# A NEW LOOK AT BER EXPRESSION OF A COHERENT DS-CDMA RECEIVER WITH WEIGHTED DESPREADING FUNCTION IN A REALISTIC FADING ENVIRONMENT 

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#### Abstract

A novel bit error rate (BER) expression for a coherent direct sequence-code division multiple access (DS-CDMA) receiver which employs exponential chip weighting waveforms with imperfect power control over a multipath Rayleigh fading channel is obtained. It is shown that the proposed expression has lower complexity than the known BER expression in the literature.


## I. INTRODUCTION

DS-CDMA has been widely recognised as a promising technique for wireless communications in both satellite and cellular mobile systems [1]. The multiple access interference (MAI), multipath fading and imperfect power control are known to be main factors that degrade the performance of DS-CDMA wireless communication systems [1, 2].

A number of studies, also few recent works by Ciftlikli and Develi have assumed that the transmitted power of each user is perfectly controlled and the waveform of despreading sequence in a receiver is the same as the spreading sequence assigned to a reference user [3-11]. However, in a more realistic environment, the power control error always exists and has a destructive effect on the performance. Moreover, in a practical DS-CDMA system over a terrestrial channel may operate with $L_{R^{-}}$ branch diversity for better performance. Thus, a novel expression has been introduced to analyse the BER performance of a DS-CDMA system with power control error over a multipath Rayleigh fading channel for the uplink [12,13]. With the purpose of MAI rejection, the RAKE receivers employ the despreading sequences weighted by adjustable exponential chip waveforms. The resulting despreading functions are determined by only one parameter and this leads to easy tuning of the despreading sequences in practice to achieve the best performance in multipath environment $[12,13]$.

Considering the importance of performance analyses of DS-CDMA systems, we concentrate here on a computationally efficient expression to calculate the BER of a coherent DS-CDMA receiver with exponentially weighted despreading sequences in a multipath Rayleigh fading channel. This is attained by indicating a simple relation among some properties of random spreading sequences. As we will demonstrate, there is no need to compute the number of transitions which occur between two consecutive chips to calculate the BER performance.

## II. SYSTEM AND CHANNEL MODELS

## A. Transmitter Model

Suppose that there are $K$ DS-CDMA users accessing the channel. User $k$ transmits a binary data sequence $b_{k}(t)$ and employs a spreading sequence $a_{k}(t)$ to spread each data bit. The spreading and data sequences for the $k$ th user are given by
$a_{k}(t)=\sum_{j=-\infty}^{\infty} a_{j}^{(k)} P_{T_{c}}\left(t-j T_{c}\right), \quad b_{k}(t)=\sum_{j=-\infty}^{\infty} b_{j}^{(k)} P_{T_{b}}\left(t-j T_{b}\right)$
where $T_{c}$ and $T_{b}$ are the chip and data durations, respectively, and $P_{x}(y)=1$, for $0<y<x$ and 0 otherwise. It is assumed that there are $N$ chips of a spreading sequence in the interval of each data bit $T_{b}$ and the spreading sequence has period equal to $N$. The transmitted signal for the $k$ th user is

$$
\begin{equation*}
S_{k}(t)=\sqrt{2 P} G_{k} b_{k}(t) a_{k}(t) \cos \left(\omega_{c} t+\theta_{k}\right) \tag{2}
\end{equation*}
$$

where the transmitted power $P$ and the carrier frequency $\omega_{c}$ are common to all users, and the parameter $\theta_{k}$ is the phase of the $k$ th user. The parameter $G_{k}$ represents the power control error for the $k$ th user and is modelled as a random variable uniformly distributed in $\left[1-\varepsilon_{m}, 1+\varepsilon_{m}\right]$
where $\varepsilon_{m}$ represents the maximum value of power control error for all users. Such a distribution for the parameter $G_{k}$ implies a complete lack of knowledge of the power control error and is a least favourable distribution [12, 13].

## B. Channel Model

In this study, a frequency-selective multipath channel is considered for the uplink [12, 13]. The equivalent complex lowpass representation of the channel for the $k$ th user is given by

$$
\begin{equation*}
h_{k}(t)=\sum_{l=0}^{L_{p}-1} \beta_{k l} \delta\left(t-\tau_{k l}\right) e^{j \eta_{k l}} \tag{3}
\end{equation*}
$$

where random variables $\beta_{k l}, \tau_{k l}$ and $\eta_{k l}$ are the $l$ th path gain, delay and phase, respectively, for the $k$ th user. Furthermore, the following assumptions are considered: (i) for different users and paths in each link, the random variables $\left\{\beta_{k l}\right\}, \quad\left\{\tau_{k l}\right\}$ and $\left\{\eta_{k l}\right\}$ are all statistically independent; (ii) the random phases $\left\{\eta_{k l}\right\}$ are uniformly distributed over $[0,2 \pi]$ and the path delays $\left\{\tau_{k l}\right\}$ are uniformly distributed over [ $0, T_{b}$ ]; (iii) for each user, the path gain $\beta_{k l}$ is a random variable with Rayleigh distribution with $E\left[\beta_{k l}^{2}\right]=2 \rho$ which is independent of $k$ and $l$.

At the central station, all user signals, after passing through their own particular channel, are added together and mixed with AWGN $n(t)$ with two-side power spectral density $N_{0} / 2$. Therefore, the input signal at the central station $r(t)$ can be represented by

$$
\begin{array}{r}
r(t)=\sqrt{2 P} \sum_{k=1}^{K} \sum_{l=0}^{L_{p}-1} G_{k} \beta_{k l} b_{k}\left(t-\tau_{k l}\right) a\left(t-\tau_{k l}\right) \cos \left(\omega_{c} t+\phi_{k l}\right) \\
+n_{c}(t) \cos \omega_{c} t-n_{s}(t) \sin \omega_{c} t \tag{4}
\end{array}
$$

where $\phi_{k l}=\theta_{k}+\eta_{k l}-\omega_{c} \tau_{k l}$, and the terms $n_{c}(t)$ and $n_{s}(t)$ are lowpass equivalent components of the AWGN $n(t)$. The random phases $\left\{\phi_{k l}\right\}$ are uniformly distributed over $[0,2 \pi]$.

## C. Receiver Model

For MAI rejection, a bank of single-path matched filters, each of which is matched to different paths, have the same impulse response matched to $2 \hat{a}_{k}(t) \cos \left(\omega_{c} t\right) P_{T_{b}}(t)$ where $\hat{a}_{k}(t)$ is the weighted despreading function with details given below. The outputs of all single-path matched filters represented by $\xi_{k l}(\kappa), l \in\left[0, L_{R}-1\right]$ where $L_{R}$ is the order of diversity, are weighted by the corresponding path gains and then summed to form a single decision variable $\xi_{k}(\kappa)$. The weighted despreading function of the user $k$ 's RAKE receiver can be expressed as

$$
\begin{equation*}
\hat{a}_{k}(t)=\sum_{j=-\infty}^{\infty} a_{j}^{(k)} w_{j}^{(k)}\left(t-j T_{c} \mid\left\{c_{j}^{(k)}, c_{j+1}^{(k)}\right\}\right) P_{T_{c}}\left(t-j T_{c}\right) \tag{5}
\end{equation*}
$$

where $c_{j}^{(k)}=a_{j-1}^{(k)} a_{j}^{(k)}$ and $w_{j}^{(k)}\left(t \mid\left\{c_{j}^{(k)}, c_{j+1}^{(k)}\right\}\right)$, for $0 \leq t \leq T_{c}$, is the $j$ th chip weighting waveform for the $k$ th receiver, conditioned on the status of three consecutive chips $\left\{a_{j-1}^{(k)}, a_{j}^{(k)}, a_{j+1}^{(k)}\right\}$. Detailed information on $w_{j}^{(k)}\left(t \mid\left\{c_{j}^{(k)}, c_{j+1}^{(k)}\right\}\right)$ can be found in [12, 13].

## D. BER Performance Expression

Suppose that $k$ th user is chosen as the reference user to analyse the performance of the RAKE receiver with coherent detection for data symbol $b_{\lambda}^{(k)}$. It is well known that the expression given below enable us to compute the BER for coherent reception at a given set of system parameters [12, 13]:
$P_{e}^{(k)}=\frac{1}{2}-\sum_{d=1}^{L_{R}} \frac{\binom{2 d-2}{d-1}}{2^{2 d} \varepsilon_{m} \overline{\operatorname{SINR}}(2 d-3)}\left\{\left(b^{2}+1\right)^{\frac{3-2 d}{2}}-\left(a^{2}+1\right)^{\frac{3-2 d}{2}}\right\}$
where $a=\left(1+\varepsilon_{m}\right) \overline{\operatorname{SINR}}$ and $b=\left(1-\varepsilon_{m}\right) \overline{\operatorname{SINR}}$. The $\overline{\operatorname{SINR}}$ is the average signal to interference plus noise ratio per channel, given by

$$
\begin{align*}
\overline{\operatorname{SINR}}= & \left\{\frac{\gamma\left[\left(\hat{N}_{k} / N\right)\left(1-e^{-\gamma}\right)+\gamma\left[1-\left(\hat{N}_{k} / N\right)\right] e^{-\gamma}\right]}{\overline{\gamma_{b}}\left[2\left(\hat{N}_{k} / N\right)\left(1-e^{-\gamma / 2}\right)+\gamma\left[1-\left(\hat{N}_{k} / N\right)\right] e^{-\gamma / 2}\right]^{2}}\right.  \tag{7}\\
& \left.+\frac{\left(K L_{p}-1\right)\left(1+\varepsilon_{m}^{2} / 3\right) \Xi^{(e)}\left(\Gamma^{\left(i c_{j}^{(i)}\right\}}, \gamma\right)}{N\left[2\left(\hat{N}_{k} / N\right)\left(e^{\gamma / 2}-1\right)+\gamma\left[1-\left(\hat{N}_{k} / N\right)\right]\right]^{2}}\right]^{-1 / 2}
\end{align*}
$$

where,
$\gamma \quad$ parameter of the exponential chip weighting waveforms,
$\hat{N}_{k} \quad$ random variable which represents the number of occurrences of $c_{j}^{(k)}=-1$ for all $j \in[0, N-1]$,
$N$ processing gain,
$\bar{\gamma}_{b} \quad$ signal to noise ratio (in dB ),
$K$ number of active users,
$\Xi^{(e)}\left(\Gamma^{\left\{c_{j}^{(k)}\right\}}, \gamma\right)$ in (7) is defined as:
$\Xi^{(e)}\left(\Gamma^{\left\{c_{j}^{(k)}\right\}}, \gamma\right)=\frac{1}{N}\left\{\Gamma_{\{-1,-1,-1\}}^{(k)}\left[4+\frac{12}{\gamma}-\frac{16 e^{\gamma / 2}}{\gamma}+\frac{4 e^{\gamma}}{\gamma}\right]\right.$
$+\left(\Gamma_{\{-1,-1,1\}}^{(k)}+\Gamma_{\{1,-1,-1\}}^{(k)}\right)\left[\frac{5}{2}-\frac{\gamma}{4}+\frac{\gamma^{2}}{24}+\frac{19}{2 \gamma}+e^{\frac{\gamma}{2}}-\frac{12 e^{\gamma / 2}}{\gamma}+\frac{5 e^{\gamma}}{2 \gamma}\right]$
$+\left(\Gamma_{\{-1,1,1\}}^{(k)}+\Gamma_{\{1,1,-1\}}^{(k)}\right)\left[-\frac{3}{2}-\frac{3 \gamma}{4}+\frac{19 \gamma^{2}}{24}-\frac{1}{2 \gamma}+e^{\gamma / 2}+\frac{e^{\gamma}}{2 \gamma}\right]$
$+\Gamma_{\{-1,1,-1\}}^{(k)}\left[-3-\frac{3 \gamma}{2}+\frac{7 \gamma^{2}}{12}-\frac{1}{\gamma}+2 e^{\gamma / 2}+\frac{e^{\gamma}}{\gamma}\right]$
$\left.+\Gamma_{\{1,-1,1\}}^{(k)}\left[1-\frac{\gamma}{2}+\frac{\gamma^{2}}{12}+\frac{7}{\gamma}+2 e^{\gamma / 2}-\frac{8 e^{\gamma / 2}}{\gamma}+\frac{e^{\gamma}}{\gamma}\right]+\Gamma_{\{1,1,1\}}^{(k)}\left[\gamma^{2}\right]\right\}$
where $\Gamma_{\left\{v_{1}, v_{2}, v_{3}\right\}}^{(k)}$ is the number of occurrence of $\left\{c_{j-1}^{(k)}, c_{j}^{(k)}, c_{j+1}^{(k)}\right\}=\left\{v_{1}, v_{2}, v_{3}\right\}$ for all $j$ in the $k$ th user's spreading sequence and each $v_{n}, n \in[1,2,3]$, takes values +1 or -1 with equal probabilities $[12,13]$. As can be seen from (8), the following computations are required on a random spreading sequence to calculate the BER of a DS-CDMA receiver:

$$
\Gamma_{\left\{v_{1}, v_{2}, v_{3}\right\}}^{(k)} \text { and } \hat{N}_{k}
$$

Clearly, it is not easy to determine these occurrences in practice when the spreading sequences have large length. In the following section the computationally efficient expression, which cancels the determination of the $\hat{N}_{k}$, is proposed.

## III. PROPOSED EXPRESSION

For the simplicity in presentation, let

$$
\Gamma_{a}^{(k)}=\Gamma_{\{-1,-1,-1\}}^{(k)}, \Gamma_{b}^{(k)}=\left(\Gamma_{\{\{1,-1,1\}}^{(k)}+\Gamma_{\{1,-1,-1\}}^{(k)}\right), \Gamma_{c}^{(k)}=\left(\Gamma_{\{-1,1,1\}}^{(k)}+\Gamma_{\{1,1,-1\}}^{(k)}\right)
$$

$$
\Gamma_{d}^{(k)}=\Gamma_{\{-1,1,-1\}}^{(k)}, \quad \Gamma_{e}^{(k)}=\Gamma_{\{1,-1,1\}}^{(k)}, \quad \Gamma_{f}^{(k)}=\Gamma_{\{1,1,1\}}^{(k)}
$$

In order to achieve a simple expression for computing the BERs, suppose that the $k$ th user's spreading sequence, $a_{k}=\left[a_{0}^{(k)}, \cdots, a_{N-1}^{(k)}\right]$, has the following form:

$$
\left.\begin{array}{ll}
a_{j}^{(k)} \neq a_{j+1}^{(k)} & \text { for } \\
a_{i}^{(k)}=a_{i+2}^{(k)} & \text { for }
\end{array} \quad i \in\{0,1,2, \ldots, N-2\}, 1,2, \ldots, N-3\right\}
$$

Based on this assumption, $a_{k}$ can be shown to be

$$
a_{k}=[0,1,0,1, \cdots, 0,1,0,1] \text { or } \quad a_{k}=[1,0,1,0, \cdots, 1,0,1,0]
$$

As expected, the number of occurrences of $c_{j}^{(k)}=-1$ for all $j \in[0, N-1]$ would be maximum and equal to $\hat{N}_{k}=N-1$. The $\left\{\Gamma_{a}^{(k)}, \Gamma_{b}^{(k)}, \ldots, \Gamma_{f}^{(k)}\right\}$ for such sequences can be obtained as
$\Gamma_{a}^{(k)}=N-3, \Gamma_{b}^{(k)}=2, \Gamma_{c}^{(k)}=0, \Gamma_{d}^{(k)}=1, \Gamma_{e}^{(k)}=0, \Gamma_{f}^{(k)}=0$
Inspecting the computations given above, there is a number of combination which equal to $N-1$. The whole alternatives are listed below:

$$
\hat{N}_{k}=\left\{\begin{array}{l}
\Gamma_{a}^{(k)}+\Gamma_{b}^{(k)}  \tag{9}\\
\Gamma_{a}^{(k)}+\Gamma_{b}^{(k)}+\Gamma_{c}^{(k)} \\
\Gamma_{a}^{(k)}+\Gamma_{b}^{(k)}+\Gamma_{e}^{(k)} \\
\Gamma_{a}^{(k)}+\Gamma_{b}^{(k)}+\Gamma_{f}^{(k)} \\
\Gamma_{a}^{(k)}+\Gamma_{b}^{(k)}+\Gamma_{c}^{(k)}+\Gamma_{e}^{(k)} \\
\Gamma_{a}^{(k)}+\Gamma_{b}^{(k)}+\Gamma_{c}^{(k)}+\Gamma_{f}^{(k)} \\
\Gamma_{a}^{(k)}+\Gamma_{b}^{(k)}+\Gamma_{e}^{(k)}+\Gamma_{f}^{(k)} \\
\Gamma_{a}^{(k)}+\Gamma_{b}^{(k)}+\Gamma_{c}^{(k)}+\Gamma_{e}^{(k)}+\Gamma_{f}^{(k)}
\end{array}\right.
$$

However the alternatives given in (9) are equal to $N-1$, they can not be generalised for a random spreading sequence because of insufficient definition on the $a_{k}$. Therefore, let us make some major modifications on the $a_{k}$ as follow.

Suppose that the $a_{k}$ takes on the form

$$
a_{j}^{(k)}=a_{j+1}^{(k)} \quad \text { for } \quad j \in\{0,1,2, \ldots, N-2\}
$$

By this modification, we have

$$
a_{k}=[0,0,0, \cdots, 0,0,0] \text { or } a_{k}=[1,1,1, \cdots, 1,1,1]
$$

$\hat{N}_{k}$ for the modified $a_{k}$ would be minimum and equal to zero. The $\left\{\Gamma_{a}^{(k)}, \Gamma_{b}^{(k)}, \ldots, \Gamma_{f}^{(k)}\right\}$ for such sequences are easily found to be
$\Gamma_{a}^{(k)}=0, \Gamma_{b}^{(k)}=0, \Gamma_{c}^{(k)}=0, \Gamma_{d}^{(k)}=0, \Gamma_{e}^{(k)}=0, \Gamma_{f}^{(k)}=N$
Based on these values, it is clear that the alternatives which include the $\Gamma_{f}^{(k)}$ can be eliminated from (9). The remainder alternatives are given by

$$
\hat{N}_{k}=\left\{\begin{array}{l}
\Gamma_{a}^{(k)}+\Gamma_{b}^{(k)}  \tag{10}\\
\Gamma_{a}^{(k)}+\Gamma_{b}^{(k)}+\Gamma_{c}^{(k)} \\
\Gamma_{a}^{(k)}+\Gamma_{b}^{(k)}+\Gamma_{e}^{(k)} \\
\Gamma_{a}^{(k)}+\Gamma_{b}^{(k)}+\Gamma_{c}^{(k)}+\Gamma_{e}^{(k)}
\end{array}\right.
$$

To proceed further, let us assume only one random chip, $a_{j}^{(k)}, j \in(1, N-2)$, in the $a_{k}$ differs from the others. If one chooses $j=1$, then the $a_{k}$ can be shown to be

$$
a_{k}=[0,1,0,0, \cdots, 0,0,0,0] \quad \text { or } a_{k}=[1,0,1,1, \cdots, 1,1,1,1]
$$

By this assumption, it is clear that the result of $\hat{N}_{k}$ would be 2. The $\left\{\Gamma_{a}^{(k)}, \Gamma_{b}^{(k)}, \ldots, \Gamma_{f}^{(k)}\right\}$ for such sequences are given by

$$
\Gamma_{a}^{(k)}=0, \Gamma_{b}^{(k)}=2, \Gamma_{c}^{(k)}=2, \Gamma_{d}^{(k)}=0, \Gamma_{e}^{(k)}=0, \Gamma_{f}^{(k)}=N-4
$$

Because $\hat{N}_{k}=2$, the alternatives which include the sum of the $\Gamma_{a}^{(k)}, \Gamma_{b}^{(k)}$ and $\Gamma_{c}^{(k)}$ can be eliminated from the remainder alternatives given in (10). Thus, we come across with two alternatives as follows:

$$
\hat{N}_{k}=\left\{\begin{array}{l}
\Gamma_{a}^{(k)}+\Gamma_{b}^{(k)}  \tag{11}\\
\Gamma_{a}^{(k)}+\Gamma_{b}^{(k)}+\Gamma_{e}^{(k)}
\end{array}\right.
$$

As a final step, let only two random consecutive chips, $a_{j}^{(k)} a_{j+1}^{(k)}, j \in(1, N-3)$ in the $k$ th user's sequence differ from the others. If we assume $j=1$, then the $a_{k}$ can be shown to be

$$
a_{k}=[0,1,1,0, \cdots, 0,0,0,0] \quad \text { or } a_{k}=[1,0,0,1, \cdots, 1,1,1,1]
$$

Since the chips which differ from the others are consecutive, the $\hat{N}_{k}$ would be equal to 2 . The $\left\{\Gamma_{a}^{(k)}, \Gamma_{b}^{(k)}, \ldots, \Gamma_{f}^{(k)}\right\}$ for such sequences are given by

$$
\Gamma_{a}^{(k)}=0, \Gamma_{b}^{(k)}=0, \Gamma_{c}^{(k)}=2, \Gamma_{d}^{(k)}=1, \Gamma_{e}^{(k)}=2, \Gamma_{f}^{(k)}=N
$$

Based on the results given above, the unique alternative that gives 2 is the second in (11). Thus, we have

$$
\begin{equation*}
\hat{N}_{k}=\Gamma_{a}^{(k)}+\Gamma_{b}^{(k)}+\Gamma_{e}^{(k)} \tag{12}
\end{equation*}
$$

Based on the relation developed, (8) can be rearranged as

$$
\begin{align*}
&{ }^{\text {new }} P_{e}^{(k)}=\frac{1}{2}-\sum_{d=1}^{L_{R}} \frac{\binom{2 d-2}{d-1}}{2^{2 d} \varepsilon_{m} \overline{S I N R}_{\text {new }}(2 d-3)} \\
&\left\{\left(b_{\text {new }}^{2}+1\right)^{\frac{3-2 d}{2}}-\left(a_{\text {new }}^{2}+1\right)^{\frac{3-2 d}{2}}\right\} \tag{13}
\end{align*}
$$

where $a_{\text {new }}=\left(1+\varepsilon_{m}\right) \overline{\operatorname{SINR}}_{n e w}$ and $b_{\text {new }}=\left(1-\varepsilon_{m}\right) \overline{\operatorname{SINR}}_{n \text { new }}$. The $\overline{\operatorname{SINR}}_{\text {new }}$ is the new version of the average signal to interference plus noise ratio per channel, given by

$$
\begin{align*}
\overline{\operatorname{SINR}}_{\text {new }}= & \left\{\frac{\gamma\left[\varsigma_{k}\left(1-e^{-\gamma}\right)+\gamma\left(1-\varsigma_{k}\right) e^{-\gamma}\right]}{\overline{\gamma_{b}}\left[2 \varsigma_{k}\left(1-e^{-\gamma / 2}\right)+\gamma\left(1-\varsigma_{k}\right) e^{-\gamma / 2}\right]^{2}}\right. \\
& \left.+\frac{\left(K L_{p}-1\right)\left(1+\varepsilon_{m}^{2} / 3\right) \Xi^{(e)}\left(\Gamma^{\left\{c_{j}^{(i)}\right\}}, \gamma\right)}{N\left[2 \varsigma_{k}\left(e^{\gamma / 2}-1\right)+\gamma\left(1-\varsigma_{k}\right)\right]^{2}}\right\} \tag{14}
\end{align*}
$$

where, $\quad \varsigma_{k}=\left(\Gamma_{a}^{(k)}+\Gamma_{b}^{(k)}+\Gamma_{e}^{(k)}\right) / N$. From the above discussion, we find that there is no need to determine the $\hat{N}_{k}$ on a random spreading sequence. Instead of, we can simply use the relation given by (12) to calculate the $\hat{N}_{k}$.

## IV. NUMERICAL EXAMPLES

In this section, we aim at numerically demonstrating the accuracy of the relation given by (12). It is widely known that, Gold sequences are useful for a multiple access system because of the large number of sequences they support. They can be chosen so that over a set of sequences available from a given generator, the crosscorrelation between the sequences is uniform and bounded [2]. Because of these fascinating features, we consider various Gold sequences of length $L=31, L=63$ and $L=127$, respectively. It is worth noting that the selection of these sequences was realised randomly.

Table I shows the calculated number of $\left\{\Gamma_{a}^{(k)}, \Gamma_{b}^{(k)}, \ldots, \Gamma_{f}^{(k)}\right\}, \quad \hat{N}_{k} \quad$ and $\quad\left(\Gamma_{a}^{(k)}+\Gamma_{b}^{(k)}+\Gamma_{e}^{(k)}\right)$ belonging to spreading sequences selected. We observe that the $\left(\Gamma_{a}^{(k)}+\Gamma_{b}^{(k)}+\Gamma_{e}^{(k)}\right)$ is equal to $\hat{N}_{k}$ for each sequence. Note that because the $\hat{N}_{k}$ can be calculated by using the sum of $\Gamma_{a}^{(k)}, \Gamma_{b}^{(k)}$ and $\Gamma_{e}^{(k)}$, there is no need to determine the $\hat{N}_{k}$ on a specific spreading sequence.

## V. CONCLUSION

A new BER performance expression of a coherent DSCDMA receiver which employs exponentially weighted despreading function for DS-CDMA communications over multipath Rayleigh fading channels is proposed.

It is emphasized that there is no need to calculate the number of transitions which occur between two consecutive chips to analyse the BER performance of a coherent receiver. As a result, the requirement on perfect definition of the spreading sequences for the signals of all active users is eliminated, successfully.

TABLE I
Calculated number of $\left\{\Gamma_{a}^{(k)}, \Gamma_{b}^{(k)}, \ldots, \Gamma_{f}^{(k)}\right\}, \hat{N}_{k}$ and $\left(\Gamma_{a}^{(k)}+\Gamma_{b}^{(k)}+\Gamma_{e}^{(k)}\right)$ for random
spreading sequences with various code length.

| $L$ | $\Gamma_{a}^{(k)}$ | $\Gamma_{b}^{(k)}$ | $\Gamma_{c}^{(k)}$ | $\Gamma_{d}^{(k)}$ | $\Gamma_{e}^{(k)}$ | $\Gamma_{f}^{(k)}$ | $\hat{N}_{k}$ | $\left(\Gamma_{a}^{(k)}+\Gamma_{b}^{(k)}+\Gamma_{e}^{(k)}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 31 | 5 | 10 | 6 | 3 | 1 | 6 | 16 | 16 |
| " | 9 | 6 | 6 | 5 | 5 | 0 | 20 | 20 |
| " | 1 | 6 | 10 | 3 | 5 | 6 | 12 | 12 |
| " | 3 | 6 | 6 | 7 | 7 | 2 | 16 | 16 |
| " | 1 | 6 | 14 | 1 | 5 | 4 | 12 | 12 |
| " | 9 | 10 | 10 | 1 | 1 | 0 | 20 | 20 |
| 63 | 2 | 12 | 12 | 10 | 10 | 17 | 24 | 24 |
| " | 8 | 16 | 16 | 8 | 8 | 7 | 32 | 32 |
| " | 4 | 8 | 24 | 4 | 12 | 11 | 24 | 24 |
| " | 4 | 24 | 16 | 8 | 4 | 7 | 32 | 32 |
| " | 16 | 16 | 8 | 12 | 8 | 3 | 40 | 40 |
| " | 12 | 24 | 8 | 12 | 4 | 3 | 40 | 40 |
| 127 | 24 | 40 | 32 | 12 | 8 | 11 | 72 | 72 |
| " | 16 | 24 | 32 | 12 | 16 | 27 | 56 | 56 |
| " | 14 | 28 | 36 | 18 | 22 | 9 | 64 | 64 |
| " | 12 | 24 | 40 | 12 | 20 | 19 | 56 | 56 |
| " | 22 | 36 | 28 | 18 | 14 | 9 | 72 | 72 |
| " | 22 | 28 | 36 | 10 | 14 | 17 | 64 | 64 |

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