Hybrid Models for SQUID Behavior

Ileana-Diana Nicolae[1], Petre-Marian Nicolae[2]

[1] University of Craiova, Faculty of Automation, Computer Science and Electronics Craiova, Romania
nicolae_ileana@software.ucv.ro
[2]University of Craiova, Faculty of Electrotechnics, Craiova, Romania
pnicolae@elth.ucv.ro

Abstract

The paper deals with the computer aided analysis of weakly coupled Josephson junctions submitted or not to applied magnetic fields. A comparative study was made between the analytical solutions provided by specialty literature for currents phase variations and those obtained after original simulation. As remarkable differences were revealed, some original formulas were deduced, validated through programs that operate symbolical (written in MATHEMATICA) and error computations were done. The original formulas provide better approximation of solutions, the approximation errors being significantly reduced.

1. Introduction

The most interesting applications of superconductor electronic devices are: SQUID magnetometers, test instruments using SQUID (e.g. susceptometers), standard Volt, mixers for millimeter and submillimeter wavelengths, detectors of various type of waves (e.g. X rays) [4],[9].

The equations describing motion equations in Josephson junctions cannot be solved by ordinary mathematic methods. They are implicit equations, nonlinear and including derivatives. Yet some efforts were made to provide analytical solutions, based on adiabatic solutions. These solutions introduce errors.

2. Fundamentals on Weak Coupled Josephson junctions

The method presented in this paper was conceived for the investigation of three SQUID cells, depicted by fig. 1, but the basic principles can be applied in a wider range [1].

The common elements in all three cases are: the supplying current $2I_0$, the loop effective inductance L and the parallel biasing schematic.

The following system is obtained for identical junctions, within the frame of a RSJ model, developing the motion equations from Eq.1:

$$\begin{cases} \dot{\phi}_1 + \sin\phi_1 = i_o - l^{-1}(\phi_1 - \phi_2 + \varphi) & \text{(a)} \\ \dot{\phi}_2 + \sin\phi_2 = i_o + l^{-1}(\phi_1 - \phi_2 + \varphi) & \text{(b)} \end{cases} \qquad (1)$$

ϕ_1 and ϕ_2 denote the Josephson phases, $i_0 = \dfrac{I_0}{I_c}$ represents the normalized biasing current (assumed as greater than 1), l represents the normalized loop inductance, $l = \dfrac{2\pi I_C L}{\phi_0}$ and $\varphi = \dfrac{2\pi\phi}{\phi_0}$ represents the external normalized flow.

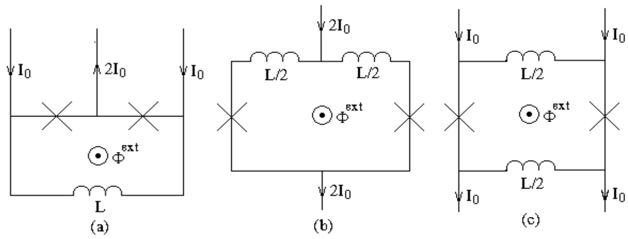

Fig. 1. SQUID cells

The derivation was made with respect to the scaled time:

$$s = 2e/\hbar \cdot I_c R_N \, t \qquad (2)$$

For a weak coupling (l>>1), the coupling can be neglected up to a zero order with respect to l^{-1} and both junctions oscillate with the Josephson phase of an overcritical free biased contact. In [2] the following analytical formula was proposed :

$$\phi_{1,2} = \arctan\left[\frac{\zeta_0}{i_0+1}\tan\left(\frac{\zeta_0 s - \delta_{1,2}}{2}\right)\right] + \frac{\pi}{2} \qquad (3)$$

where δ_1 and δ_2 are constant phases and $\zeta_0 = \sqrt{i_0^{\,2} - 1}.$

3. Numerical Solution in the Absence of an Applied Magnetic Flow

An original MATLAB program was conceived, in order to solve the differential equations system (1). The corresponding block diagram is depicted by fig. 2.

If the junction applied magnetic flow is assumed to be 0, the initial conditions used for integration are identical and the value for the block named "flux" is zero.

A 5-th order Runge-Kutta method was used, with a minimum step of 10^{-4} and a tolerance of 10^{-5}. The numerical values of parameters are $i_0 = 1.5$ (widely used in practice) and $l^{-1} = 0.001$ (weak coupling). The curve representing the numerical solution of ϕ_1 required a preliminary Spline interpolation, because the values used for its construction were provided by the bloc Phi1_, at different time steps, as yielded by the internal calculation process.

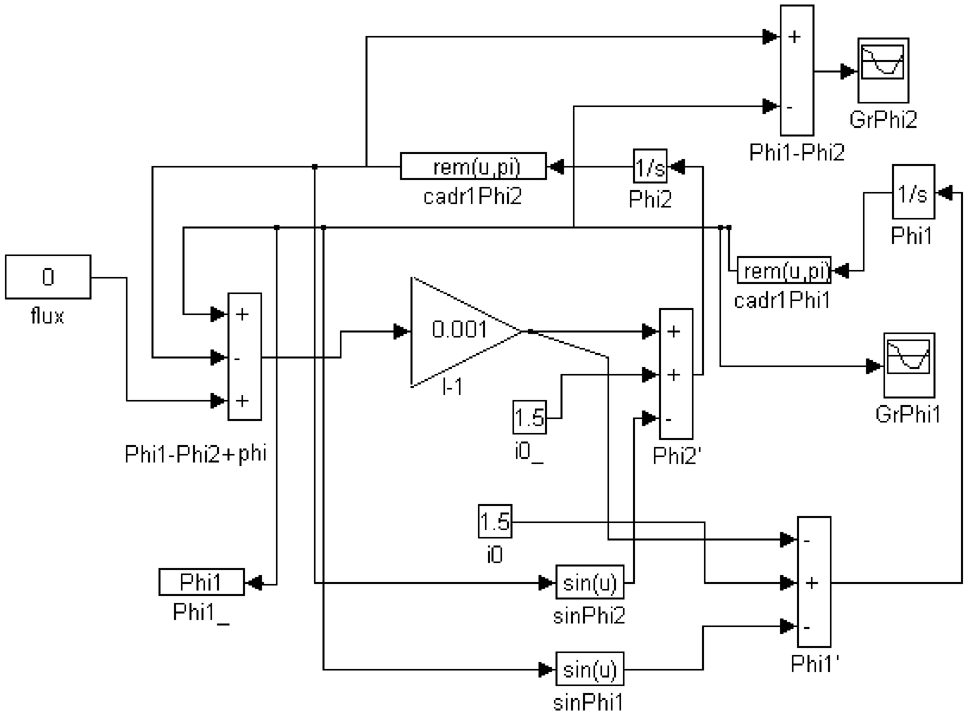

Fig. 2. MATLAB block diagram used to simulate a SQUID with low coupling

For comparison, fig. 3 depicts two curves: the curve represented by full line relies on numerical data and the other one represents the analytical solution , as described by (3).

The periods of the represented curves are obviously different. To determine the variation of the numerical curve period, the program was employed for various values of i_0 (in the range 1.3 - 1.6) and the results were interpolated in the least squares sense.

The following original formula was deduced for the period:

$$pernum(i_0) = -18.45 \cdot i_0^{3} + 94.12 \cdot i_0^{2} - 163.74 \cdot i_0 + 100.2 \quad (4)$$

Based on the above expression, we can now use a new formula, as follows:

$$\phi_1(s) = \arctan\left(\frac{\xi_0}{i_0+1} * \tan\left(\frac{2*\pi}{pernum} * \frac{s}{2}\right)\right) + \frac{\pi}{2} \quad (5)$$

Fig. 4 depicts the difference between the analytical solution (3) and the curve (5).

To reduce this difference, one must introduce a correction factor, of sine shape. Its amplitude is deduced after an interpolation in the least squares sense of the data generated for various values of i_0 (in the range 1.3 - 1.6), as follows:

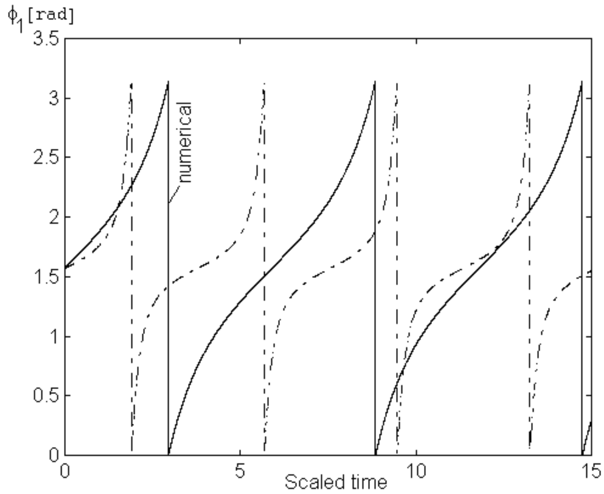

Fig. 3. Numerical and analytical solutions

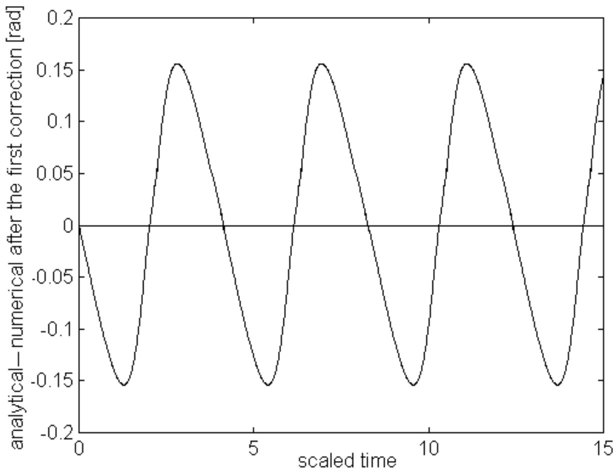

Fig. 4. Difference after the first correction

$$amp(i_0) = 4.16 \cdot i_0^4 - 26.67 \cdot i_0^3 + 63.46 \cdot i_0^2 - 66.68 \cdot i_0 + 26.31 \quad (6)$$

The numerical formula after the second correction is:

$$\phi_1(s) = \arctan\left(\frac{\xi_0}{i_0+1} * \tan\left(\frac{2*\pi}{pernum} * \frac{s}{2}\right)\right) + \frac{\pi}{2} + $$
$$+ amp * \sin\left(\frac{4*\pi}{pernum} * \frac{s}{2}\right) \quad (7)$$

where pernum and amp are calculated with the interpolation polynoms presented above.

In order to validate the results and to compare the errors introduced by the analytical formula and respectively by the original numerical formula, a MATHEMATICA program was conceived.

Figure 5 depicts the deviation from 0 of the equation (1.a) using both formulas, for the most usual case (i0=1.5).

An analysis of the curves from fig. 5 emphasizes that the deviation introduced by the original formula presented in this paper presents a maximum absolute value of 0.12 and its values alternate around 0, having a mean value of almost 0, whereas the deviation of the analytical formula has a double absolute value (0.25), only negative values and consequently a negative mean value.

One can conclude that the numerical solution provides a better solution.

4. Numerical Solution in the Presence of an Applied Magnetic Flow

When the magnetic flow applied on the cell is nonzero, it influences the variation curves periods. Moreover, in this case the relative inductivity of the cell has an influence that cannot be neglected, the right term becomes non trivial.

In order to consider the influence of the above factors, the MATLAB program (fig. 2) was used to generate different sets of data, corresponding to different values of the relative inductivities and of applied magnetic flow , for a current i_0=1.5.

Table 1. Periods calculated for different inductances and applied magnetic fields

l^{-1}	Calculated period			
	φ=0	φ=1	φ=2	φ=3
0.01	4.12	4.17	4.23	4.31
0.1	4.12	4.8	5.93	7.6
0.15	4.12	5.22	7.75	9.2

Based on this data, the second order interpolation polynomials (in the least square sense) were deduced for each curve, considering l^{-1} as constant. Three interpolation polynomials were obtained. Then each set of polynomial coefficients was interpolated with respect to l^{-1}. As a consequence, the supplementary term calculation introduced by the presence of a magnetic flow has the formula:

$$per\_supl(l,\varphi) = coef_1(l) \cdot \varphi^2 + coef_2(l) \cdot \varphi + coef_3(l) \quad (8)$$

where:

$$coef_1(l) = 47.7381 \cdot 1/l^2 - 2.5845 \cdot 1/l + 0.0286 \quad (9)$$

$$coef_2(l) = -33.89 \cdot 1/l^2 + 7.8842 \cdot 1/l - 0.035 \quad (10)$$

$$coef_3(l) = -0.9603 \cdot 1/l^2 + 0.1501 \cdot 1/l - 0.0009 \quad (11)$$

Consequently the following formula is proposed:

$$\phi_1(s) = \arctan\left(\frac{\xi_0}{i_0+1} * \tan\left(\frac{\pi*(s-\varphi)}{pernum + per\_supl}\right)\right) + \frac{\pi}{2} \quad (12)$$

where the initial phases are constant and their difference equals the applied magnetic field (δ_1 - $\delta_2 \approx \varphi$).

Using a MATHEMATICA program, a comparative study of errors was performed (fig. 6) . This specialized software is used because it provides symbolic calculations.

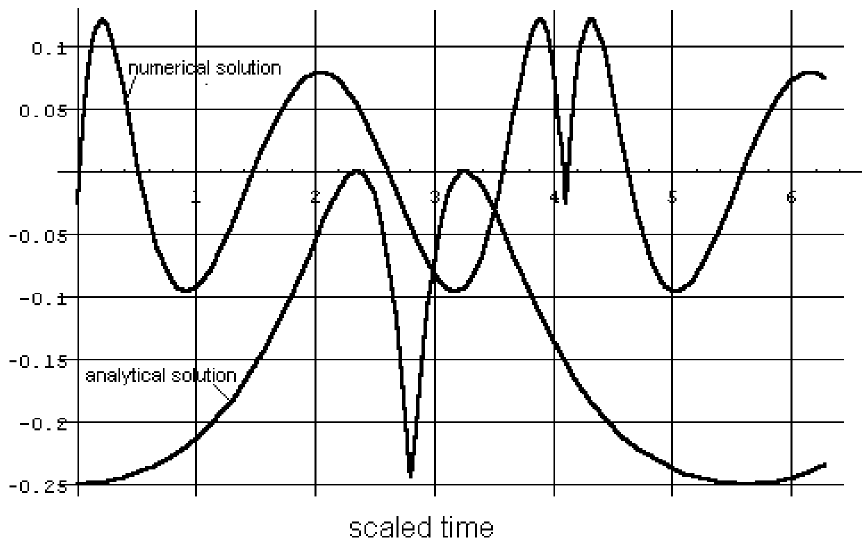

Fig. 5.Comparative study of errors

The curves represent the deviation from 0 of (1.a) for l^{-1}=0.1 and φ=2.5. One can notice an obvious decrease of the absolute error, from 2 (analytical curve) to 0.2 (numerical curve).

5. Conclusions

The system of motion equations describing SQUID cells behavior in the weak coupling case consists of implicit equations, nonlinear and including derivatives. Present mathematic methods do not provide analytical solutions for this system.

Using the adiabatic method, an analytical solution was deduced and proposed in the specialty literature. A comparison with a numerical solution reveals significant differences.

After repeated simulations and interpolations performed on different sets of input parameters, original hybrid solutions were deduced.

The errors calculation proves the superiority of the original formulas.

ACKNOWLEDGMENT

The authors thank to ANCS who supported this research through the 4th Program – "Partnership in priorities domains" under the Contract 71-145/2007 and "IDEI" Program under the Contract 695/2008.

6. References

[1] Basler M., Krech W., *et.a.* Theory of Phase Locking in Simple Two-Dimensional Josephson Junction Arrays with Small Inductances, *IEEE Trans. on Appl. Sup.*, vol. 7, no. 2, p. 3130, 1997

[2] Basler, M., Krech W., *et.a.*. - Theory of Phase Locking in Small Josephson Junctions Cells, Ph. Rev. B, p.7504, vol. 5, no. 2, 1995

[3] J. Clarke, A. I. Braginski, *The SQUID Handbook. Fundamentals and Technology of SQUIDs* ed. Wiley-VCH, 2006

[4] Duzzer, T.,Turner, C.- Principles of Superconductive Devices and Circuits, Elsevier North Holland, Inc. New York, 1981

[5] Jain, A. K., Likharev, K.K., Saubageua, J.E.- Mutual Phase-Locking in Josephson Junction Arrays, Nort-Holland Physics Publishing Amsterdam, 1980

[6] Kornev, V.K. a.o.- Basic Cells for Phase-Locked 1 and 2-D Josephson Junction Arrays, IEEE Trans. on Appl. Sup., vol. 5, nr. 4, p. 3527, 1995

[7] J. B. Majer,, J. R. Butcher, and J. E. Mooij, Simple phase bias for superconducting circuits, *Applied Physics Letters* , Vol. 80, No. 19, pp.3638-3641, 2002

[8] Pinker D. S. , Sahoo L. K., *et.a.*, Investigation of YBCO SQUID-s with gold damping resistors, *IEEE Trans. on Appl. Sup* vol. 15 (1), no. 2 , pp. 789-792 , 2005.

[9] Ruggiero,S.,Rudman,D. Superconducting Devices, Acad. Press, Inc., 1990

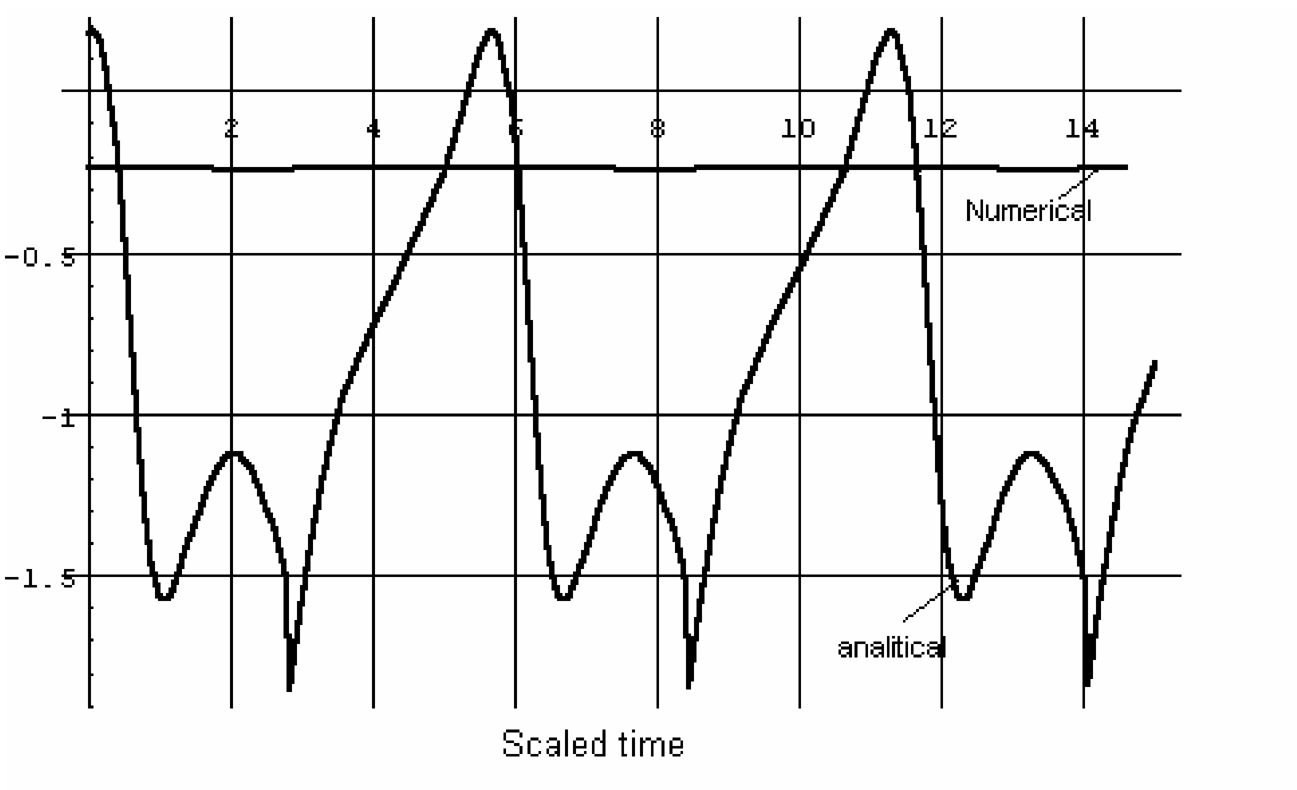

Fig. 6. Errors study for a nonzero applied magnetic field