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Input Impedance of Electrically Thin and Thick Rectangular Microstrip Antennas

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Abstract

Using the cavity model, a method is presented for calculating the input impedances of electrically thin and thick rectangular microstrip antennas. The effect of the surface waves on the input impedance has been determined. The theoretical results obtained in this study have been found to be in good agreement with the experimental and theoretical results available in the literature.

Key Terms: *Rectangular microstrip antenna, input impedance*

1. Introduction

In recent years, microstrip antennas have aroused great interest in both theoretical research and engineering applications due to their low profile, light weight, conformal structure, and ease in fabrication and integration with solid-state devices [1-3]. Microstrip antennas have been used in various configurations such as square, rectangular, circular, ring and elliptical . A number of methods [1-3] have been developed to determine the input impedance of rectangular microstrip antennas, as this configuration is one of the popular and convenient ones. The main methods used are the transmission line model, the cavity model and the integral equation method. The transmission line model in its original form is limited to rectangular or square patches where extension to other shapes is also possible. The integral-equation method is perhaps the most general: it can treat arbitrary patch shapes as well as thick substrates. However, it requires considerable computational effort and provides little physical insight. The cavity model [4-7] offers both simplicity and physical insight. It also appears to yield results accurate enough for many engineering purposes.

The cavity model has also been successfully used to calculate the input impedance of microstrip antennas with the substrates ranging in thickness from $0.005\lambda_s$ to $0.02\lambda_s$, where λ_s is the wavelength in the substrate. For thicker substrates, however, the cavity model, as well as other simpler models [8] have been abandoned in favor of the more rigorous moment method computations. The need for a theoretical analysis of the microstrip antennas with electrically thick substrates is motivated by several major factors. Among these is the fact that microstrip antennas are currently being considered for use in millimeter-wave systems. The substrates proposed for such applications often have high relative dielectric constants and, hence, appear electrically thick. The necessity for greater bandwidth is another major reason for studying thick substrate microstrip antennas. Consequently, this problem, and in particular efficiency, resonant frequency, and input impedance aspects of it have received considerable attention in recent years. In [9] the effects of surface waves on the efficiency of electrically thick rectangular microstrip antennas have been studied. In [10] a new fringing field extension expression for the resonant frequency of rectangular microstrip antennas has been proposed. The theoretical resonant frequency results obtained by using this new fringing field extension expression are in very good agreement with the experimental results available in the literature [5,11]. In this paper, the effects of surface waves on the input impedance of rectangular microstrip antennas are studied. An analytical expression for the input impedance of electrically thin and thick rectangular microstrip antennas excited by a coaxial probe is presented using the cavity model and the equivalent resonant circuits. Unlike the approaches in literature the surface wave radiation resistance, which is ignored by the conventional cavity model has been used to compute the input impedance of coax-fed electrically thick rectangular microstrip antennas.

2. Analysis

2.1. Input Impedance

Consider a rectangular patch of width W and length L over a ground plane with a substrate of thickness h and a dielectric constant ϵ_r, as shown in Figure 1. Carver and Coffey [4] showed that the lumped-element model of a single-port single-mode cavity can be used to determine the input impedance, resonant frequency, and efficiency of the rectangular microstrip patches. It has been shown that the equivalent circuit for a single spectrally-isolated resonant mode of the microstrip antenna can be represented by a parallel RLC lossy resonant circuit which is in series with an inductive term representing both the influence of the feed and also the residual net magnetic energy associated with all the higher order modes. The residual effects of evanescent and higher-order modes, together with any error in the calculated effective dielectric constant are accounted for in the edge extension term. It is also possible to consider either the dominant mode or the complete spectrum of modes. The rectangular microstrip patch antenna can be considered in the fundamental mode, modelled by a simple resonant parallel RLC circuit [5], as shown in Figure 2. In order to take the coax-feed probe into account, it is necessary to modify the input impedance by an inductive reactance term [3]:

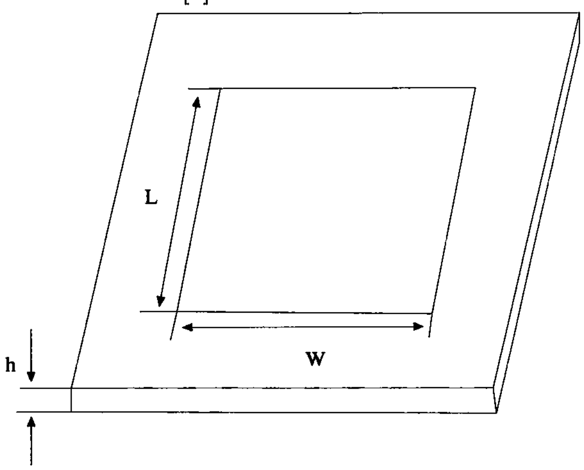

Figure 1. Geometry of rectangular microstrip antenna

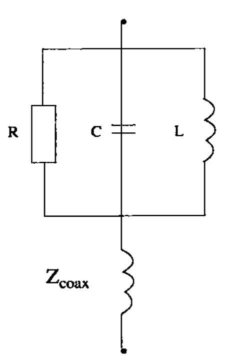

Figure 2. Equivalent resonant parallel RLC circuit

$$Z_{coax} = 60\left[(k_0 h)^2\sqrt{\varepsilon_r} + jk_0\, h\; arcsin\;\frac{2h}{r_0} + \frac{r_0 - (r_0^2 + 4h^2)^{1/2}}{2h}\right] \tag{1}$$

where r_0 is the radius of the inner conductor, $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$: propagation constant in free space, μ_0 and ε_0 are the permeability and permittivity of free space respectively. The following equation can be used to calculate the input impedance of rectangular microstrip antenna.

$$Z(f) = \frac{R}{1 + Q_T^2\left[\frac{f}{f_r} - \frac{f_r}{f}\right]^2} - j\left\{\frac{RQ_T\left[\frac{f}{f_r} - \frac{f_r}{f}\right]}{1 + Q_T^2\left[\frac{f}{f_r} - \frac{f_r}{f}\right]^2}\right\} + Z_{coax} \tag{2}$$

where R is the resonant resistance of the resonant parallel RLC circuit, f_r is the resonant frequency, and Q_T is the total quality factor associated with system losses including radiation, the loss associated with surface wave propagation on a dielectric coated conductor, the loss due to heating in the conducting elements and the ground plane, and the loss due to heating within the dielectric medium. For the resonant frequency of electrically thin and thick rectangular microstrip antennas, we used equations given in Appendix I [10]. The total quality factor Q_T is calculated from the formulas given in Appendix II.

Equation (2) shows explicitly the dependence of the input impedance on the characteristic parameters of a patch antenna, and is valid for electrically thin and thick substrates [12].

The resonant resistance R in equation (2) can be written as

$$R = R_r + R_d + R_c + R_s \tag{3}$$

where the four terms represent the radiation resistance, the equivalent resistance of the dielectric loss, the equivalent resistance of the copper loss and the surface wave radiation resistance. Although R_r, R_d and R_c, which are given in Appendix II, are easily found, the surface wave radiation resistance R_s has to be obtained using the complicated Green function method.

2.2. Surface Wave Radiation Resistance

In general, if the thickness of the substrate on which the antenna is etched is very small compared to the wavelength of interest, the power propagated via the surface wave is negligible so that the effects of the surface waves on input impedance may be ignored. Most of the results obtained using the cavity model are for electrically thin substrates. Thus, in the original cavity model, the surface wave radiation resistance is neglected. However, it was shown in [9] that the power carried by surface waves is an important parameter for the designer of electrically thick rectangular microstrip antennas. This power has to be considered as a loss because it is trapped in the dielectric substrate. Moreover, unwanted radiation results when the surface wave encounters a discontinuity (e.g. the edge of substrate). The surface wave power has also a very important effect on the input impedance of electrically thick microstrip antennas. For this reason, in this study the following surface wave radiation resistance expression is used to account for the influence of surface waves on the input impedance of rectangular microstrip antennas.

$$R_s = \frac{Z_c^2}{2P_{su}} \tag{4}$$

where Z_c given in Appendix II is the characteristic impedance for the patch, and P_{su} is the power radiated in surface waves.

2.3. Surface Wave Power

Surface wave power P_{su} can be obtained by using the Green function method. In this work, the method [13] presented for the determination of power radiated via the space wave and the surface wave from the aperture of an arbitrarily shaped microstrip antenna is used.

The equivalent magnetic current density at the periphery of the microstrip patch antenna can be written as

$$\vec{J}_m = \left\{ \vec{a}_x m_x(x,y) + \vec{a}_y m_y(x,y) \right\} \delta(z) \tag{5}$$

where $\vec{a}_x, \vec{a}_y$ are the unit vectors, and $\delta(z)$ is the Dirac delta function of z. For an arbitrarily shaped microstrip patch antenna, the following expression for the complex power radiated by the magnetic current source is obtained by using the general approach.

$$P_r = \frac{j\omega}{4\pi^2} \int\int_{-\infty}^{+\infty} \frac{L_1(k_x,k_y)}{y_{in}(k_x^2+k_y^2)} dk_x\, dk_y + \frac{1}{j\omega 4\pi^2} \int\int_{-\infty}^{+\infty} \frac{L_2(k_x,k_y)}{y'_{in}(k_x^2+k_y^2)} dk_x\, dk_y \tag{6a}$$

with

$$L_1(k_x,k_y) = k_y^2|M_x|^2 + k_x^2|M_y|^2 - k_x k_y (M_x M_y^* + M_x^* M_y) \tag{6b}$$

$$L_2(k_x,k_y) = k_x^2|M_x|^2 + k_y^2|M_y|^2 + k_x k_y (M_x M_y^* + M_x^* M_y) \tag{6c}$$

$$y_{in} = jy_{01}\tan(k_{1z}d) + y_{01}\frac{y_{02} + jy_{01}\tan(k_{1z}d)}{y_{01} + jy_{02}\tan(k_{1z}d)} \tag{6d}$$

$$y'_{in} = jy'_{01}\tan(k_{1z}d) + y'_{01}\frac{y'_{02} + jy'_{01}\tan(k_{1z}d)}{y'_{01} + jy'_{02}\tan(k_{1z}d)} \tag{6e}$$

where

$$\begin{aligned} d &= h/2, \quad y_{01} = jk_{1z}/\varepsilon, \quad y_{02} = jk_{0z}/\varepsilon_0, \\ k_{1z}^2 &= \omega^2\mu\varepsilon - k_x^2 - k_y^2 \text{ and } k_{0z}^2 = \omega^2\mu_0\varepsilon_0 - k_x^2 - k_y^2 \\ y'_{01} &= -j\mu/k_{1z}, \quad y'_{02} = -j\mu_0/k_{0z} \end{aligned} \tag{6f}$$

In eqns. (6), M_x and M_y are the double Fourier transforms of $m_x(x,y)$ and $m_y(x,y)$, respectively where $*$ denotes the complex conjugate. For integrating (6a), the change of variables $k_x = k_\rho \cos\varphi$ and $k_y = k_\rho \sin\varphi$, is used. The limits for k_ρ and φ will be 0 to ∞ and 0 to 2π, respectively. Moreover, $dk_x dk_y$ will be replaced by $k_\rho dk_\rho d\varphi$.

The following equation is obtained using the change of variables above.

$$P_r = \frac{j\omega}{4\pi^2} \int_0^\infty \int_0^{2\pi} \frac{L_1(k_\rho,\varphi)}{y_{in}k_\rho} dk_\rho d\varphi + \frac{1}{j\omega 4\pi^2} \int_0^\infty \int_0^{2\pi} \frac{L_2(k_\rho,\varphi)}{y'_{in}k_\rho} dk_\rho d\varphi$$

Space wave power can be determined by integrating the equation (7) in the range $0 < k_\rho < k_0$, since in this range of k_ρ, k_z is real. However, for $k_\rho > k_0$, the values of k_z are imaginary, indicating that the field decays rapidly as it progresses away from the substrate. Moreover for $k_\rho > k_0$, the integrals in equation (7) are purely imaginary except at the singular points of the integrals. The singularity of the integrals lying between k_0 and k is associated with the surface wave modes. Power propagated via the surface waves can be determined by evaluating the integral in the neighborhood of singular points. The lowest order surface wave mode arises from the first zero of y_{in}. For small substrate thickness there is only one singularity and this singularity can be approximated as in [14].

$$k_{\rho 0} = k_0 + 0.5 \left(k_0^3 h^2 / k^4 \right) \left(k^2 - k_0^2 \right)^2 / \left(1 - d^2 k^2 + d^2 k_0^2 \right)^2 \tag{8}$$

In the neighborhood of the singular point $k_{\rho 0}$, the first integral in equation (7) can be evaluated using the singularity extraction technique [15]. By using this technique, the surface wave power P_{su} is obtained as

$$P_{su} = \frac{\omega \varepsilon_0 F}{4\pi k_{\rho 0}^2} \int_0^{2\pi} L_1(k_{\rho 0}, \varphi) d\varphi \tag{9a}$$

where

$$F = \frac{1 + \varepsilon_r d \sqrt{k_{\rho 0}^2 - k_0^2}}{(2h/\varepsilon_r)(k_{\rho 0}^2 - k_0^2)^{-1/2}} \tag{9b}$$

For large substrate thickness there exist a number of singular points arising from the zeros of y_{in} and y'_{in}. The singular points associated with the zeros of y_{in} are due to the transverse magnetic (TM) surface wave modes and those from y'_{in} are due to the transverse electric (TE) surface wave modes. The surface wave power carried by all such modes can be determined using the singularity extraction techniques as above.

It can be assumed that for a rectangular patch antenna operating in the TM_{10} mode, the aperture electric field on the radiating aperture is given as $E_z = E_0$. The magnetic current density can also be expressed as $\vec{J}_m = E_0 \delta(z) \vec{a}_y$. Comparing with equation (5), we have $m_x(x,y) = 0$ and $m_y(x,y) = E_0$. Taking the Fourier transforms of $m_x(x,y)$ and $m_y(x,y)$, and using the above described method, the surface wave power is obtained as follows:

$$P_{su} = \frac{E_0^2 \pi \omega \varepsilon_0 W_e^2 \Delta L^2}{\pi^2} F \int_0^{2\pi} \cos^2 \varphi S_n^2 \left(\frac{k_{\rho 0} W_e \sin \varphi}{2} \right) S_n^2 \left(\frac{k_{\rho 0} \Delta L \cos \varphi}{2} \right) cos^2 \left(\frac{k_{\rho 0} L_c \cos \varphi}{2} \right) d\varphi \tag{10}$$

where $S_n(x) = \sin(x)/x, W_e$ which is shown in Figure 3 is the effective width, ΔL is the edge extension and $L_c := L + \Delta L$ is the centre distance between the equivalent slots. P_{su} given by equation (10) is different from the equation given in [13]. In order to take into consideration the influence of fringing field effectively, we used W_e, L_c and ΔL instead of W, L and h, respectively.

For small W and ΔL, after a number of mathematical manipulations [16], the above expression for the surface wave power P_{su} can be approximated as

$$P_{su} \cong \frac{32}{15\pi}\frac{E_0^2 k_0 F}{k_{\rho 0}^4} J_0^2\left(\frac{k_{\rho 0}L_c}{2}\right) J_1^2\left(k_{\rho 0}W_e/2\right) J_1^2\left(k_{\rho 0}\Delta L/2\right) \tag{11}$$

Here $J_0(x)$ and $J_1(x)$ are Bessel functions of the first kind and order zero and one, respectively.

The surface wave radiation resistance R_s is obtained by substituting equation (10) or (11) into equation (4). Thus, the effect of surface wave power on the input impedance can be determined by substituting equation (3) and equation (4) into equation (2).

3. Numerical Results and Discussions

In this section, the numerical results for the input impedance obtained from the above method are compared with previously presented measurements and numerical results.

In rectangular microstrip antenna design, it is important to determine the resonant frequencies of the antenna accurately because microstrip antennas have narrow bandwidths and can only operate effectively in the vicinity of the resonant frequency. Furthermore, the input impedance of these antennas depends firmly on the resonant frequency of the antenna. This dependence is clear from equation (2). In this work, the resonant frequency of electrically thin and thick rectangular microstrip antennas is calculated from the equations proposed in [10], which are given in Appendix I . This is because the resonant frequency results obtained from these equations are in very good agreement with the experimental results [5,11] which, in turn, leads to good accuracy in the calculation of the input impedance. It is noted that the equations (17a) and (17b) are valid for electrically thin and thick rectangular microstrip antennas, respectively.

The effective dimensions L_e and W_e instead of the physical dimensions L and W, respectively, and the effective permittivity constant ε_e instead of the relative dielectric constant ε_r are used to take into consideration effectively the influence of the fringing fields at the edges, and the dielectric inhomogeneity of electrically thin and thick rectangular microstrip antennas.

For the effective relative dielectric constant, we used the equations (15)-(16) since the accuracy of the results given by these equations is claimed to be better than 2.5 % for the range of normalized widths $0.01 \leq W/h \leq 100$ and $\varepsilon_r \leq 50$.

In the literature, for the effective dimensions L_e and W_e the empirical formulas proposed in [3, page 121] are often used to account for the fringing fields at the perimeter of the patch. However, it can be easily shown that the resonant frequencies calculated by using these formulas are not in good agreement with the experimental results [5,11] both for electrically thin and thick rectangular microstrip antennas. Therefore, these formulas are only very crude approximations for the effective dimensions and will lead to poor accuracy in calculating the resonant frequency, and therefore input impedance. In this work, the equations given in Appendix I are used for the effective dimensions because the resonant frequencies calculated from these equations are also in very good agreement with the experimental results [5,11].

The position of the feed point of rectangular patch antennas is shown in Figure 4. The dimensions of the antennas, which are used in Figures 5-10, are given in Table 1.

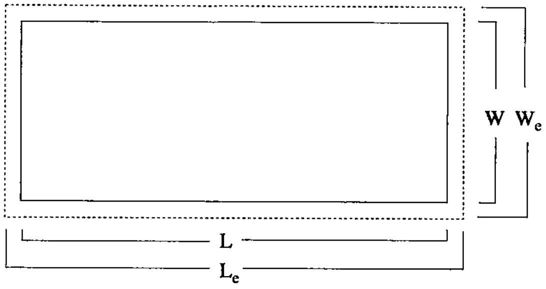

Figure 3. Effective dimensions of rectangular microstrip antenna

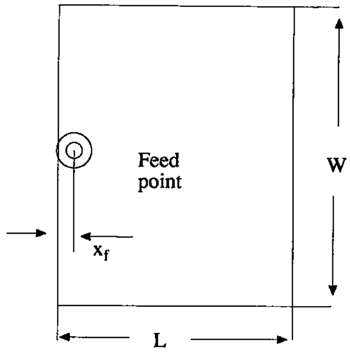

Figure 4. Geometry of feed point

Figure 5 compares the measured input impedance results of Lo et al. [17], the calculated results published by Pozar [18], and calculated results obtained from the method presented. In Figure 6, the calculations using this method are also compared with the calculated and measured results of Carver and Coffey [4]. As shown in Figures 5-6, our results agree somewhat better with the experimental results over the frequency range of interest, than do the previously presented results. In Figures 5-6, the antennas are the electrically thin microstrip antennas. Thus, the power propagated via the surface wave modes in negligible [9] so that the effects of surface waves on the input impedance may be ignored.

To illustrate the effect of surface waves on the input impedance of a coax-probe excited square microstrip antenna on thick substrate, a comparison of the calculated results by this method with the calculated results and measurements of Deshpande and Bailey [19] is given in Figure 7. We observe that our calculated values are in good agreement with the measured values. It is also evident from Figure 7 that our results are better than those predicted by Deshpande and Bailey [19]. In Figures 8-10, the input impedances calculated from the model proposed here are also compared with the previously presented measurements of electrically thick rectangular microstrip antennas of Schaubert et al. [20]. In these figures the input impedances calculated from the model with and without the inclusion of the surface wave are plotted for comparison. As shown in the figures, the prediction was improved considerably when the surface wave was included. It was found that while the dielectric and copper losses become relatively insignificant the surface wave power increases with thickness h and relative dielectric constant ε_r and as much as 30 percent of the power can be lost through surface wave excitation. For this reason, the effect of the surface wave power on the input impedance of electrically thick microstrip patch antennas must not be ignored.

Table 1. The Dimensions of Microstrip Patch Antennas

Case	ε_r	h (cm)	L (cm)	W (cm)	x_f (cm)	$\tan\delta$
1	2.59	0.1588	13.97	20.45	0.635	0.003
2	2.50	0.1524	4.14	6.858	0.0	0.002
3	2.55	0.159	2.01	2.01	0.13	0.002
4	10.2	0.254	1.90	3.00	0.65	0.0024
5	10.2	0.254	0.90	1.50	0.32	0.0024
6	2.22	0.152	2.50	4.00	0.40	0.0009

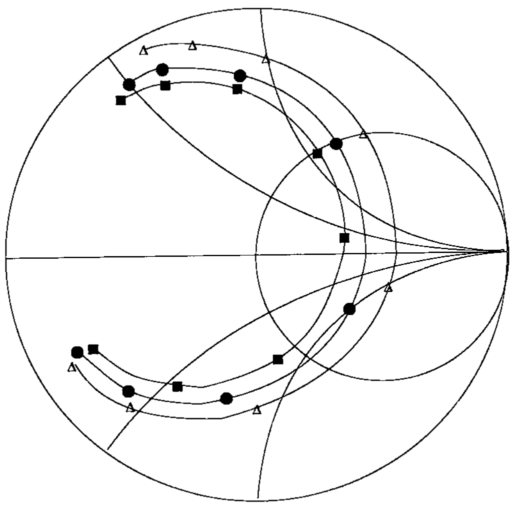

Figure 5. Input impedance of rectangular microstrip antenna for case 1, and frequency = 0.640 – 0.675 GHz, $\Delta f = 0.005$

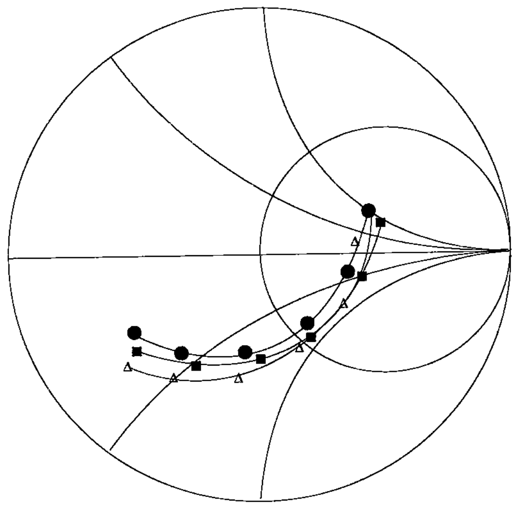

Figure 6. Input impedance of rectangular microstrip antenna for case 2, and frequency $= 2.2 - 2.3$ GHz, $\Delta f = 0.02$

We also observed that as the substrate thickness increases, analysis of input impedance data reveals interesting trends. The expected shift [21] of the impedance locus to the inductive side of the chart, as h/λ_s increases, is clearly observed in Figures 8-9. In Figure 9, the inductive shift was large enough so that the impedance loci did not cross the real axis, and so a resonant frequency or a resistance could not be defined. These results suggest that the inductive shift is caused primarily by the use of a thick substrate.

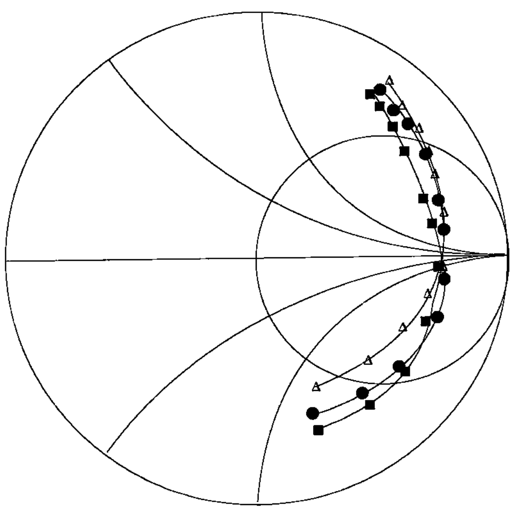

Figure 7. Input impedance of rectangular microstrip antenna for case 3, and frequency = 4.15-4.65 GHz, $\Delta f =$ 0.05

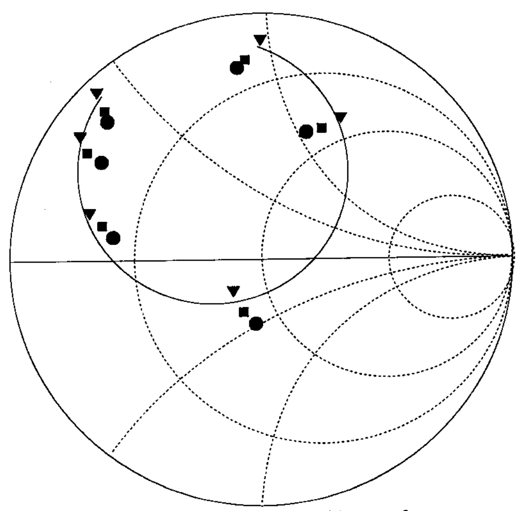

Figure 8. Contribution of surface waves to the input impedance of rectangular microstrip antenna for case 4, and frequency = 2.18-2.38 GHz, $\Delta f = 0.04$

For thick substrates, in reference [9] we also showed that the radiation efficiency of the antenna can fall below an acceptable level. This is due to the fact that as the thickness increases, the surface wave power increases and power via the space wave gets reduced. Thus, it appears that for broadside radiation, very thick microstrip antennas, although having broad-band impedance characteristics, may not be desirable from the radiation efficiency point of view.

As shown in Figures 5-10, the theoretical input impedance results calculated by using the proposed method in this work are in good agreement with the experimental results which justifies our method.

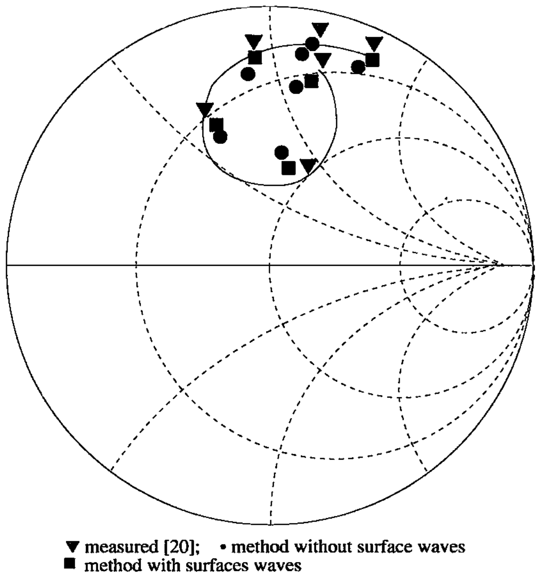

Figure 9. Contribution of surface waves to the input impedance of rectangular microstrip antenna for case 5, and frequency =4.1-4.85 GHz, Δf=0.15

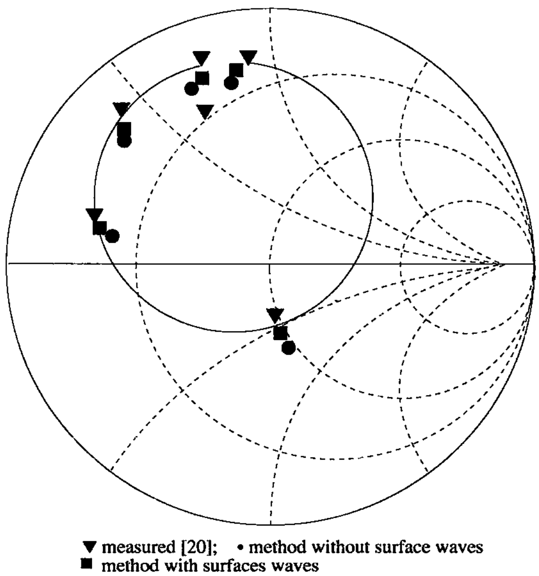

Figure 10. Contribution of surface waves to the input impedance of rectangular microstrip antenna for case 6, and frequency = 3.7-4.5 GHz, Δf = 0.2

The discrepancy between the model presented here and experimental results in Figures 5-10 can be explained by the tolerances on the structural parameters. A large degree of uncertainty is introduced when fabricating microstrip antennas. Even though the same mask is used to produce the patches, the resulting patches all have slightly different dimensions and geometries. The substrate thicknesses and relative permittivities are specified within 5% or even 10% by some manufacturers. Fabrication tolerances can greatly effect the resonant frequency and consequent input impedance of rectangular microstrip antennas. Since the microstrip antenna is a narrow bandwidth device slightest deviations in dimensions, relative dielectric permittivity of the substrate material or nonuniformity in the substrate thickness, can lead to discrepancies between theory and practice. For example, for the antenna given in Figure 8, the almost perfect agreement with experimental results was obtained using ε_r = 10.4 instead of 10.2, and if the losses were somewhat less. Therefore, the fabrication tolerances of the patch and substrate dimensions play an important role in the final accuracy of the predicted input impedances.

4. Conclusion

A method for calculating the input impedance of rectangular microstrip antennas is presented which properly accounts for the effects of dielectric constant, dielectric losses, conductor losses, substrate thickness, surface waves, radiation from the walls, and the dimensions of the patch. The agreement between the measured and computed results supports the validity of this method. Since this method only takes a few milliseconds to produce the input impedance of electrically thin and thick rectangular microstrip antennas, even on IBM personal computer, it is useful for computer aided design of rectangular microstrip antenna arrays.

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Appendix I:

The resonant frequency of rectangular microstrip antennas can be calculated from the following formula proposed in [10]:

$$f_{mn} = \frac{c}{2(\varepsilon_e)^{1/2}} \sqrt{(m/L_e)^2 + (n/W_e)^2} \tag{12}$$

where ε_e is the effective relative dielectric constant for the patch, c is the velocity of electromagnetic waves in free space, m and n take integer values, and L_e and W_e are the effective dimensions as shown in Figure 3. To calculate the resonant frequency of rectangular patch antenna driven at its fundamental TM_{10} mode, eqn. (12) is written as

$$f_{10} = \frac{c}{2(\varepsilon_e)^{1/2} L_e} \tag{13}$$

The effective length L_e can be defined as follows:

$$L_e = L + 2\Delta L \tag{14}$$

The role of the nonuniform medium and the fringing fields at each end of the patch are accounted for by the effective relative dielectric constant ε_e and the edge extension ΔL which is the effective length to which the fields fringe at each end of the patch. The following effective dielectric constant formula proposed by Hammerstad and Jensen [22] is used in eqn. (13)

$$\varepsilon_e = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left[1 + \frac{10h}{W}\right]^{-ab} \tag{15}$$

where

$$a = 1 + \frac{1}{49} \ln\left\{\frac{(W/h)^4 + W^2/(52h)^2}{(W/h)^4 + 0.432}\right\} + \frac{1}{18.7} \ln\left\{1 + \left[\frac{W}{18.1h}\right]^3\right\} \tag{16a}$$

$$b = 0.564 \left[\frac{\varepsilon_r - 0.9}{\varepsilon_r + 3}\right]^{0.053} \tag{16b}$$

The following closed-from expressions proposed in [10] are used for the edge extension ΔL of rectangular microstrip antennas.

$$\frac{\Delta L}{h} = \frac{21.4075 + k_0 h(184.6614 - 1.1475\varepsilon_r + 8.5k_0 h) - 1.35\varepsilon_r}{18(1 + 10.85k_0 h + 8.5k_0^2 h^2)}, \text{ for } h/\lambda_s \leq 0.11 \tag{17a}$$

$$\frac{\Delta L}{h} = \frac{6.8955 + k_0 h(61.062 - 0.3315\varepsilon_r + 8.5k_0 h) - 0.39\varepsilon_r}{5.2(1 + 10.85k_0 h + 8.5k_0^2 h^2)}, \text{ for } h/\lambda_s > 0.11 \tag{17b}$$

where $k_0 = 2\pi/\lambda_0$ and $\lambda_0 = 2.08L\sqrt{\varepsilon_r}$. If ε_r is used for the calculation of free-space wavelength, this wavelength can be calculated from $\lambda_0 = 2.08L\sqrt{\varepsilon_r}$. This equation is also used by Poraz [23] for the computer-aided design of rectangular microstrip antennas. The resonant frequency is then obtained by substituting eqns. (14)-(17) into eqn. (13).

The effective width can be written as follows:

$$W_e = W + 2\Delta W \tag{18}$$

ΔW can be obtained similarly as ΔL by replacing L by W in all of the above formulas given in this section.

Appendix II:

The total quality factor can be written as

$$Q_T = 2\pi f_r R C \tag{19}$$

The capacitance C of the TM_{10} mode is given below [5]

$$C = \frac{\varepsilon_e \varepsilon_0 W_e L_e}{2h} \cos^{-2} \frac{\pi x_f}{L_e} \tag{20}$$

where x_f is feed point. In equation (20) we used the effective dimensions L_e and W_e instead of the physical dimensions L and W, respectively, to take into consideration the energy stored in the fringing fields.

R_c, R_d, and R_r are given by [5].

$$R_c = 0.00027\sqrt{f_r}\frac{L}{W}Q_r^2 \quad (f_r \text{ in GHz}) \tag{21a}$$

$$R_d = \frac{30\tan\delta}{\varepsilon_r}\frac{h\lambda_0}{LW}Q_r^2 \tag{21b}$$

$$R_r = \frac{Q_r}{2\pi f_r C} \tag{21c}$$

where $\tan\delta$ is the substrate loss tangent, and Q_r is the radiation quality factor [2] given in equations (22), in which we replace the relative dielectric constant ε_r by the effective dielectric constant ε_e.

$$Q_r = \frac{3}{16}\frac{\varepsilon_e}{\eta Y_c}\frac{\lambda_0^2}{h^2}, \qquad W_{equ} < 0.35\lambda_0 \tag{22a}$$

$$Q_r = \frac{\pi Y_c}{\frac{4\pi Y_c}{\sqrt{\varepsilon_e}}\frac{h}{\lambda_0} - \frac{1}{15\pi^2}}, \qquad 0.35\lambda_0 \le W_{equ} \le 2\lambda_0 \tag{22b}$$

$$Q_r = \frac{\sqrt{\varepsilon_e}}{4}\frac{\lambda_0}{h}, \qquad 2\lambda_0 < W_{equ} \tag{22c}$$

where $\eta = 120\pi$ Ohm, $Y_c = 1/Z_c$, and W_{equ}, which is the equivalent dimension obtained from the planar model [24], given by

$$W_{equ} = \frac{120\pi h}{Z_c\sqrt{\varepsilon_e}} \tag{23}$$

For calculating Z_c for a given value of W/h, the following expression is used [24].

$$Z_c = \frac{\eta}{\sqrt{\varepsilon_e}}\left[\frac{W}{h} + 1.393 + 0.667\ln\left(\frac{W}{h} + 1.444\right)\right]^{-1} \quad \text{for } (W/h \ge 1) \tag{24}$$

Elektriksel İnce ve Kalın Dikdörtgen Mikroşerit Antenlerin Giriş Empedansı

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Özet

Elektriksel ince ve kalın dikdörtgen mikroşerit antenlerin giriş empedansını hesaplamak için boşluk rezonatör modeline dayalı bir yöntem kullanılmıştır. Yüzey dalgalarının giriş empedansı üzerindeki etkileri belirlenmiştir. Bu calışmada elde edilen kuramsal sonuçların, literatürdeki kuramsal ve deneysel sonuçlar ile iyi bir uyum sağladığı gözlenmiştir.

Anahtar Sözcükler: *Dikdörtgen mikroşerit anten, giriş empedansı*