A PATTERN SYNTHESIS METHOD FOR PLANAR ARRAYS WITH INDEPENDENT CONTROL OF SIDELOBE LEVEL AND BEAMWIDTH IN TWO PRINCIPAL PLANES

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ABSTRACT

This paper presents a method for the synthesis of planar array patterns, which enables a relatively independent control of sidelobe level and beamwidth in two principal planes. The method is based on a 'more conventional' definition of Chebyshev planar arrays, where the pattern is of Chebyshev type in only two cross sections, and employs a modified-Chebyshev technique that optimizes a few parameters leading to beamwidth adjustment in these planes for the specified sidelobe level. The design is suitable for applications that require low sidelobes and a controllable beamwidth, such as wireless applications. An expression for the excitation coefficients of the array elements is given and numerical examples are used to illustrate the properties of the proposed arrays.

I. INTRODUCTION

Among the different antenna array configurations, planar arrays provide additional variables that can be used to control and shape the pattern of the array. They can provide more symmetrical patterns with lower sidelobes. Their applications include tracking radar, search radar, remote sensing, wireless communications, and many others [1].

The conventional Chebyshev planar arrays [2, 3] are well known for providing sidelobes of equal magnitude. They are optimal in the sense that they provide the narrowest beam for a specified sidelobe level (SLL), but they suffer from directivity saturation when the number of array elements becomes large. Very narrow beams are required in some applications such as radars. Other applications, such as wireless applications, rather prefer the independent control of both the beamwidth and the SLL. In [4], the authors proposed a new design method for linear and circular arrays that enables the beamwidth to be enlarged to any desired degree from the minimum achieved with the classic Dolph-Chebyshev design [5]. This method, which is based on a modification of the Dolph-Chebyshev design, was extended in [6] to suit the design of planar arrays and in [7] to design wideband arrays.

In the work in [6] and [7], however, the beamwidth control was possible in only one principal plane of the planar array pattern. The beam in other cross sections remained narrower than prescribed. In this paper, a method is presented for the design of planar arrays that allows for the relatively independent control of the SLL and the beamwidth in two principal planes. This method relies on a "more conventional" definition of Chebyshev planar arrays, where the pattern is of Chebyshev behavior only in two principal sections, and uses an approach similar to that in [4] to enlarge the beamwidth in these sections. With the proposed method, the SLL and beamwidth in one principal plane can be different from those in the other plane, and in each the control of the beamwidth and sidelobe level is relatively independent.

II. A MORE CONVENTIONAL CHEBYSHEV PLANAR ARRAY

The normalized array factor of an $L \times L$-element conventional Chebyshev planar array is given by [3]

$$F_c(\theta,\phi) = \frac{1}{R} T_{L-1}\left(x_0 \cos u \ \cos v\right) \qquad (1)$$

where $T_N(x)$ is the N^{th} order Chebyshev polynomial, x_0 is such that $\left|T_{L-1}(x_0)\right| = R$, R is the sidelobe level ratio (SLR), and u and v are given by:

$$\begin{aligned} u &= \pi \frac{d}{\lambda}\left(\sin\theta\cos\phi - \sin\theta_0 \cos\phi_0\right) \\ v &= \pi \frac{d}{\lambda}\left(\sin\theta\sin\phi - \sin\theta_0 \sin\phi_0\right). \end{aligned} \qquad (2)$$

In (2), d is the inter-element spacing of the array, λ the wavelength, θ and ϕ are respectively the elevation and azimuth angles, and θ_0 and ϕ_0 determine the steering angle. A “more conventional” Chebyshev planar array has the following array factor [3]:

$$F(\theta,\phi)=\frac{1}{R_u}T_{L-1}\left(x_u\cos v\right)\cdot\frac{1}{R_v}T_{L-1}\left(x_v\cos u\right). \quad (3)$$

For this array, the Chebyshev pattern occurs only in the planes corresponding to $u=0$ and $v=0$. The SLR is R_u in the plane $u=0$ and R_v in $v=0$. The parameters x_u and x_v, which are responsible for the SLL in each plane are calculated from

$$\begin{aligned} x_u &= \cosh\left(\operatorname{acosh}\left(R_u\right)/\left(L-1\right)\right) \\ x_v &= \cosh\left(\operatorname{acosh}\left(R_v\right)/\left(L-1\right)\right) \end{aligned}. \quad (4)$$

The sidelobes in the other cross sections containing the direction of maximum radiations are also of equal magnitude, but with a different SLR equal to R_uR_v.

To derive the equation for the beamwidth in the section $u=0$, let θ_R and θ_L denote respectively the closest angles to the right and the left of the steering angle θ_0 such that $F(\theta_R,\phi_0)=F(\theta_L,\phi_0)=1/R_u$, and define the beamwidth BW as the width of the main lobe bounded by θ_R and θ_L. Therefore, BW is given by

$$BW=\begin{Bmatrix} \theta_R-\theta_L & if\ \theta_s\le\theta_{0\max} \\ \pi-\theta_R+\theta_L & if\ \theta_s>\theta_{0\max} \end{Bmatrix}, \quad (5)$$

where θ_R and θ_L, when ϕ_0 is a multiple integer of $\pi/2$, are given by [6]

$$\left.\begin{aligned} \theta_L &= \operatorname{asin}\left(\frac{-\operatorname{acos}\left(x_u^{-1}\right)}{\pi d}+\sin\theta_s\right) \\ \theta_R &= \operatorname{asin}\left(\frac{\operatorname{acos}\left(x_u^{-1}\right)}{\pi d}+\sin\theta_s\right) \end{aligned}\right\}\ \text{if } \theta_s\le\theta_{0\max}$$

$$\left.\begin{aligned} \theta_L &= \operatorname{asin}\left(\frac{-\operatorname{acos}\left(-x_u^{-1}\right)}{\pi d}+\sin\theta_s\right) \\ \theta_R &= \operatorname{asin}\left(\frac{-\operatorname{acos}\left(x_u^{-1}\right)}{\pi d}+\sin\theta_s\right) \end{aligned}\right\}\ \text{if } \theta_s>\theta_{0\max}, \quad (6)$$

and $\theta_{0\max}$ is given by

$$\theta_{0\max}=\operatorname{asin}\left(1-\frac{\operatorname{acos}\left(x_u^{-1}\right)}{\pi d}\right) \quad (7)$$

In (5) and (6), $\theta_s\triangleq\operatorname{acos}\left(\cos\left(\theta_0\right)\right)$. In the section $v=0$, the beamwidth is also given by (5) but with subscript *u* in (6) and (7) replaced by *v*.

For the case $\phi_0=(2n+1)\pi/4$, where n = 0,1,2,3, θ_R and θ_L can be easily derived, and for other values of ϕ_0, a numerical method should be used to obtain θ_R and θ_L.

III. PROPOSED DESIGN

A Chebyshev-like pattern is obtained using the function [4]

$$G_N(x,\alpha,\beta)=\begin{cases} \cos\left(N\left(\beta-e^{\alpha|x|}\right)\operatorname{acos}(x)\right) & \text{if } |x|<1 \\ \cosh\left(N\left(\beta-e^{\alpha|x|}\right)\operatorname{acosh}(x)\right) & \text{if } |x|\ge1 \end{cases} \quad (8)$$

where α and β are real parameters to be determined. Clearly, for $\alpha=0$ and $\beta=0$, (9) reduces to an N^{th} order Chebyshev polynomial.

The theoretical equation for the array factor of the proposed planar array will be

$$\begin{aligned} AF(\theta,\phi)=&\frac{1}{R_u}G_{L-1}\left(x_{pu}\cos v,\alpha_u,\beta_u\right) \\ &\cdot\frac{1}{R_v}G_{L-1}\left(x_{pv}\cos u,\alpha_v,\beta_v\right) \end{aligned} \quad (9)$$

In each principal section ($u=0$ or $v=0$), x_p is related to the sidelobe level ratio *R* and the beamwidth. Knowing x_p, α is iteratively obtained from [4]

$$\alpha_{k+1}=e^{\ln P-\alpha_k|x_p|} \quad (10)$$

where

$$P=\alpha_k e^{\alpha_k|x_p|}=\frac{\operatorname{acosh}(R)}{(L-1)\operatorname{acosh}^2(x_p)\sqrt{x_p^2-1}} \quad (11)$$

and β from the closed-form expression

$$\beta=e^{\alpha|x_p|}+\frac{\operatorname{acosh}(R)}{(L-1)\ \operatorname{acosh}\left(x_p\right)} \quad (12)$$

Subscript *u* or *v* should be added in (10-13) to denote the corresponding plane.

In terms of the excitation coefficients, the array factor of the proposed array is:

$$AF(\theta,\phi)=\begin{cases}4\sum_{m=1}^{L/2}\sum_{n=1}^{L/2} I_{mn}\cos(2m-1)u\cdot\cos(2n-1)v & L \text{ even}\\ \sum_{m=1}^{\frac{L+1}{2}}\sum_{n=1}^{\frac{L+1}{2}}\varepsilon_m\varepsilon_n I_{mn}\cos 2(m-1)u\cdot\cos 2(n-1)v & L \text{ odd}\end{cases}. \tag{13}$$

In (13), ε_m and ε_n are equal to 1 for $m=n=1$, to 2 for $m,n\neq 1$, and $I_{mn}=I_m I_n$ where I_m is

$$I_m=\begin{cases}\frac{2}{L\,R_v}\sum_{q=1}^{L/2}\Big\{G_{L-1}\Big[x_{pv}\cos\Big(\frac{\pi}{L}\big(q-\frac{1}{2}\big)\Big),\alpha_v,\beta_v\Big]\\ \qquad\cdot\cos\Big[\frac{2\pi}{L}\big(m-\frac{1}{2}\big)\ \big(q-\frac{1}{2}\big)\Big]\Big\}, & L \text{ even}\\ \frac{1}{L\,R_v}\sum_{q=1}^{\frac{L+1}{2}}\Big\{\varepsilon_q G_{L-1}\Big[x_{pv}\cos\Big(\frac{\pi}{L}(q-1)\Big),\alpha_v,\beta_v\Big]\\ \qquad\cdot\cos\Big[\frac{2\pi}{L}(m-1)\ (q-1)\Big]\Big\}, & L \text{ odd.}\end{cases} \tag{14}$$

I_n is obtained from (14) by replacing the subscript *v* by u.

When $N\left(\beta-e^{\alpha|x|}\right)\notin\mathbb{N}$, equation (8) does not correspond to a polynomial, and as a result, the array factor obtained from the excitations given in (14) is a truncated Fourier series of (9). Hence, the sidelobes in the two cross sections will not exactly be of equal magnitude. When ϕ_0 is a multiple integer of $\pi/2$, $\theta_{0\max}$ and the angles θ_R and θ_L in each section are as given in (7) and (6) respectively, but with x_0 replaced by x_p with the correct subscript.

DESIGN ALGORITHM FOR $\theta_s \leq \theta_{0\,max}$

To find the excitation currents of the proposed array, given by (14), the parameters α, β and x_p need to be determined. An algorithm is needed to optimize the value of x_p in each plane. This algorithm was given in [4] for the case of linear and circular array, and was adopted with few modifications in [6] to work with planar arrays. For convenience, this algorithm, for the case when ϕ_0 is a multiple integer of $\pi/2$ and $\theta_s \leq \theta_{0\max}$, is given below. For $\theta_s > \theta_{0\max}$, it can be found in [6], and the case $\phi_0=(2n+1)\pi/4$, where n = 0,1,2,3, can be easily derived.

For $\theta_s \leq \theta_{0\max}$, the beamwidth in each of the two principal planes, as defined in (5), can be written as

$$BW=\operatorname{asin}(A)-\operatorname{asin}(B) \tag{15}$$

where

$$A=\operatorname{acos}\left(x_p^{-1}\right)/\pi d+\sin(\theta_s)\,; \tag{16}$$

$$B=-\operatorname{acos}\left(x_p^{-1}\right)/\pi d+\sin(\theta_s)\,. \tag{17}$$

Equations (16) and (17) respectively yield:

$$x_p=1/\cos\left(\pi d\left(A-\sin(\theta_s)\right)\right); \tag{18}$$

$$x_p=1/\cos\left(\pi d\left(B-\sin(\theta_s)\right)\right). \tag{19}$$

Since $A+B=2\sin(\theta_s)$, as deduced from (16) and (17), the following iterative forms for *A* and *B* are obtained making use of (15):

$$A_{k+1}=2\sin(\theta_s)-A_k\cos(BW)+\sqrt{1-A_k^2}\,\sin(BW) \tag{20}$$

$$B_{k+1}=2\sin(\theta_s)-B_k\cos(BW)-\sqrt{1-B_k^2}\,\sin(BW)\,. \tag{21}$$

A recursive procedure to compute the x_p necessary to obtain the desired beamwidth for the specified steering angle is inferred from the above equations as follows:

1. Start with $x_p=x_0$ as computed from (4). x_0 is equal to x_u in the plane $u=0$, and to x_v in the plane $v=0$.
2. Compute B_0 from (17).
3. Update *B* using (21).
4. Recalculate x_p using (19).
5. Compute A_0 from (16).
6. Update *A* using (20).
7. Recalculate x_p using (18).
8. Return to step 3 unless there is convergence.

The maximum value of x_p corresponds to $\theta_s=0$, giving $A+B=0$, or $A=\sin(BW/2)$. Thus

$$x_{p\max}=1/\cos\left(\pi d\sin(BW/2)\right). \tag{22}$$

x_p is then upper-bounded by $x_{p\max}$ and lower-bounded by x_0, which makes the convergence of the above algorithm faster and more stable.

The above design was carried out and equations given assuming the same number of elements *L* and same spacing *d* in the two dimensions of the array. Minor changes are needed to account for the case where the configuration is rectangular with differing number of elements and inter-element spacing.

IV. RESULTS

As an illustrative example, we first consider a planar array with $L = 23$, $d = 0.5\lambda$, and $\theta_0 = \phi_0 = 0$. In the plane $\phi = 0$ (v = 0), the SLL is set to -30 dB $\left(R_v = 10\sqrt{10}\right)$, and in the plane $\phi = 90^0$ (u = 0), $R_u = 10$ (-20 dB SLL). The Dolph-Chebyshev design results in the smallest beamwidth of 13.7^0 in the first plane and 9.9^0 in the second plane, respectively. For a desired beamwidth $BW_v = 25^0$ in (v = 0), the following parameter values were obtained: $x_{pv} = 1.0607$, $\alpha_v = 0.051734$, and $\beta_v = 1.6$. The pattern in this plane is shown in Figure 1, where the pattern of the Dolph-Chebyshev design is also plotted. Figure 1 shows an enlargement in the main beam accompanied by a decrease in the number of sidelobes. In fact, the desired beamwidth is achieved and the specified SLL is respected.

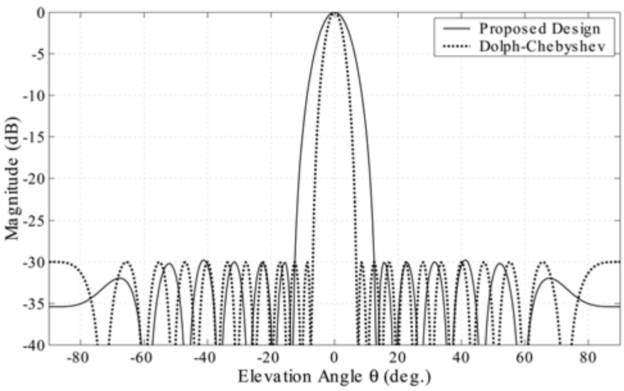

Figure 1. Patterns in the cross section $\phi = 0^0$ ($L = 17$, $\theta_0 = \phi_0 = 0^0$, $d = 0.5\lambda$, $R_v = 10^{3/2}$, and $BW_v = 25^0$)

For $BW_u = 20^0$, we have $x_{pu} = 1.0384$, $\alpha_u = 0.009046$ and $\beta_u = 1.502$. The patterns in this plane, for our design and the Dolph-Chebyshev design, are shown in Figure 2. Also in this plane, it is seen that the desired beamwidth is attained, and the sidelobe level is generally maintained.

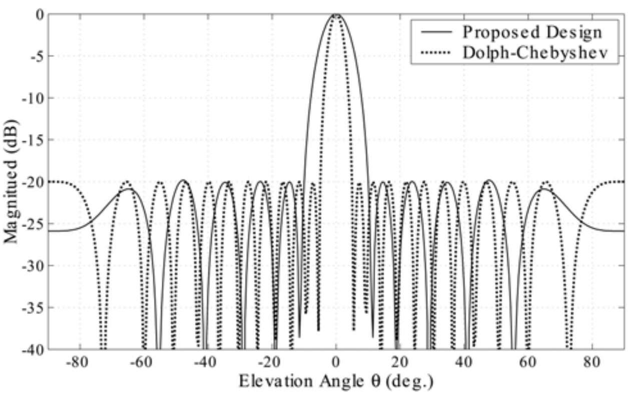

Figure 2. Patterns in the cross section $\phi = 90^0$ ($L = 17$, $\theta_0 = \phi_0 = 0^0$, $d = 0.5\lambda$, $R_u = 10$, and $BW_v = 20^0$)

For the next example, we take an array with $L = 15$, $\theta_0 = 35^0$ and $\phi_0 = 90^0$. We let $R_v = 10\sqrt{10}$, $R_u = 10$, $BW_v = 35^0$, and $BW_u = 30^0$. The 3D array factor plot is given in Figure 3. It is concluded that the sidelobes in the planes (v = 0) and (u = 0) are as prescribed, although not exactly of the same magnitude. In the other cross sections containing the direction of maximum radiation, the sidelobes are also *quasi*-ripple, but their SLL is -50 dB. The SLL in these directions is the product of the SLLs in the two principal planes (SLL is -30 dB for the first plane and -20 dB for the second). The main beam width in the two planes was adjusted as specified. This property of the proposed design enables to choose the size (area) of the region to be covered by the main beam while keeping radiation in the other directions below a desired level.

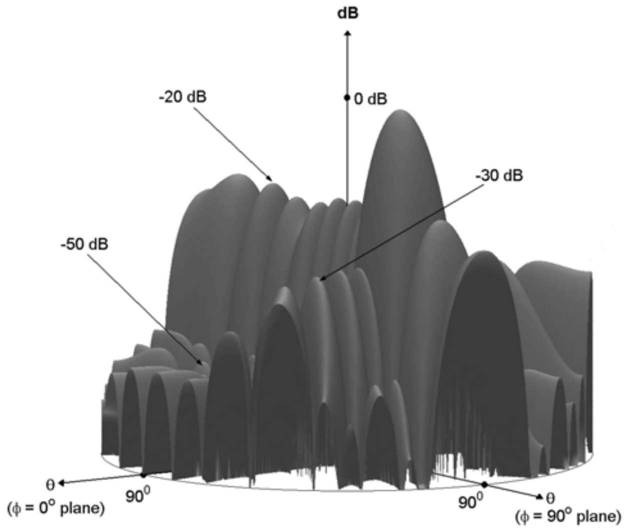

Figure 3. Three-dimensional pattern of the proposed planar array for $L = 15$, $d = 0.5\lambda$, $\theta_0 = 35^0$, $\phi_0 = 90^0$, $R_v = 10^{3/2}$, $R_u = 10$, $BW_v = 35^0$ and $BW_u = 30^0$

V. CONCLUSION

This paper presented a design method for planar arrays that permits a relatively independent control of the SLL and the beamwidth in the two principal planes corresponding to $u = 0$ and $v = 0$. Using a "more conventional" definition of Chebyshev planar arrays, where the pattern is of Chebyshev behavior only in two principal sections, the presented method employs a modified-Chebyshev technique to enlarge the beamwidth in these sections. The design is carried out separately for each of the two planes, and in each the beamwidth and sidelobe level can be adjusted with relative independence. The properties possessed by the resulting arrays make them suitable for many uses such as wireless applications.

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