APPROXIMATION OF CONTROL FUNCTION BY USING ADEQUATE FUZZY MODEL

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Introduction

Fuzzy control at the executive level can be interpreted as an approximation technique for a control function based on typical, imprecisely specified input-output tuples that are represented by fuzzy sets [1-4]. The basic motivation of using fuzzy rule-based system for control purposes is to deduce simple and fast approximations of the unknown or too complicated models. The use of linguistic variables and terms for values is an attempt to mimic human ways of solving control problems by rule-type experience. Behind these intuitive methods there are however clear mathematical formulas, and by analyzing them, some further conclusions for the applicability of fuzzy control can be obtained.

In this study using adequate fuzzy model for approximation of control function is proposed. The adequate fuzzy models have been investigated in [5]. There was found conditions which support adequacy for fuzzy model represented by if-then rules. Founded conditions demand non-intersection of membership functions of each input parameter of fuzzy system. For calculation crisp output crisp input parameter fuzzified by symmetrical triangle membership function with kernel which equal to the crisp input parameter [6,7]. Then by using inference mechanism we calculate fuzzy output and defuzzify them. Proposed method is tested and simulated for subway train problem [8].

Single input - single output fuzzy system

Let U is finite universe of discourse. We define fuzzy sets A as $\mu_A : U \to [0,1]$. Also we define support of fuzzy set $supp(A) = \{u \in U \,/\, \mu_A(u) > 0\}$, and kernel of fuzzy set $kern(A) = \{u \in U \,/\, \mu_A(u) = 1\}$. Fuzzy set is normal, if exist point $u_0 \in U$, where $\mu(u_0) = 1$. If kernel of fuzzy set contains only single point, we say that fuzzy set is unimodal. In this study we shall consider only unimodal normal fuzzy sets.

Let we have universe of discourse $U = \{u_1, u_2, \dots, u_n\}$ for input parameter and $V = \{v_1, v_2, \dots, v_m\}$ for output parameter. We have fuzzy if-then rule base in following form:

$$if\ x = A^l\ then\ y = B^l \quad (l = \overline{1,s}), \qquad (1)$$

where

$$A^l = \sum_{j=1}^{n} \mu_{A^l}(u_j) / u_j$$

and

$$B^l = \sum_{i=1}^{m} \mu_{B^l}(v_i) / v_i$$

are unimodal normal fuzzy sets and s is number of rules. For arbitrary fuzzy input $A = \sum_j \mu_A(u_j) / u_j$ we calculate defuzzified output by formula

$$y = \frac{\sum_{l=1}^{s} \alpha_l b_l}{\sum_{i=1}^{s} \alpha_l},$$

where

$$\alpha_l = \max_j \min\{\mu_A(u_j), \mu_{A^l}(u_j)\} \qquad (2)$$

and b_l is point where $\mu_{B^l}(b_l) = 1$.

In [4] is proved that for adequate fuzzy model we have crisp output $y = b_l$ if on input we have fuzzy sets A^l given in the l-th rule of rule base (1) and necessary condition is non-intersection of membership functions of input parameter.

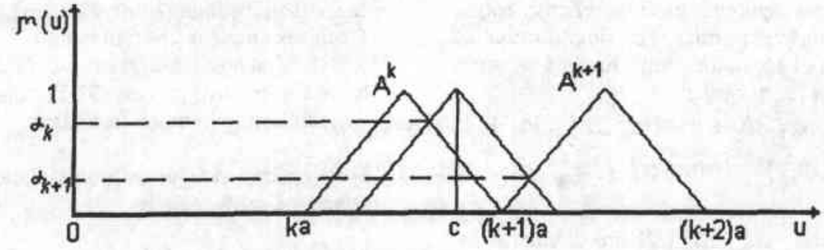


Fig. 1. Explanation to approximation with adequate fuzzy model

Let us study approximation process for adequate fuzzy model . Support of every membership function is equal to a. We have following formula for membership function of k-th linguistic term (fig. 1):

$$\mu_{A_k}(x) = \begin{cases} 0, & if\ x < ka \\ \dfrac{x-ka}{0.5a}, & if\ x \geq ka\ and \\ & x \leq (k+0.5)a \\ 1-\dfrac{x-(k+0.5)a}{0.5a}, & if\ x \geq (k+0.5a) \\ & and\ x \leq (k+1)a \\ 0, & if\ x > (k+1)a \end{cases}$$

and so for every measuring crisp value c after fuzzification according formula (2) we have:

$$\alpha_k = \frac{(k+1.5a)-c}{a} \quad (3)$$

$$\alpha_{k+1} = \frac{c-(k+0.5)a}{a} \quad (4)$$

The other α-s is equal to zero, so we calculate crisp output by following simple formula:

$$y = \alpha_k b_k + \alpha_{k+1} b_{k+1},$$

where b_k and b_{k+1} are kernels for output fuzzy sets in k-th and $(k+1)$-th rules.

Multiple input - single output fuzzy system

Let us consider fuzzy rule-based system with n inputs and single output. Every input x_i is defined on universe of discourse $U_i = \{u_1, u_2, \dots, u_{n_i}\}$ and output y is defined on universe of discourse $V = \{v_1, v_2, \dots, v_m\}$. The parameter x_i takes its values from term-set $A(x_i)$ and the parameter y takes its values from term-set $B(y)$. Finite number of rules is given below:

$$\text{if } (x_1 = A_l^1 \ \&\ x_2 = A_l^2 \ \&\dots\&\ x_n = A_l^n) \text{ then } y = B_l \quad (l = \overline{1,s}). \quad (5)$$

Again we suppose, that fuzzy model is adequate, i.e.

$\forall i$ for any two A_l^i, $A_k^i \in A(x_i)$ if $k \neq l$, then $supp(A_l^i) \cap supp(A_k^i) = \varnothing$. For arbitrary fuzzy input $A_0 = (A_0^1, A_0^2, \dots, A_0^n)$ we calculate defuzzified output by formula

$$y = \frac{\sum_{l=1}^{s} \alpha_l b_l}{\sum_{l=1}^{s} \alpha_l}$$

where

$$\alpha_l = \min_j \max_{k_j} \min\{\mu_{A_l^j}(u_{k_j}^j), \mu_{A_0^j}(u_{k_j}^j)\} \quad (6)$$

Where $\mu_{A_0^j}(u_{k_j}^j)$ is membership function of term A_0^j and $\mu_{A_l^j}(u_{k_j}^j)$ is membership function of terms A_l^j used in rules, b_l is point where $\mu_{B^l}(b_l) = 1$.

If we have full combination of rules, then for n inputs fuzzy system we shall use only n^2 rules for calculation of crisp output. For fuzzy system with two inputs for calculation crisp output we shall use only four rules. For simplification of representation of approximation formula we write it for two input fuzzy system.

$$y = \min(\alpha^1_{k_1}, \alpha^2_{k_2})b_1 + \min(\alpha^1_{k_1}, \alpha^2_{k_2+1})b_2 +$$
$$+ \min(\alpha^1_{k_1+1}, \alpha^2_{k_2})b_3 + \min(\alpha^1_{k_1+1}, \alpha^2_{k_2+1})b_4$$

where $\alpha^i_{k_i}$ and $\alpha^i_{k_i+1}$ $(i=1,2)$ are calculated by formulas (3) and (4), b_l $(l=\overline{1,4})$ are kernels of appropriate outputs in fuzzy rule base (5).

By analogy we can write approximation formula for fuzzy system with n output.

Simulation

Let consider application of adequate fuzzy model to the problem of control approaching subway train to a station [8]. The inputs are the distance from the station (D) and the speed of the train (S). The output is new speed (NS) after brakes power. We choice following linguistic term-sets for input and output parameters (fig. 2,3): T(D)={Very close (VC), Close (C), Less than medium (LM), Medium (M), More than medium (MM), Far (F)}, T(S)=T(SN)={Very slow (VS), Slow (S), Less than medium (LM), Medium (M), More than medium (MM), Fast (F)}. The rule base for this example is given in table 1.

Usually realization of fuzzy control is based on iterative process [9]. For our case we can write following iteration formulas:

S(new)=fuzzy_rule_base(S(old), D(old))
D(new)=D(old)-S(new)t

where t is any time step.

On fig. 4 is given result of simulation of problem with some initial location of train and value of speed.

Conclusion

Fuzzy sets theory allows approximate control function for many control problems. There are many attempts to simplify this procedure. Proposed method is based on constructing adequate fuzzy model, which decrease number of fired rules in every calculation. The other advantage of proposed method is possibility decreasing number of linguistic terms that define input parameters. Testing proposed method for subway train control problem has give successful results.

Speed \ Distasnce	VC	C	LM	M	MM	F
VS	VS	VS	VS	VS	VS	VS
S	VS	VS	S	S	VS	VS
LM	VS	S	LM	LM	LM	LM
M	VS	LM	M	M	M	M
MM	VS	LM	M	MM	MM	MM
F	VS	LM	M	MM	MM	F

Table 1. The rule base

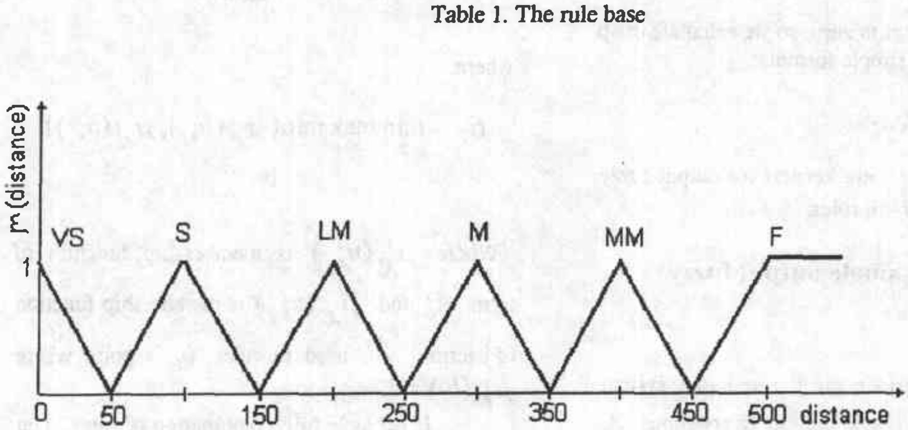


Fig. 2. Membership functions for distance

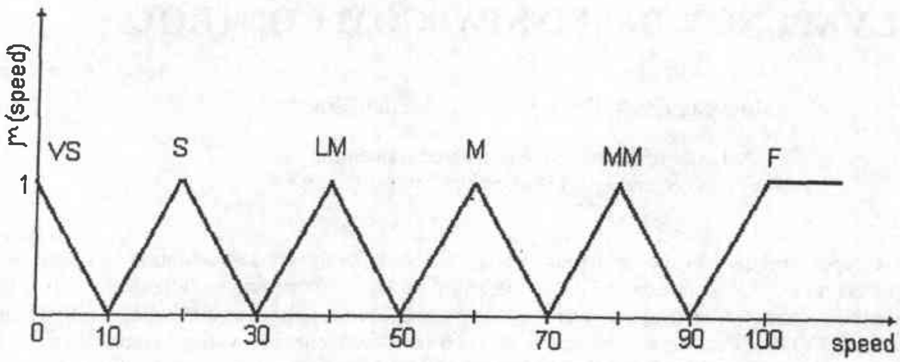
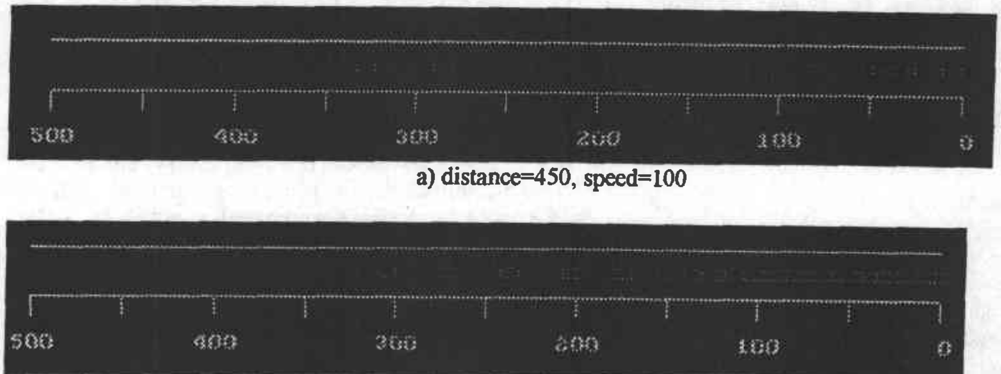


Fig. 3. Membership functions for speed



a) distance=450, speed=100

b) distance=300, speed=70

Fig. 4. Simulation examples

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