Robust Attitude Tracking Control for a Small–Scaled Unmanned Model Helicopter

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***Abstract*— This work focuses on the robust attitude tracking control problem for a small–scaled unmanned model helicopter. To realize this purpose, the actual system inputs, namely the elevator servo input, the aileron servo input and the rudder servo input, are used as the control inputs. In the dynamic model of the small–scaled unmanned model helicopter, rotor dynamics and rigid body dynamics are combined in the same dynamic model by expressing the input torque as a function of the actual system inputs. In this expression, vector of the control inputs is premultiplied with a non–symmetric matrix which makes the input gain matrix is a non–symmetric matrix. Compensating the mentioned non–symmetry is the one of the main motivations of this study. In this study, two different type of robust controllers are proposed to reach this aim. One of these controllers is a full–state feedback robust controller while the other one is an output feedback robust controller. Performance of the proposed controllers are demonstrated via computer based numerical simulation studies.**

I. Introduction

High maneuverability and their versatility make helicopters important aerial vehicles for many applications. However their highly nonlinear and uncertain flight dynamics and strong coupling effects and natural instability in their system dynamics make the control design problem a challenging task. The control system of the unmanned helicopters can be divided into two parts; an inner–loop level control and an outer–loop level control which are related with attitude and position control, respectively. Since the position tracking can be ensured via attitude control, designing a controller for the attitude control of these type of vehicles is considered as the main control objective in many studies.

There are lots of control studies related with the attitude control of helicopters available in the literature [1], [2], [3], [4], [5], [6], [7]. These studies were usually interested in linearized dynamics. A more realistic approach can be provided by designing a controller for the nonlinear dynamics. Some examples about the these type of studies can also be found in the literature [8], [9], [10], [11], [12], [13].

In this work, a full–state feedback robust controller and an output feedback robust controller are presented for the attitude tracking control scheme for a small–scaled unmanned model helicopter. A velocity observer is proposed to compensate the lack of velocity measurement in the output feedback control design. In the proposed approaches, actual inputs namely the elevator servo input, the aileron servo input and the rudder servo input are used as control inputs. Rigid body and the rotor dynamics are combined in the same dynamic model of the helicopter to express the effects of these inputs on the system dynamics [14]. Input torque is expressed as a function of actual inputs and the vector of actual inputs is premultiplied with a non–symmetric input gain matrix to obtain this expression. Designing a controller that is based on actual inputs can be seen as a necessity for the applicability and the realisticity of the designed controller. However, symmetric structure of the input gain matrix is a critical case for the Lyapunov–based control designs. Realizing robust control designs that can provide the attitude control of a small–scaled unmanned model helicopter by taking into account the non–symmetric structure of the input gain matrix is the main motivation of this study. The stability of the closed–loop error dynamics of the proposed controllers are proven via Lyapunov–based arguments. The performance of the designed controllers are then demonstrated via numerical simulations.

II. System Model and Its Properties

The dynamic model of the small–scaled unmanned model helicopter is given as [14]

$$M_h\ddot{x} + C_h\dot{x} + G_h = \tau_i \tag{1}$$

where $x(t) = [\phi,\, \theta,\, \psi]^T \in \mathbb{R}^3$ is a position vector that contains yaw angle $\phi(t) \in \mathbb{R}$, roll angle $\theta(t) \in \mathbb{R}$ and pitch angle $\psi(t) \in \mathbb{R}$. The first and second time derivatives of this vector are denoted by $\dot{x}(t)$ and $\ddot{x}(t) \in \mathbb{R}^3$, respectively. In addition to these, inertia matrix, matrix of Coriolis–centrifugal forces and vector of conservative forces are denoted by $M_h(x)$, $C_h(x, \dot{x}) \in \mathbb{R}^{3\times3}$ and $G_h(x) \in \mathbb{R}^3$, respectively. The vector of input torque, denoted by $\tau_i(t)$, is expressed as

$$\tau_i = S_h^{-T}(Av_c + B) \tag{2}$$

where $S_h(\eta) \in \mathbb{R}^{3\times3}$ is a velocity transformation matrix from from the body frame to the inertia frame and defined as

$$S_h \triangleq \begin{bmatrix} 1 & s_\phi s_\theta/c_\theta & c_\phi s_\theta/c_\theta \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi/c_\theta & c_\phi/c_\theta \end{bmatrix} \tag{3}$$

where s_ϕ, s_θ, c_ϕ and c_θ denote $\sin(\phi)$, $\sin(\theta)$, $\cos(\phi)$ and $\cos(\theta)$, respectively. At this point it should be stated that, due to the possible value interval of θ, the term $\cos(\theta)$ does not equal to 0. Thanks to the this situation, it does not lead to the indefiniteness of the terms that it is used as a denominator. For a detailed information of the possible value intervals of yaw, pitch and roll angles reader can refer [14], [15] and [16]. In (2), $\upsilon_c(t) \in \mathbb{R}^3$ is a vector that is expressed as $\upsilon_c = \begin{bmatrix} a & b & T_T \end{bmatrix}^T$ where $a(t), b(t) \in \mathbb{R}$ are the flapping angles and $T_T(t) \in \mathbb{R}$ is the tail rotor thrust. Moreover, $A \in \mathbb{R}^{3\times3}$ and $B \in \mathbb{R}^3$ denote a constant invertible matrix and a constant vector. A simplified model for flapping angles and tail rotor thrust at hovering flight conditions is expressed as follows [15]

$$\begin{aligned} a &= A_b b - A_{lon}\delta_{lon} \\ b &= -B_a a + B_{lat}\delta_{lat} \\ T_T &= K_{ped0}\delta_{ped} \end{aligned} \quad (4)$$

where A_b, A_{lon}, B_a, B_{lat} and $K_{ped0} \in \mathbb{R}$ denote the constant terms relate with the helicopter dynamics and the vector $\tau = \begin{bmatrix} \delta_{lon} & \delta_{lat} & \delta_{ped} \end{bmatrix}^T$ contains, the elevator servo input $\delta_{lon}(t)$, the aileron servo input $\delta_{lat}(t)$ and the rudder servo input $\delta_{ped}(t)$, respectively. As a result, a simplified rotor model can be expressed as

$$\tau_i = S^{-T}(AC_\delta\tau + B) \quad (5)$$

where $C_\delta \in \mathbb{R}^{3\times3}$ is defined as

$$C_\delta = \begin{bmatrix} -\frac{A_{lon}}{A_b B_a + 1} & \frac{A_b B_{lat}}{A_b B_a + 1} & 0 \\ \frac{B_{lat}}{A_b B_a + 1} & \frac{B_a A_{lon}}{A_b B_a + 1} & 0 \\ 0 & 0 & K_{ped0} \end{bmatrix}. \quad (6)$$

The dynamic model can be rearranged as follows by substituting (5) in (1)

$$M\ddot{x} + C\dot{x} + f_x = \tau \quad (7)$$

where $M(x)$, $C(x, \dot{x}) \in \mathbb{R}^{3\times3}$ and $f_x(x) \in \mathbb{R}^3$ are defined as

$$\begin{aligned} M &\triangleq \left(S_h^{-T}AC_\delta\right)^{-1} M_h \\ C &\triangleq \left(S_h^{-T}AC_\delta\right)^{-1} C_h \\ f_x &\triangleq G_h - S_h^{-T}B. \end{aligned} \quad (8)$$

As it can be seen clearly from the above equations that, the non–symmetric structure of the input gain matrix makes $M(x)$ is a non–symmetric matrix.

Dynamic model in (7) can be rearranged to compensate the non–symmetric structure of the matrix denoted by M. This rearrangement is utilized to simplify the subsequent stability analysis. The non–symmetric matrix is decomposed as [17], [18]

$$M = SU \quad (9)$$

where $S(\cdot)$ and $U(\cdot) \in \mathbb{R}^{3\times3}$ denote a symmetric, positive definite matrix and an upper triangular matrix with diagonal entries 1. A symmetric, positive definite matrix can be obtained by premultiplying the non–symmetric matrix in (9) with $U^T(\cdot)$. The rearranged model can be obtained as follows by applying this multiplication to the dynamic model given in (7)

$$\bar{M}(x)\ddot{x} = \bar{C}(x, \dot{x})\dot{x} - U^T f_x + U^T\tau \quad (10)$$

where $\bar{M}(\psi)$ and $\bar{C}(x, \dot{x}) \in \mathbb{R}^{3\times3}$ are defined as

$$\begin{aligned} \bar{M} &\triangleq U^T SU & (11) \\ \bar{C} &\triangleq -U^T C. & (12) \end{aligned}$$

The following inequality is provided for the symmetric and positive definite matrix $\bar{M}(\psi)$

$$\lambda_1 \|\zeta\|^2 \leq \zeta^T \bar{M}\zeta \leq \lambda_2 \|\zeta\|^2, \ \forall\zeta \in R^3 \quad (13)$$

where λ_1, $\lambda_2 \in \mathbb{R}$ are positive bounding terms.

III. Control Development

In this section, a full–state feedback controller that needs both position and velocity measurements for the control and an output feedback controller that needs only position measurement for the control are presented, respectively.

A. Full–State Feedback Control

The control development in this subsection is based on the assumption that both velocity and position measurements of the small–scaled unmanned model helicopter are available.

1) Error System Development: The tracking error signal, $e_1(t) \in \mathbb{R}^3$, is defined as

$$e_1 \triangleq x_d - x \quad (14)$$

where $x_d(t) \in \mathbb{R}^3$ is a desired trajectory. The desired trajectory and its first and second time derivatives will be designed bounded ($x_d(t)$, $\dot{x}_d(t), \ddot{x}_d(t) \in \mathcal{L}_\infty$) for the subsequent stability analysis. To simplify the stability analysis, two auxiliary error terms denoted by $e_2(t)$ and $r(t) \in \mathbb{R}^3$ are defined as

$$\begin{aligned} e_2 &\triangleq \dot{e}_1 + e_1 & (15) \\ r &\triangleq e_1 + e_2 & (16) \end{aligned}$$

The following equation can be obtained by substituting the second time derivative of (14) and the first time derivative (15) in the first time derivative of (16)

$$\dot{r} = \ddot{x}_d - \ddot{x} + 2\dot{e}_1. \quad (17)$$

The following expression can be obtained by premultiplying the both sides of (17) by $\bar{M}(\psi)$ and substituting (10) in this multiplication

$$\bar{M}\dot{r} = \bar{M}\ddot{x}_d - \bar{C}\dot{x} - U^T f_x - U^T\tau + 2\bar{M}\dot{e}_1. \quad (18)$$

The above expression can be rearranged as follows by adding and subtracting $0.5\dot{\bar{M}}(\psi)r(t)$, $e_1(t)$ and $\tau(t)$ to the right–hand–side of it

$$\bar{M}\dot{r} = N - \tau - \frac{1}{2}\dot{\bar{M}}r - e_2 - (U^T - I_3)\tau \quad (19)$$

where the auxiliary term denoted by $N(\cdot) \in \mathbb{R}^3$ is defined as

$$N \triangleq \bar{M}\ddot{x}_d - \bar{C}\ddot{x} + 2\bar{M}\dot{e}_1 + \frac{1}{2}\dot{\bar{M}}r - U^T f_x + e_2. \quad (20)$$

The open–loop error dynamics in (19) can be rewritten as follows to substantiate the control development

$$\bar{M}\dot{r} = \tilde{N} + N_d - \frac{1}{2}\dot{\bar{M}}r - e_2 - \begin{bmatrix} 0 \\ 0 \\ \Lambda \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ \Phi_d \end{bmatrix} - \tau \quad (21)$$

where $\Lambda(t)$, $\Phi_d(t) \in \mathbb{R}$ are defined as

$$\begin{bmatrix} 0 \\ 0 \\ \Lambda + \Phi_d \end{bmatrix} \triangleq U^T - I_3\tau. \quad (22)$$

It should be noted that, since the matrix denoted by $U(\cdot)$ is an upper triangular matrix with diagonal entries 1, the term $(U^T - I_3)$ is a lower triangular matrix and the first and second rows of $R(U^T - I_3)$ equals to zero. The auxiliary terms $N_d(\cdot)$, $\tilde{N}(\cdot) \in \mathbb{R}^3$ in (21) can be defined as

$$N_d \triangleq N\,|_{x=x_d,\ \dot{x}=\dot{x}_d,\ \ddot{x}=\ddot{x}_d} \quad (23)$$

$$\tilde{N} \triangleq N - N_d \quad (24)$$

From (23), it can be seen that the auxiliary term $N_d(\cdot)$ is a function of the desired trajectory and its time derivatives. Since the desired trajectory and its time derivatives are designed as bounded terms, $N_d(\cdot)$ is also a bounded term.

The auxiliary term $\tilde{N}(\cdot)$ in (24) can be upper bounded as follows by utilizing Mean Value Theorem [19]

$$\left\|\tilde{N}\right\| \leq \rho_N(\|z\|)\,\|z\|\,. \quad (25)$$

where $\rho_N(\cdot) \in \mathbb{R}$ represents a positive definite, globally invertible and non–decreasing function while $z(t) \in \mathbb{R}^9$ denotes the combined error vector and defined as

$$z \triangleq \left[e_1^T,\, e_2^T,\, r^T\right]^T. \quad (26)$$

2) Control Design: Based on the open–loop error dynamics in (21), the control input is designed as [20], [21]

$$\tau \triangleq Kr + \hat{f} \quad (27)$$

where the constant, positive definite, diagonal matrix of control gains is denoted by $K \in \mathbb{R}^{3\times3}$ while $\hat{f}(t) \in \mathbb{R}^3$ represents the feedforward term used for compensating uncertain $N_d(t)$ and $\Phi_d(t)$ terms. Its bounded design necessity is the only constraint on the design of $\hat{f}(\cdot)$ ($\hat{f}(\cdot) \in \mathcal{L}_\infty$). It should be noted that the feedforward term $\hat{f}(\cdot)$ in (27) is not specified in this study. However it can be obtained in applications by utilizing a series of method include neural networks.

The closed–loop error system is obtained as follows by substituting the control design in (27) into (21)

$$\bar{M}\dot{r} = \Pi + \Psi_d - \frac{1}{2}\dot{\bar{M}}r - e_2 - Kr \quad (28)$$

where auxiliary signals denoted by $\Pi(t)$, $\Psi_d(t) \in \mathbb{R}^3$ are defined as

$$\Pi \triangleq \tilde{N} - \begin{bmatrix} 0 \\ 0 \\ \Lambda \end{bmatrix},\ \Psi_d \triangleq N_d - \begin{bmatrix} 0 \\ 0 \\ \Phi_d \end{bmatrix} - \hat{f}. \quad (29)$$

At this point it should be noted that, $\Lambda(\cdot)$ and $\Phi_d(\cdot)$ terms are dependent on the first two diagonals of K.

3) Stability Analysis:

Theorem 1: The full–state feedback controller designed in (27) provides a semi–global, uniformly distributed and exactly bounded tracking result given as

$$\|e_1(t)\| \leq \epsilon,\ \forall t \in [t_0, \infty) \quad (30)$$

where $\epsilon \in \mathbb{R}$ is a positive constant that can be adjusted as small as desired by using control gains.

Proof: It should be stated that the aforementioned controller is a different version of the controller that was developed in the previous study. Its semi–global, uniformly distributed and exactly bounded tracking result was proven in [20], [21]. The detailed stability analysis is not given in this study by considering the page limitation. However, reader can refer the mentioned studies for the detailed stability analysis. ■

B. Output Feedback Control

The control development in this subsection is based on the assumption that only the position measurement of the small–scaled unmanned model helicopter is available.

1) Observer Design: The high–gain observers denoted by $\hat{e}_1(t)$, $\hat{r}_2(t) \in \mathbb{R}$ are designed as

$$\dot{\hat{e}}_1 = \hat{r}_2 - 2\hat{e}_1 + \frac{\alpha_1}{\varepsilon}(e_1 - \hat{e}_1) \quad (31)$$

$$\dot{\hat{r}}_2 = \frac{\alpha_2}{\varepsilon^2}(e_1 - \hat{e}_1) \quad (32)$$

where α_1, α_2, $\varepsilon \in \mathbb{R}$ are positive observer gains.

2) Controller Design: Based on the open–loop error dynamics in (21), the control input is designed as [21]

$$\tau \triangleq sat\{K\hat{r}\} + \hat{f} \quad (33)$$

where $sat\{\cdot\} \in \mathbb{R}^3$ denotes vector saturation function while the feedforward term is denoted by $\hat{f}(t) \in \mathbb{R}^3$.

The closed–loop error system is obtained as follows by substituting the control design in (33) into (21)

$$\bar{M}\dot{r} = \Pi + \Psi_d - \frac{1}{2}\dot{\bar{M}}r - e_2 - sat\{K\hat{r}\} \quad (34)$$

where $\Pi(\cdot)$ and $\Psi_d(\cdot)$ are used as defined in (29).

3) Stability Analysis:

Theorem 2: The output feedback controller designed in (33) provides a semi–global, uniformly distributed and exactly bounded tracking result given as

$$\|e_1(t)\| \leq \epsilon,\ \forall t \in [t_0, \infty) \quad (35)$$

where $\epsilon \in \mathbb{R}$ is a positive constant that can be adjusted as small as desired by using control gains.

Proof: It should be stated that the aforementioned controller is a different version of the controller that was developed in the previous study. Its semi–global, uniformly distributed and exactly bounded tracking result was proven in [20], [21]. The detailed stability analysis is not given in this study by considering the page limitation. However, reader can refer the mentioned studies for the detailed stability analysis. ■

IV. Numerical Simulation Results

Numerical simulations are utilized to demonstrate the performance of the proposed controllers. All system parameters are obtained from experimental modeling studies [14], [15] and [16]. The mathematical model of the helicopter in (1) was utilized with the inertia matrix that have following form

$$M_h = \begin{bmatrix} c_0 & 0 & 0 \\ 0 & c_1 + c_2 \cos(c_3\psi) & c_4 \\ 0 & c_4 & c_5 \end{bmatrix}$$

The Coriolis–centrifugal forces matrix and vector of conservative forces have the following forms

$$\begin{aligned} C_h &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & c_6 \sin(2c_3\psi)\,\dot{\psi} & c_6 \sin(2c_3\psi)\,\dot{\theta} \\ 0 & -c_6 \sin(2c_3\psi)\,\dot{\theta} & 0 \end{bmatrix}, \\ G_h &= \begin{bmatrix} c_7 \cos(\phi) & 0 & 0 \end{bmatrix}^T. \end{aligned}$$

The constant parameters are

$$\begin{aligned} c_0 &= 7.5,\ c_1 = 0.4305,\ c_2 = 3 \times 10^{-4},\ c_3 = -4.143, \\ c_4 &= 0.108,\ c_5 = 0.4993,\ c_6 = 6.214 \times 10^{-4}, \\ c_7 &= -73.58. \end{aligned}$$

The simplified rotor dynamics in (5) are given as

$$\begin{aligned} A &= \begin{bmatrix} c_8\dot{\psi}^2 & 0 & 0 \\ 0 & c_{11}\dot{\psi}^2 & 0 \\ c_{12}\dot{\psi} + c_{13} & 0 & c_{15}\dot{\psi}^2 \end{bmatrix}, \\ B &= \begin{bmatrix} c_9\dot{\psi} + c_{10} & 0 & c_{14}\dot{\psi}^2 + c_{15} \end{bmatrix}^T \end{aligned}$$

with the constant parameters that are given as

$$\begin{aligned} c_8 &= 3.411,\ c_9 = 0.6004,\ c_{10} = 3.679, \\ c_{11} &= -0.1525,\ c_{12} = 12.01,\ c_{13} = 10^5, \\ c_{14} &= 1.204 \times 10^{-4},\ c_{15} = -2.642. \end{aligned}$$

The numerical values of other parameters are used as

$$\begin{aligned} A_{lon} &= -0.1,\ A_{lat} = 0.0313,\ A_b = -0.189 \\ B_{lon} &= 0.0138,\ B_{lat} = 0.14,\ B_a = 0.368 \\ K_{ped} &= 2.16. \end{aligned}$$

The reference position $x_d(t)$ was selected as

$$x_d(t) = \begin{bmatrix} 10\sin(0.1t) \\ 15\sin(0.1t) \\ 20\sin(0.1t) \end{bmatrix} \text{(deg)}.$$

A. Full–State Feedback Control

The matrix of control gains is selected as $K = diag\left\{ 140 \quad 145 \quad 110 \right\}$ via trial–and–error method for the full–state feedback control. Tracking results are shown in Figure 1 while the tracking errors and the control inputs are shown in Figures 2 and 3, respectively. From Figures 1 and 2, it can be seen that the tracking control objective was met.

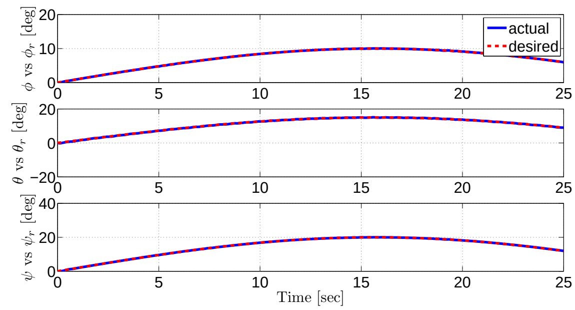

Fig. 1. Tracking Results for Full–State Feedback Control

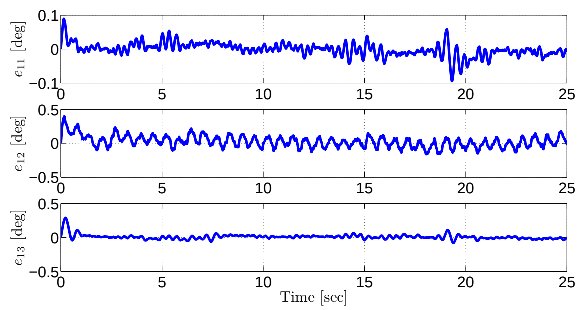

Fig. 2. Tracking Errors for Full–State Feedback Control

B. Output Feedback Control

The matrix of control gains is selected as $K = diag\left\{ 130 \quad 145 \quad 165 \right\}$ via trial–and–error method for the full–state feedback control. The observer gains are selected as $\alpha 1 = 10$, $\alpha_2 = 10$ and $\varepsilon = 0.1$ while the saturation limits are specified as ± 100. Tracking results are shown in Figure 4 while the tracking errors and the control inputs are shown in Figures 5 and 6, respectively. From Figures 4 and 5, it can be seen that the tracking control objective was met.

V. Conclusions

Proposing a solution for the attitude tracking control problem of a small–scaled unmanned model helicopter is aimed in this study. First of all, the overall problem is transformed into a second order system by utilizing some reasonable simplifications for the rotor model under the hovering flight

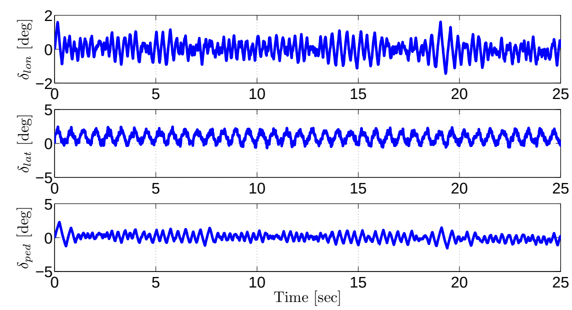

Fig. 3. Control Inputs for Full–State Feedback Control

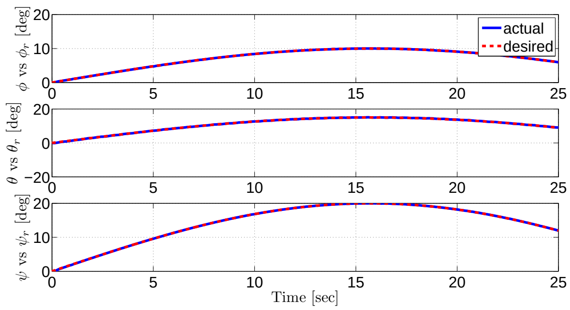

Fig. 4. Tracking Results for Output Feedback Control

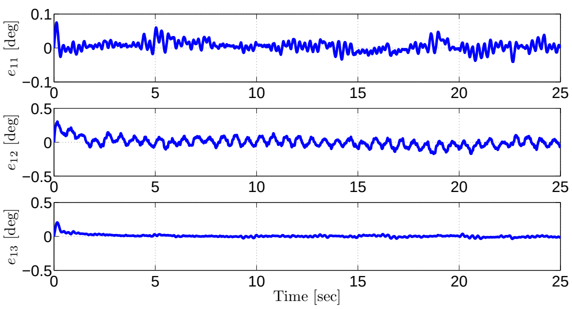

Fig. 5. Tracking Errors for Output Feedback Control

conditions. Then, the dynamical model is rearranged to get rid of the non–symmetry in the input gain matrix. Two nonlinear continuous robust controller are proposed. One of these controllers is a full–state feedback robust controller while the other one is an output feedback robust controller. High-gain observer structure is also proposed to compensate the lack of velocity measurement for the output feedback robust controller. The overall results are supported by Lyapunov based arguments. The performance of the designed controllers are demonstrated via computer based numerical simulation studies.

The most important aspects of these designs can be summarized as:

- Thanks to the robust controller of the designed controllers highly uncertain flight dynamics, strong coupling effects and the natural instability of the small–scaled unmanned model helicopters are coped with.
- Thanks to the nonlinear structure of the designed controllers highly nonlinear dynamics the small–scaled unmanned helicopters are taken into account.
- Thanks to the approach used to design controllers the non–symmetric structure of the input gain matrix of the small–scaled unmanned model helicopter is compensated and actual control inputs dependent control designs are realized.
- Thanks to the output feedback structure of the one of the control approaches a velocity measurement device necessity is removed.

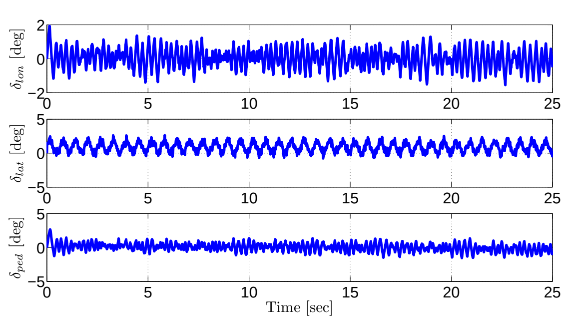

Fig. 6. Control Inputs for Output Feedback Control

It can be mentioned that the realisticity and suitability for the real time applications of small–scaled unmanned helicopters of the designed controllers by considering all of these aspects.

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