, is evaluated for each set of parameters. These values are plotted for different frequencies and they are given in Figure 2. As can be seen from these results the optimal range for the electrical parameters of coupling medium ranges from $10 < \epsilon_r < 70$ for $0.5GHz$, whereas it becomes restricted to interval of $30 < \epsilon_r < 40$ as the frequency increases. Another point that must be stressed is the norm of the error between reconstructed and the exact profiles are not affected much from the changes in the conductivity. Lastly, it is noticable that the minimum mean square error levels increase with the increasing frequency. This is in fact an expected phenomenon due to lower penetration depth with the increasing frequency.

Finally, in Figure 3 and Figure 4 the obtained reconstructions for the coupling mediums of $\epsilon_r = 36$, $\sigma = 0.1$ are given for all frequencies. As can be seen from these results the stroke can be easily identified for the frequencies lower than 1.5 GHz, especially from the conductivity images. Therefore, it can be concluded that the optimal operation of frequencies are also between 0.5 - 1.5 GHz.

IV. Conclusion

In this work, we have presented an analysis for optimal coupling medium for the brain stroke imaging. Our results indicate that the detection of the stroke is possible for the frequencies between 0.5-1.5 GHz. Furthermore the optimal range of the electrical parameters of the coupling medium is calculated for several frequencies in the 0.5-1.5 GHz interval. According to the presented results the conductivity of the coupling medium is not too much effective whereas a relative dielectric parameter between 30-50 will yield the optimal imaging results, at least for the contrast source inversion algorithm.

Acknowledgment

This work was supported by Turkish Scientific and Research Council (TUBITAK) under the project number 113E977.

References

[1] B. Gebauer, M. Hanke, A. Kirsch, W. Muniz, and C. Schneider, "A sampling method for detecting buried objects using electromagnetic scattering," *Inverse Problems*, vol. 21, no. 6, p. 2035, 2005.

[2] C. Fischer, A. Herschlein, M. Younis, and W. Wiesbeck, "Detection of antipersonnel mines by using the factorization method on multistatic ground-penetrating radar measurements," *Geoscience and Remote Sensing, IEEE Transactions on*, vol. 45, no. 1, pp. 85–92, 2007.

[3] M. N. Akıncı and M. Çayören, "Microwave subsurface imaging of buried objects under a rough air–soil interface," *Remote Sensing Letters*, vol. 5, no. 8, pp. 703–712, 2014.

[4] C. Eyraud, A. Herique, J.-M. Geffrin, and W. Kofman, "Imaging of a scaled comet model from lab experiments," in *Antenna Measurements & Applications (CAMA), 2014 IEEE Conference on*. IEEE, 2014, pp. 1–2.

[5] A. Abubakar, P. M. Van den Berg, and J. J. Mallorqui, "Imaging of biomedical data using a multiplicative regularized contrast source inversion method," *Microwave Theory and Techniques, IEEE Transactions on*, vol. 50, no. 7, pp. 1761–1771, 2002.

[6] Z. Q. Zhang and Q. H. Liu, "Three-dimensional nonlinear image reconstruction for microwave biomedical imaging," *Biomedical Engineering, IEEE Transactions on*, vol. 51, no. 3, pp. 544–548, 2004.

[7] R. Scapaticci, L. Di Donato, I. Catapano, and L. Crocco, "A feasibility study on microwave imaging for brain stroke monitoring," *Progress In Electromagnetics Research B*, vol. 40, pp. 305–324, 2012.

[8] M. Abbak, M. N. Akıncı, and . Akduman, "Novel compact microwave antenna for biomedical imaging applications," in *Antenna Measurements & Applications (CAMA), 2015 IEEE Conference on*, Submitted.

[9] S. Y. Semenov and D. R. Corfield, "Microwave tomography for brain imaging: Feasibility assessment for stroke detection," *International Journal of Antennas and Propagation*, vol. 2008, 2008.

[10] B. J. Mohammed, A. M. Abbosh, S. Mustafa, and D. Ireland, "Microwave system for head imaging," *Instrumentation and Measurement, IEEE Transactions on*, vol. 63, no. 1, pp. 117–123, 2014.

[11] M. Lazebnik, E. L. Madsen, G. R. Frank, and S. C. Hagness, "Tissue-mimicking phantom materials for narrowband and ultrawideband microwave applications," *Physics in medicine and biology*, vol. 50, no. 18, p. 4245, 2005.

[12] S. Romeo, L. Di Donato, O. M. Bucci, I. Catapano, L. Crocco, M. R. Scarfì, and R. Massa, "Dielectric characterization study of liquid-based materials for mimicking breast tissues," *Microwave and Optical Technology Letters*, vol. 53, no. 6, pp. 1276–1280, 2011.

[13] P. M. van den Berg, A. Van Broekhoven, and A. Abubakar, "Extended contrast source inversion," *Inverse Problems*, vol. 15, no. 5, p. 1325, 1999.

[14] S. Gabriel, R. Lau, and C. Gabriel, "The dielectric properties of biological tissues: Iii. parametric models for the dielectric spectrum of tissues," *Physics in medicine and biology*, vol. 41, no. 11, p. 2271, 1996.