# A Simple Method to Place Amplifiers in a WDM LAN/MAN 

Ignacio de Miguel, Juan Carlos Aguado, Patricia Fernández, Rubén M. Lorenzo, Evaristo J. Abril, Miguel López<br>E.T.S.I. de Telecomunicación, Universidad de Valladolid<br>Campus "Miguel Delibes", 47011 Valladolid (Spain)<br>E-mail: ignmig@tel.uva.es


#### Abstract

We propose a simple method to place amplifiers in wavelength division multiplexing (WDM) lo$\mathrm{cal} /$ metropolitan area networks (LAN/MAN). The objective is to reduce the ASE noise at the receiver when the total gain to be supplied and the number of amplifiers per link are known. A comparison with another method called ALAP (As Late As Possible) is performed, showing that our method always reduces ASE noise when compared to it. For instance, for a sample network studied the ASE noise power reduction ranges from $1.87 \%$ to $35.92 \%$.


## 1 Introduction

Wavelength Division Multiplexing (WDM) allows the transmission of several data channels through the same optical fiber using different wavelengths. Therefore, WDM allows to use the high bandwidth the fiber provides. On the other hand, it is possible to take advantage of wavelength to perform functions such as routing, switching and service segregation in networks [1].

The advent of optical amplifiers has been very important in optical communication systems and networks, since they increase the repeater spacing. Erbium-Doped Fiber Amplifiers (EDFA) are of great importance as they provide low noise, high gain, wide bandwidth, and polarization independency [2]. Anyway, it is important to minimize the number of amplifiers used in networks due to cost, noise, maintenance and fault-tolerance considerations. Several studies about this topic are [3]- [7].

Our starting point is the work by Ramamurthy et al. [6]. Their study is focused on WDM LAN/MANs as the example shown in Fig. 3. The network consists of several stations and "nonreflective" passive optical stars. For example, signals flowing through link 4 are split in star 2 towards links 2 and 5, but not towards link 3, that is the meaning of "nonreflective". Stations use dedicated wavelengths (lightpaths) to broadcast data to all other stations of the network. Each station has a fixed-wavelength transmitter and a tunable receiver or a receiver array. As the signal travels through the network, it is attenuated, so it may be necessary to add amplifiers to some of the links of the network. The goal of
their study is to find the minimum number of amplifiers required to operate the network and to determine their exact placements at each link.

The amplifier model used in their work and the one we are using in our study is [6]:

$$
\begin{equation*}
\frac{P_{i n}}{P_{s a t}}=\frac{1}{G-1} \ln \left(\frac{G_{0}}{G}\right) \tag{1}
\end{equation*}
$$

where $P_{\text {in }}$ is the total input power to the amplifier, $P_{s a t}$ is the internal saturation power and $G_{0}$ is the smallsignal gain, all of them in absolute scale, not dB. Two constraints are applied to this model. First of all, the maximum small-signal gain of the amplifier is $G_{\max }$, therefore $G_{0} \leq G_{m a x}$. Secondly, the maximum output power a transmitter or an amplifier may supply is $P_{\max }$, hence $G P_{i n} \leq P_{\max }$. According to these constraints, the amplifier gain model used is represented in Fig. 1. It is assumed that the amplifier has a flat gain over the bandwidth of interest and that all wavelengths contribute equally to the gain saturation of the amplifier.


Figure 1: Amplifier gain model used in [6] and in this study, for different values of the small-signal gain $\left(G_{0}\right)$. The dashed lines show the model of ec. (1). Since there is a constraint on the maximum output power, the model used is given by the solid lines.

Another important constraint is related to $p_{\text {sen }} . p_{\text {sen }}$ is the minimum signal power at the receiver, and signals should never fall below this value in any point of the network. Amplified spontaneous emission (ASE) and crosstalk are not considered, and are assumed to be incorporated in this parameter. Several fixed parameters used in our study and their values are shown in Table 1.

| Symbol | Parameter | Value used |
| :--- | :--- | :--- |
| $P_{s a t}$ | Internal saturation power <br> of the amplifier | 1.298 mW |
| $G_{m a x}$ | Maximum small-signal <br> gain | $100(20 \mathrm{~dB})$ |
| $\alpha_{d B}$ | Fiber attenuation | $0.2 \mathrm{~dB} / \mathrm{Km}$ |
| $p_{\text {sen }}$ | Minimum signal power <br> at receiver | $1 \mu \mathrm{~W}$ |
| $n_{s p}$ | Spontaneous emission <br> factor | 1.4 |
| $h$ | Planck constant | $6.625 \cdot 10^{-34} \mathrm{Js}$ |
| $f_{c}$ | Optical carrier frequency | 193.41 THz |

Table 1: Fixed parameters and their values in this study

The solution approach in [6] consists of four modules. The first three modules determine how many amplifiers are required in each link ( $N$ ), the total gain they must supply, and the power at the beginning of the link ( $P_{t x}$ ). The fourth module splits the total gain among the amplifiers and determines where to place them. Note that as the power at the beginning of the link, the total gain and the link distance (and hence attenuation) are given, the power at the end of the link is fixed and it is independent of how the amplifiers are placed (as long as ASE is not considered).

Two methods are proposed in [6] for placing amplifiers: ASAP (As Soon As Possible) and ALAP (As Late As Possible), but the second one is the method implemented in their work. With that method, each link is traversed downstream and each amplifier is placed only after the power level on each of the signals has fallen to its minimum acceptable value ( $p_{\text {sen }}$ ), unless the end of the link is reached. Although in [6], ASE noise is implicitly incorporated in $p_{\text {sen }}$, different placing solutions may increase or decrease the ASE noise, and hence they affect the quality of the link. Several studies have been made about the optimal placing of amplifiers in links. In [8], it is briefly discussed how to place a single amplifier to get the maximum Q-factor, and also how to place equidistant amplifiers. A method to calculate the lengths of the transmission fibers and the doped fibers of the amplifiers to get the minimum bit error rate (BER) is shown in [9]. These studies deal with trying to get the best quality, but are not restricted by the total gain that must be achieved or do not consider the maximum output power constraint. Therefore, in this paper we propose an algorithm for placing amplifiers reducing the ASE noise at the end of the link taking into account the constraints given in [6].

In section 2, some considerations about ASE noise in WDM network are discussed. In section 3, we provide a simple method for situating amplifiers in a the link. This method is optimal when there is a single amplifier in the link. In section 4, we obtain numerical results of our method when applied to the network of Fig. 3, and a comparison with the ALAP method is performed. Section 5 summarizes our results.

## 2 Considerations about ASE noise in WDM networks

An optical amplifier amplifies the input signal but it also adds amplified spontaneous emission (ASE) noise to the output. Considering there are two fundamental polarization modes in the fiber, ASE noise power is [10]:

$$
\begin{equation*}
P_{A S E}=2 P_{n}(G-1) B_{o}, \tag{2}
\end{equation*}
$$

where $P_{n}=n_{s p} h f_{c}$ (see Table 1) and $B_{o}$ is the optical bandwidth. ASE noise propagates together with the amplified signal, being attenuated by the fiber and amplified by other amplifiers. It should also be kept in mind that as the gain of the amplifier depends on the input power, it will depend on the ASE noise introduced by previous amplifiers. Therefore the best way to evaluate ASE noise propagating through a cascade of amplifiers is by means of an iterative algorithm.

In a network as the one considered in this study (e.g. Fig. 3), some links start at a transmitter and so, there is no initial ASE at the beginning of the link, but other links start at a star, and hence they may have some initial ASE coming from another link. In our case, we are dealing with a WDM network, so each station is assigned a different wavelength. We will suppose wavelengths are separated by 50 GHz (as in the AON testbed [10]), and that stations belonging to the same group $i$, that is, connected to the same star, are assigned a wavelength belonging to a band $B_{i}$. So, in Fig. 3 stations at group 1 will transmit using wavelengths in $B_{1}$, group 2 in $B_{2}$, and group 3 in $B_{3}$. If a group $i$ has $M$ stations, we suppose the bandwidth of $B_{i}$ is $B W_{i}=50 \times M$. Each link of the network will carry signals from one or more of that groups, and therefore, a filter after each amplifier should remove ASE noise out of the bands of interest. For instance, in link 1 only signals within $B_{1}$ are transmitted, so a filter must be placed with optical bandwidth $B W_{1}$. Link 3 will be traversed by signals contained in bandwidths $B_{1}$ and $B_{3}$, hence, the filters should allow signals and ASE within these bands to propagate and eliminate other bands. Therefore, the optical bandwidth at link 3 is $B_{o}=B W_{1}+B W_{3}$. For transmitter to star, and star to receiver links the optical bandwidth $B_{o}$ is the optical bandwidth assigned to a single channel. Note that in a star to receiver link, the component that limits the optical bandwidth is the receiver and not the amplifier. Since signals in bands $B_{i}$ travel through different links, the ASE noise power within each band is different. Therefore, ASE noise at a receiver depends on the band it is listening to.

## 3 A simple method for placing amplifiers

We make the same assumptions as in [6]. Dispersion is not considered and ASE is not supposed to contribute to saturate the amplifiers. Besides these, we suppose that all the amplifiers have the same value of $n_{s p}$. First
of all, we present an optimal method for one amplifier and later we extend the method for its use in links with more amplifiers. In our analysis, we use absolute units (not dB) unless otherwise stated.

### 3.1 Link with one amplifier

We start analyzing an optimal method for placing an amplifier in a link subject to the constraints imposed by the model of [6]. Let $P_{t x}$ be the power at the beginning of the link, $L_{T}$ the total distance of the link, and $\alpha$ the fiber attenuation. Note that $\alpha$ is related to the customary parameter $\alpha_{d B}(\mathrm{~dB} / \mathrm{Km})$ by $\alpha=(\ln (10) / 10) \alpha_{d B}$. We are placing an amplifier that must provide a gain $G$ and we must determine where it should be located in order to minimize ASE noise. Let us suppose the amplifier is situated $l_{0}$ kilometers downstream, as shown in Fig. 2


Figure 2: Link from a star/station to a star/station with only one amplifier. Given data are in normal font, and variables are in bold face.

The input power to the amplifier, $P_{i n}$, is given by $P_{i n}=P_{t x} \exp \left(-\alpha l_{0}\right)$. Using (1) we get

$$
\begin{equation*}
l_{0}=-\frac{1}{\alpha} \ln \left[\frac{P_{s a t}}{P_{t x}(G-1)} \ln \left(\frac{G_{0}}{G}\right)\right] . \tag{3}
\end{equation*}
$$

But we have to take into account that output power, $P_{\text {out }}=G P_{\text {in }}$, should not be higher than $P_{\text {max }}$, so

$$
\begin{equation*}
G P_{t x} \exp \left(-\alpha l_{0}\right) \leq P_{\max } \tag{4}
\end{equation*}
$$

Substituting (3) into (4) and solving for $G_{0}$ yields

$$
\begin{equation*}
G_{0} \leq G \exp \left(\frac{G-1}{G}\right) \tag{5}
\end{equation*}
$$

Therefore, and considering that $G_{0} \leq G_{m a x}$ as well,

$$
\begin{equation*}
G_{0, \max }=\min \left\{G_{\max }, G \exp \left(\frac{G-1}{G}\right)\right\} \tag{6}
\end{equation*}
$$

The ASE noise power at the end of the link, which should be minimized is given by

$$
\begin{equation*}
P_{A S E}=2 P_{n}(G-1) B_{0} \exp \left(-\alpha l_{1}\right) \tag{7}
\end{equation*}
$$

As $2 P_{n}$ and $B_{o}$ are constants, and $G$ is a fixed parameter (the total gain to be supplied in the link), minimizing $P_{A S E}$ is equivalent to minimize $\exp \left(-\alpha l_{1}\right)$ and this is achieved when $l_{1}$ is maximum. Since the total length of the link is $L_{T}=l_{0}+l_{1}$, maximizing $l_{1}$ is equivalent to minimize $l_{0}$. To do so, and according to
(3), $G_{0}$ should be maximized, that is, the value given by (6) should be used. Other constraints related to the distance of the link, and to the minimum power in any point of the link need to be considered. Therefore, the optimal method for placing only one amplifier in a link subject to the constraints of [6] and minimizing ASE is the following one.

1. There is a lower bound for the input power to the amplifier. So, the maximum distance that the signal can travel without being amplified is given by

$$
\begin{equation*}
l_{\max }=\frac{1}{\alpha} \ln \left(\frac{P_{t x}}{p_{s e n} \cdot|\lambda|}\right) \tag{8}
\end{equation*}
$$

where $|\lambda|$ is the number of wavelengths in the link. Note that $l_{\max } \leq L_{T}$ is always satisfied because if $L_{T}>l_{\max }$, the amplifier would not be necessary.
2. Calculate the value of $G_{0, \max }$ using (6) and substitute the value obtained in

$$
\begin{equation*}
l_{0}=-\frac{1}{\alpha} \ln \left[\frac{P_{\theta a t}}{P_{t x}(G-1)} \ln \left(\frac{G_{0, \max }}{G}\right)\right] . \tag{9}
\end{equation*}
$$

3. Consider the constraints imposed by the length of the link and the minimum power in the link. Hence, the optimum value of $l_{0}$ is given by

$$
l_{0, o p t}= \begin{cases}0, & \text { if } l_{0}<0  \tag{10}\\ l_{\max }, & \text { if } l_{0}>l_{\max } \\ l_{0,}, & \text { otherwise }\end{cases}
$$

Note that if $l_{0, \text { opt }}=0$, then $G_{0}=$ $G \exp \left[\left(P_{t x} / P_{s a t}\right)(G-1)\right]$ and if $l_{0, o p t}=l_{\max }$, then $G_{0}=G \exp \left[\left(p_{s e n}|\lambda| / P_{s a t}\right)(G-1)\right]$.

### 3.2 Extension of the method

Finding an optimal method for every possible number of amplifiers is a difficult problem, so we are going to propose a simple algorithm although it is not optimal. ASE noise power in a link with a cascade of $N$ amplifiers is

$$
\begin{align*}
P_{A S E}= & 2 P_{n} B_{o} G_{N} \exp \left(-\alpha l_{N}\right) \sum_{i=1}^{N-1} T_{i} \\
& +2 P_{n} B_{0}\left(G_{N}-1\right) \exp \left(-\alpha l_{N}\right)  \tag{11}\\
T_{i}= & \frac{\left(G_{i}-1\right)}{G_{i}} \prod_{j=i}^{N-1} G_{j} \exp \left(-\alpha l_{j}\right) \tag{12}
\end{align*}
$$

Since $G_{N}$ and $\exp \left(-\alpha l_{N}\right)$ appear in all the terms of (11), we will set them as low as possible. Therefore, the previous $N-1$ amplifiers should provide as much gain as possible, and the last amplifier should be placed using the algorithm presented in the previous subsection. The complete method is therefore as follows:

1. Place $N-1$ amplifiers using the ALAP method. With this method, all the amplifiers but the last one provide as much gain as possible.
2. Use the method given in the previous subsection setting $G$ to the gain the last amplifier must supply, $L_{T}$ to the distance between the $N-1$ amplifier and the end of the link, and $P_{t x}$ to the output power of the $N-1$ amplifier. The method provides the distance from the $N-1$ amplifier to the point where the last amplifier must be located.

## 4 Numerical results an comparison with ALAP method

We have applied our method to the sample network shown if Fig. 3, so that we can compare with the results of the ALAP method used in [7] The results are given in Tables 2 and 3. ASE noise has been calculated according to the guidelines given in section 2.

ASE noise obtained with our method is always equal or less than with the ALAP method. In links from a station to a star (aggregated links 7,9 and 11) there are no amplifiers and there is no initial ASE since the links start at the stations. Then, there is no difference between any method used. In links between stars (links 1 to 6), the placement of the amplifiers with both methods is the same save the link 1 (Table 2). This is because the last amplifier placed with the ALAP method provides $P_{\text {max }}$, and therefore it cannot be placed before. Hence, link 1 is the only one that benefits directly from our method reducing ASE at the end of it ( $27.7 \%$ ). Anyway, as ASE noise coming from link 1 to star 2 is split to links 3 and 5 , these links also benefit from our method ( $10.64 \%$ and $11.30 \%$ respectively) as their incoming ASE is lower than when using the ALAP method. ASE noise is specially important at the receivers (Table 3). Receivers in the sample network are placed at the end of aggregated links 8,10 and 12 and all of them benefit directly from our method. The signal received may come from the same group or for another group. ASE noise depends on the number of amplifiers the signal has traversed. Then, signals coming from different groups go accompanied of different ASE noise powers. If the signal comes from another station of the same group, it will be affected less by ASE, and it is observed that our method gets less improvement in these cases. Our method gets an ASE reduction ranging from $1.87 \%$ in the case of signal received at group 3 when transmitted from a station of group 3, to $35.92 \%$ for signals received at group 2 from stations of group 1. As the received signal power is the same with both algorithms (or even greater in our case since ASE will contribute less to the saturation of amplifiers in the real scenario), our method improves the BER when compared to ALAP.

## 5 Conclusion

We have presented a simple method to place amplifiers in WDM LAN/MANs. The algorithm is based on the work by Ramamurthy et al. [6], and it only deals with the fourth module of their method. Our objective is to
reduce the ASE noise at the receiver when the total gain to be supplied and the number of amplifiers per link are known. First of all, we made some considerations about the evaluation of ASE noise in WDM networks. Then, we showed an optimal method when there a single amplifier in the link and extended it to links with more amplifiers. Finally, we performed a comparison of our method and the ALAP method used in [6], finding out that ASE noise power reduction is achieved with our method, and therefore a BER improvement.

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\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multirow[t]{2}{*}{Link} \& \multirow[t]{2}{*}{Gain (dB)} \& \multicolumn{2}{|r|}{Distance (Km.)} \& \multirow[t]{2}{*}{\[
\begin{gathered}
P_{\text {ASE }}(\mathrm{W}) \\
\text { ALAP }
\end{gathered}
\]} \& \multirow[t]{2}{*}{\begin{tabular}{l}
\[
\overline{P_{A S E}(W)}
\] \\
Our method
\end{tabular}} \& \multirow[t]{2}{*}{\[
\begin{gathered}
P_{A S E} \\
\text { reduction (\%) }
\end{gathered}
\]} \\
\hline \& \& ALAP \& Our method \& \& \& \\
\hline 1 \& \[
\begin{aligned}
\& G_{1}=16.99 \\
\& G_{2}=13.50 \\
\& \hline
\end{aligned}
\] \& \[
\begin{aligned}
\& \hline l_{0}=3.28 \\
\& l_{1}=84.94 \\
\& l_{2}=11.78 \\
\& \hline
\end{aligned}
\] \& \[
\begin{aligned}
\& \hline l_{0}=3.28 \\
\& l_{1}=67.43 \\
\& l_{2}=29.29 \\
\& \hline
\end{aligned}
\] \& \(8.91 \times 10^{-6}\) \& \(6.44 \times 10^{-6}\) \& 27.71 \% \\
\hline 2 \& \[
\begin{aligned}
\& G_{1}=13.67 \\
\& G_{2}=11.87 \\
\& \hline
\end{aligned}
\] \& \multicolumn{2}{|c|}{\[
\begin{gathered}
l_{0}=40.67 \\
l_{1}=59.33 \\
l_{2}=0
\end{gathered}
\]} \& \multicolumn{2}{|c|}{\(1.64 \times 10^{-4}\)} \& 0\% \\
\hline 3 \& \[
\begin{aligned}
G_{1} \& =13.19 \\
G_{2} \& =13.19 \\
G_{3} \& =8.68
\end{aligned}
\] \& \(l_{0}=\)
\(l_{1}=\)
\(l_{2}=\)
\(l_{3}\)

$l^{\circ}$ \& 0.67
5.94
3.39
0 \& $1.74 \times 10^{-4}$ \& $1.55 \times 10^{-4}$ \& 10.64 \% <br>

\hline 4 \& $$
\begin{aligned}
& G_{1}=17.47 \\
& G_{2}=17.47 \\
& G_{3}=4.77 \\
& \hline
\end{aligned}
$$ \& \multicolumn{2}{|c|}{\[

$$
\begin{gathered}
l_{1}=87.36 \\
l_{2}=55.51 \\
l_{3}=0
\end{gathered}
$$
\]} \& \multicolumn{2}{|c|}{$7.14 \times 10^{-6}$} \& $0 \%$ <br>

\hline 5 \& $$
\begin{aligned}
& G_{1}=14.56 \\
& G_{2}=11.87 \\
& \hline
\end{aligned}
$$ \& \multicolumn{2}{|c|}{\[

$$
\begin{gathered}
l_{1}=59.33 \\
l_{2}=0 \\
\hline
\end{gathered}
$$
\]} \& $2.07 \times 10^{-4}$ \& $1.83 \times 10^{-4}$ \& 11.30 \% <br>

\hline 6 \& $$
\begin{aligned}
& G_{1}=15.53 \\
& G_{2}=15.53 \\
& \hline
\end{aligned}
$$ \& \multicolumn{2}{|c|}{\[

$$
\begin{aligned}
& l_{1}=77.64 \\
& l_{2}=21.92
\end{aligned}
$$
\]} \& \multicolumn{2}{|c|}{$6.98 \times 10^{-6}$} \& 0 \% <br>

\hline
\end{tabular}

Table 2: Results of the ALAP method and ours for star to star links. $P_{A S E}$ has been calculated at the end of the link.

| Link(Group) | Gain <br> (dB) | Distance (Km) |  | Listening to | $\begin{gathered} \hline \bar{P}_{\text {ASE }}(W) \\ \text { ALAP } \end{gathered}$ | $\bar{P}_{A S E}(\mathrm{~W})$ <br> Our method | $\begin{gathered} P_{A S E} \\ \text { reduction (\%) } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ALAP | Our method |  |  |  |  |
| $\begin{array}{\|c\|} \hline 8 \\ (\mathrm{Gr} .1) \end{array}$ | $\begin{gathered} G_{1}= \\ 3.35 \end{gathered}$ | $\begin{gathered} l_{0}=3.28 \\ l_{1}=16.72 \end{gathered}$ | $\begin{aligned} & \hline i_{0}=0 \\ & l_{1}=20 \end{aligned}$ | Gr. 1 | $1.72 \times 10^{-10} \cdot B_{c}$ | $1.48 \times 10^{-10} \cdot B_{c}$ | 13.89 \% |
|  |  |  |  | Gr. 2 | $3.70 \times 10^{-8} \cdot \bar{B}_{c}$ | $3.18 \times 10^{-8} \cdot \bar{B}_{c}$ | 13.95 \% |
|  |  |  |  | Gr. 3 | $3.59 \times 10^{-8} \cdot \bar{B}_{c}$ | $3.09 \times 10^{-8} \cdot \bar{B}_{c}$ | 13.95\% |
| $\begin{gathered} 10 \\ \text { (Gr. 2) } \end{gathered}$ | $\begin{gathered} \hline G_{1}= \\ 2.57 \end{gathered}$ | $\begin{gathered} l_{0}=\overline{7} .13 \\ l_{1}=12.87 \end{gathered}$ | $\begin{aligned} & l_{0}=0 \\ & l_{1}=20 \end{aligned}$ | Gr. 1 | $5.02 \times 10^{-9} \cdot B_{c}$ | $3.22 \times 10^{-9} \cdot B_{c}$ | 35.92 \% |
|  |  |  |  | Gr. 2 | $1.58 \times 10^{-10} \cdot B_{c}$ | $1.14 \times 10^{-10} \cdot B_{c}$ | 27.84 \% |
|  |  |  |  | Gr. 3 | $4.89 \times 10^{-9} \cdot B_{c}$ | $3.18 \times 10^{-9} \cdot B_{c}$ | 34.92\% |
| $\begin{gathered} 12 \\ \text { (Gr. 3) } \end{gathered}$ | $\begin{gathered} G_{1}= \\ 3.91 \end{gathered}$ | $\begin{gathered} l_{0}=0.44 \\ l_{1}=19.56 \end{gathered}$ | $\begin{aligned} & \hline l_{0}=0 \\ & l_{1}=20 \end{aligned}$ | Gr. 1 | $3.30 \times 10^{-9} \cdot B_{c}$ | $2.87 \times 10^{-9} \cdot B_{c}$ | 12.93 \% |
|  |  |  |  | Gr. 2 | $5.85 \times 10^{-9} \cdot \bar{B}_{c}$ | $5.14 \times 10^{-9} \cdot B_{c}$ | 12.15\% |
|  |  |  |  | Gr. 3 | $2.10 \times 10^{-10} \cdot \bar{B}_{c}$ | $2.06 \times 10^{-10} \cdot B_{c}$ | 1.87\% |

Table 3: Results of the ALAP method and ours for star to station links. $P_{A S E}$ has been calculated at the end of the link. $B_{c}$ is the optical bandwidth of a channel in GHz .


Figure 3: Sample network used in this study and in [6].

