A MODEL OF HOMOPOLAR SYNCHRONOUS GENERATOR FEEDING 3-PHASE BRIDGE RECTIFIER

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Abstract:

In this paper, a homopolar synchronous generator is modeled using the generalized machine theory. Then, this model is combined with a model of a three-phase uncontrolled bridge rectifier. The whole non-linear model is solved to predict the performance of the system.

Keywords: Homopolar synchronous generator, battery charger

1. Introduction

The synchronous generators are widely used to generate ac electrical power to supply the demand of costumers. These generators are commonly designed with slip rings in order to use the rotating field terminals with a stationary dc power supply. The carbon brushes are the most critical parts of the generator that may need maintenance. Therefore, the brushless types of generators are usually preferred for critical loads. These generators are usually consisting of permanent magnets in order to eliminate the rotating winding. While the permanent magnet eliminates the carbon brushes and slip rings, also it eliminates the winding and its terminals, which would be used as the control terminals. But the homopolar synchronous generator, as being different from the permanent magnet machines, has the field and armature winding terminals available and stationary.

Although the field winding in an ordinary synchronous generator is located on the rotor and the field winding of a homopolar synchronous generator is located on the stator, both machines share the same terminal characteristics, and therefore can be described with the same lumped parameters [3-5]. In the homopolar synchronous generator, the field winding is fixed to the stator and generally encircles the rotor rather than being placed on the rotor. The rotor pole faces on the upper part of the rotor are offset from the pole faces on the lower part, as it is shown in Figure 1.

In this machine, there are 6 poles at the upper and 6 poles at the lower parts of the rotor, with the lower poles rotated 30 degrees with respect to the upper

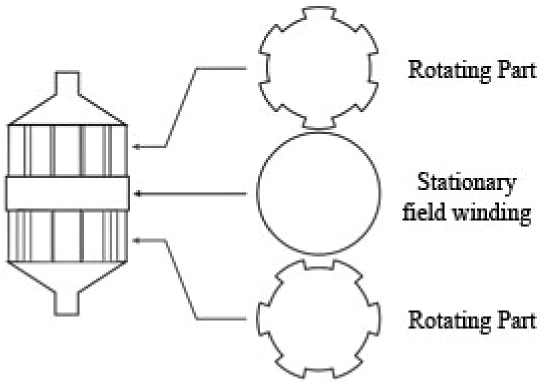

Figure 1 View of rotor structure

poles. The center portion of the rotor is cylindrical, and the field winding encircles this portion of the rotor. The 6 upper poles are all the same magnetic polarity (N), and the flux returns down through the back core iron to the lower set of poles (S). With this structure, this machine is also called “homopolar synchronous machine”. This machine has totally 12 poles and no saliency assumed, since larger part of the rotor encircled by cylindrical wound field coil.

Recently, the transportation vehicles have been equipped with electrical appliances totally over 5 kW. This power is supplied from a battery charged up by a generator coupled to the engine shaft. In this paper, a system is considered having a homopolar generator and uncontrolled bridge rectifier feeding power to a battery through a current limiting resistor. The generator terminal voltage is usually controlled via field current.

The rectifier input current harmonics and overlap angle are neglected. The input power factor is assumed to be unity. The entire system is modeled in rotor reference frame. The estimated values of stator currents, field current, rotor speed are obtained.

2. Model of the Generator

The homopolar synchronous machine is operated as a generator, therefore, it is convenient to assume that the direction of positive stator current is out of the terminals. This machine does not have any damper windings. With this convention the voltage equations in generator may be expressed in matrix form (1) and (2).

$$v_{abcs} = -R_s i_{abcs} + p\lambda_{abcs} \quad (1)$$

$$v_f = R_f i_f + p\lambda_f \quad (2)$$

The stator variables are transformed into the rotor reference frame which eliminates the time-varying inductances in the voltage equations. The speed of the reference frame is set to the rotor speed (ω=ω_r).

Here, the currents are selected as independent variables, the flux linkages are expressed in terms of currents, therefore, and the voltage equations (3) are obtained from the synchronous machine model given in [1].

$$\begin{bmatrix} v_{qs}^r \\ v_{ds}^r \\ e_{xf}^{\prime r} \end{bmatrix} = \begin{bmatrix} -R_s pL_s & -\omega_r L_s & \omega_r L_m \\ \omega_r L_s & -R_s pL_s & pL_m \\ 0 & -\frac{X_m}{R_f'}(pL_m) & \frac{X_m}{R_f'}(R_f' + pL_f') \end{bmatrix} \begin{bmatrix} i_{qs}^r \\ i_{ds}^r \\ i_f^{\prime r} \end{bmatrix} \quad (3)$$

where;

$$e_{xf}^{\prime r} = -\frac{X_m}{R_f'}(pL_m) i_{ds}^r + \frac{X_m}{R_f'}(R_f' + pL_f') i_f^{\prime r} \quad (4)$$

The electromagnetic torque equation is given below.

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(L_m i_{qs}^r i_f^{\prime r}\right) \quad (5)$$

The torque balance equation is

$$T_e = T_L + Jp\omega_r$$

2.1. Mathematical Model of Generator with Rectifier

The circuit representation of the system is given in Figure 2. The dynamic model of rectifier is obtained by considering the conservation of average real power at the input and output of rectifier. The overlap angle is neglected and the input power factor of rectifier is assumed to be unity. The rectifier input currents are assumed to be sinusoidal.

The equation (6) can be written at the output of rectifier.

$$v_{dc} = l_f p\, i_{dc} + r_f i_{dc} + v_i \quad (6)$$

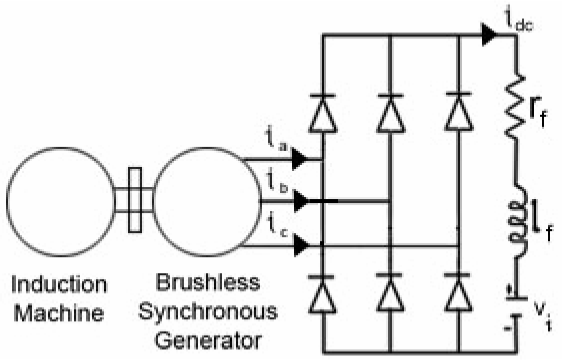

Figure 2 The generator with rectifier load

where, $v_{dc} = \frac{3\sqrt{3}}{\pi} v_s$

and $v_s = \left(v_{qs}^{r\,2} + v_{ds}^{r\,2}\right)^{1/2}$

As shown in the above equations, v_{dc} is the average rectifier output voltage and r_f, l_f, and v_i are the load resistance, inductance and battery voltage respectively. The average real power at the input and output of rectifier is given below;

$$\frac{3}{2}\left(v_{qs}^r i_{qs}^r + v_{ds}^r i_{ds}^r\right) = v_{dc} i_{dc} \quad (7)$$

The input reactive power of rectifier is

$$v_{ds}^r i_{qs}^r - v_{qs}^r i_{ds}^r = 0 \quad (8)$$

With the assumption that the rectifier input current is sinusoidal and the power factor is unity, the relation between the rectifier input currents and voltages may be given in terms of resistance.

$$v_{qs} = R\, i_{qs} \text{ and } v_{ds} = R\, i_{ds} \quad (9)$$

and

$$i_{dc} = \frac{\pi}{3\sqrt{3}} \frac{3}{2} \sqrt{i_{qs}^{r\,2} + i_{ds}^{r\,2}} \quad (10)$$

Taking the derivative of dc link current with respect to time,

$$\frac{d\, i_{dc}}{dt} = K\left(i_{qs}^{r\,2} + i_{ds}^{r\,2}\right)^{-1/2} \left(i_{qs}^r \frac{d\, i_{qs}^r}{dt} + i_{ds}^r \frac{d\, i_{ds}^r}{dt}\right) \quad (11)$$

hence,

$$i_{qs}^{r}\frac{d\,i_{qs}^{r}}{dt}+i_{ds}^{r}\frac{d\,i_{ds}^{r}}{dt}=\frac{\sqrt{{i_{qs}^{r}}^{2}+{i_{ds}^{r}}^{2}}}{K}\left(\frac{d\,i_{dc}}{dt}\right) \qquad (12)$$

where,

$$K=\frac{\pi}{3\sqrt{3}}\frac{3}{2}$$

The derivative of the dc current can be obtained from (6) as follows;

$$\frac{d\,i_{dc}}{dt}=\frac{1}{l_{f}}\left[\frac{3\sqrt{3}}{\pi}R\sqrt{{i_{qs}^{r}}^{2}+{i_{ds}^{r}}^{2}}-r_{f}i_{dc}-v_{i}\right] \qquad (13)$$

By solving the equivalent resistance R from (12) and (13), that might replace the rectifier operated at unity power factor, the following relation can be obtained.

$$R=\left(i_{qs}^{r}\frac{d\,i_{qs}^{r}}{dt}+i_{ds}^{r}\frac{d\,i_{ds}^{r}}{dt}\right)\frac{1}{{i_{qs}^{r}}^{2}+{i_{ds}^{r}}^{2}}l_{f}\frac{\pi^{2}}{18}+r_{f}\frac{\pi^{2}}{18}+\frac{v_{i}}{\sqrt{{i_{qs}^{r}}^{2}+{i_{ds}^{r}}^{2}}}\frac{\pi}{3\sqrt{3}} \qquad (14)$$

Derivative of the stator currents i_{qs}^{r} and i_{ds}^{r} from (3) are derived and given in (15) and (16).

$$\frac{d\,i_{qs}^{r}}{dt}=-\frac{1}{L_{s}}\left(R\,i_{qs}^{r}\right)-\frac{1}{L_{s}}\left(R_{s}i_{qs}^{r}+\omega_{r}L_{s}i_{ds}^{r}-\omega_{r}L_{m}{i_{f}'}^{r}\right) \qquad (15)$$

$$\frac{d\,i_{ds}^{r}}{dt}=\left(\frac{L_{f}'}{L_{s}L_{f}'-L_{m}^{2}}\right)\left(R\,i_{ds}^{r}\right)+\left(\frac{L_{f}'}{L_{s}L_{f}'-L_{m}^{2}}\right)\left(-\omega_{r}L_{s}i_{qs}^{r}+R_{s}i_{ds}^{r}\right)+\left(\frac{L_{m}R_{f}'}{L_{s}L_{f}'-L_{m}^{2}}\right)\left(e_{xf}'-X_{m}{i_{f}'}^{r}\right) \qquad (16$$

Finally, the equations (15) and (16) are substituted into (14) and the value of R is solved in terms of the system parameters and rectifier input current.

$$R=\left\{\frac{\pi^{2}}{18}l_{f}\frac{1}{{i_{qs}^{r}}^{2}+{i_{ds}^{r}}^{2}}\left[-\frac{i_{qs}^{r}}{L_{s}}\left(R_{s}i_{qs}^{r}+\omega_{r}L_{s}i_{ds}^{r}-\omega_{r}L_{m}{i_{f}'}^{r}\right)-\frac{L_{f}'}{L_{s}L_{f}'-L_{m}^{2}}i_{ds}^{r}\left(-\omega_{r}L_{s}i_{qs}^{r}+R_{s}i_{ds}^{r}\right)+\frac{L_{m}R_{f}'}{X_{m}\left(L_{s}L_{f}'-L_{m}^{2}\right)}i_{ds}^{r}\left(e_{xf}'-X_{m}{i_{f}'}^{r}\right)\right]r_{f}\frac{\pi^{2}}{18}+\frac{v_{i}}{\sqrt{{i_{qs}^{r}}^{2}+{i_{ds}^{r}}^{2}}}\frac{\pi}{3\sqrt{3}}\right\}\Bigg/\left(1+\frac{{i_{qs}^{r}}^{2}}{L_{s}}\frac{1}{{i_{qs}^{r}}^{2}+{i_{ds}^{r}}^{2}}l_{f}\frac{\pi^{2}}{18}+\frac{L_{f}'}{L_{s}L_{f}'-L_{m}^{2}}\frac{{i_{ds}^{r}}^{2}}{{i_{qs}^{r}}^{2}+{i_{ds}^{r}}^{2}}l_{f}\frac{\pi^{2}}{18}\right) \qquad (17)$$

If the smoothing inductance is neglected, then the equivalent resistance may be obtained as follows;

$$\Rightarrow\; R=\frac{\frac{3}{2}r_{f}}{\left(\frac{27}{\pi^{2}}\right)}+\frac{\left(\frac{3\sqrt{3}}{\pi}V_{i}\right)}{\left(\frac{27}{\pi^{2}}\right)}\frac{1}{\sqrt{{I_{qs}^{r}}^{2}+{I_{ds}^{r}}^{2}}}$$

When the rectifier output is connected to just one resistance, i.e. battery and inductance are both omitted, then

$$R=\frac{\pi^{2}}{18}r_{f}$$

3. Torque-Load Angle Equation of Generator with Rectifier

In order to obtain torque-load angle equation of generator with rectifier at steady state, the voltage equations are obtained at synchronously rotating reference frame [4]. The stator voltage equations are given in (18) and (19).

$$v_{qs}=\left(-R_{s}-pL_{s}\right)i_{qs}-\omega L_{s}i_{ds}+\left(\omega L_{m}\mathrm{Cos}\left(\theta_{r}-\theta\right)+pL_{m}\mathrm{Sin}\left(\theta_{r}-\theta\right)\right)i_{f}' \qquad (18)$$

$$v_{ds}=\left(-R_{s}-pL_{s}\right)i_{ds}+\omega L_{s}i_{qs}+\left(-\omega L_{m}\mathrm{Sin}\left(\theta_{r}-\theta\right)+pL_{m}\mathrm{Cos}\left(\theta_{r}-\theta\right)\right)i_{f}' \qquad (19)$$

Where $\theta=\int_{0}^{t}\omega(\xi)d\xi+\theta(0)$ and ω is the speed of reference frame which is the synchronous speed, $\theta_{r}=\int_{0}^{t}\omega_{r}(\xi)d\xi+\theta_{r}(0)$ and ω_{r} is the speed of rotor.

The q axis of the stator windings is positioned such that it coincides with the positive peak value of stator phase a voltage. Therefore, the v_{qs} component of phase voltage is equal to this peak value and v_{ds} is zero. At steady state operation, the differential portions are canceled out, v_{ds} and i_{ds} become zero at unity power factor. The quadrature axis current is obtained from (19) as follows:

$$\omega L_{s}i_{qs}=\omega L_{m}i_{f}'\mathrm{Sin}\delta \quad \text{where } \delta=\theta_{r}-\theta$$

hence,

$$i_{qs}=\frac{L_{m}}{L_{s}}i_{f}'\mathrm{Sin}\delta$$

Then, this current is substituted into the torque equation in (20) obtained at synchronously rotating reference frame in [4].

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left[L_m i'_f \left(i_{qs}\cos\delta - i_{ds}\sin\delta\right)\right] \qquad (20)$$

Hence,

$$T_e = \frac{3}{2}\frac{P}{2} L_m i'_f i_{qs} \cos\delta$$

Finally, the electromagnetic torque equation is obtained in terms of load angle.

$$T_e = \frac{1}{2}\frac{3}{2}\frac{P}{2}\frac{e_f^2}{\omega X_s}\sin 2\delta$$

where e_f is the peak value of back emf per phase and

$$e_f = \omega L_m i'_f$$

The maximum load angle is 45 degrees for the stable operation of the system.

4.Simulation Results

The model given in (3), (9) and (17) is solved for a 3-phase, 2.5 kW, 28 V, 12 poles, 3000 rpm machine having the following parameters:

R_s= 0.0303 Ω L_s= 0.000318 H L_m= 0.000237 H

$R'_f = 0.00318\ \Omega$ $L'_f = 0.000726$, J= 0.048 J.s^2

$l_f = 0$, $r_f = 1\Omega$, $v_i = 24\ V$

The rotor is driven at no-load by a prime mover at 3000 rpm. The field current is set to a value such that when the rectifier is connected to the stator terminals, the generator demands 8.0 N.m of torque. The stator current, field current, rotor speed, load angle and electromagnetic torque are predicted under this loading condition during first 3 seconds. After then, the shaft torque is reduced to 4.0 N.m. The predicted stator q-axis current, stator d-axis current and field current are given in figures 3, 4 and 5. By reducing the load torque from 8 to 4 N.m, the stator current decreases. The field current changes due to the mutual coupling between stator and rotor, when the load torque is varied suddenly as it can be seen from figure 5.The rotor speed decreases as the input torque of the generator is reduced, as being shown in figure 6. T he load angle in figure 7 decreases as the input torque is reduced. The maximum load angle in the stable operating region is 45 electrical degrees. The electromagnetic torque, given in figure 8, follows the shaft torque.

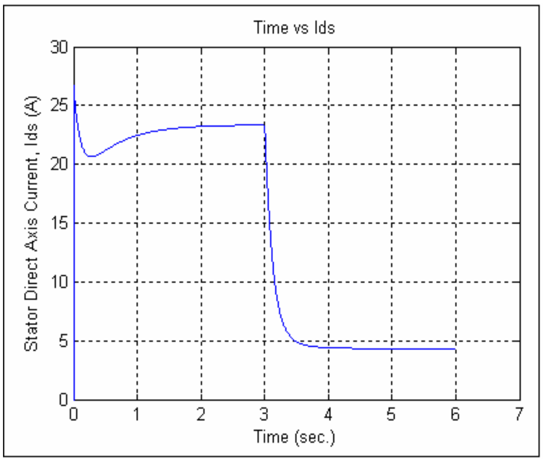

Figure 3 Stator quadrature axis current, Iqs

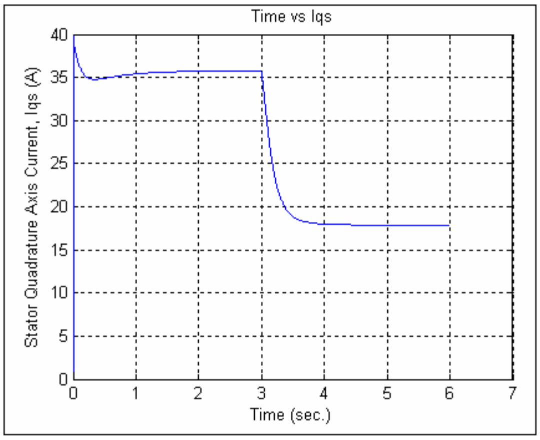

Figure 4 Stator direct axis current, Ids

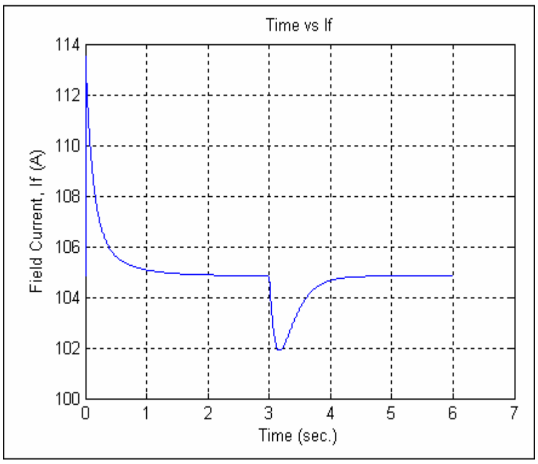

Figure 5 Field Current, If

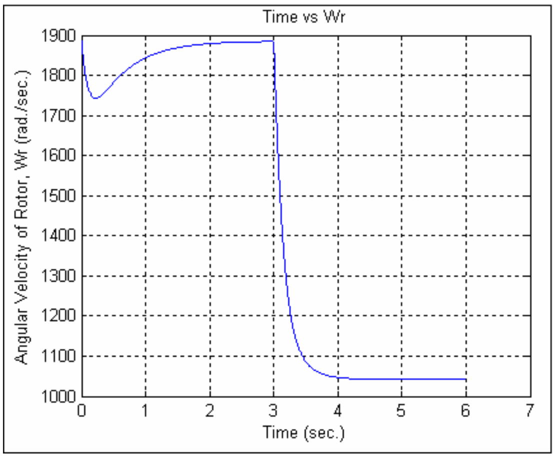

Figure 6 Angular velocity of rotor, Wr

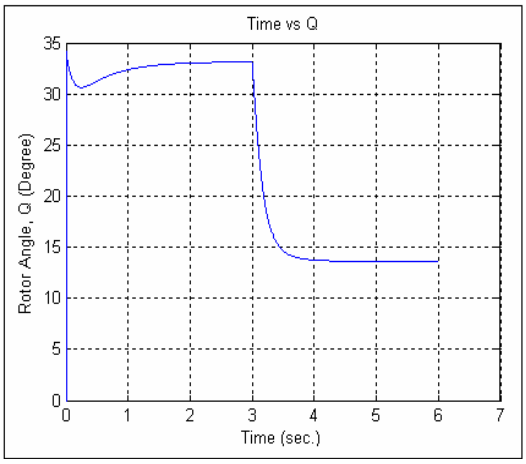

Figure 7 Rotor Angle, Q

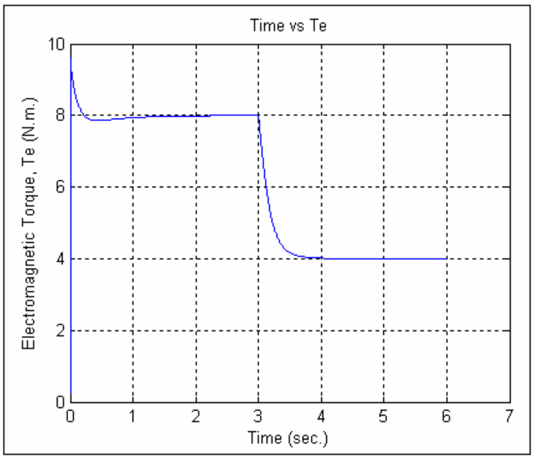

Figure 8 Electromagnetic Torque, Te

5. Conclusions

In this article, a homopolar synchronous generator has been modeled and analyzed. In order to obtain the model of machine, the standard abc/qd model of cylindrical rotor synchronous machine has been used. The mathematical model of the rectifier has been derived and combined with machine model. The overlap angle and rectifier input current harmonics are neglected. The simulation results during the transient and steady-state operation have been obtained for the system. Also, the model yields a formulation for the steady-state analysis.

References

[1] Krause, P.C.,& Wasynczuk, O.,& Sudhoff, S.C. Analysis of electric machinery. New York: McGraw-Hill Book Company: IEEE Press, 1995.

[2] Krause, P.C.,& Wasynczuk, O. Electromechanical motion devices. New York: McGraw-Hill Book Company, 1989.

[3] Tsao, P.,& Senesky, M.,& Sanders, S.R. "An Integrated Flywheel Energy Storage System With Homopolar Induction Motor/Generator and High Frequency Drive", IEEE Transactions on IA, Vol. 39, No: 6, Dec. 2003.

[4] Budakçı, D. "Brushless Synchronous Generator with Wound Field and Its Modeling", M. Sc Thesis, DEÜ, December 2004.

[5] Tsao, P.,& Senesky, M.,& Sanders, S.R. "A Homopolar Machine for High Speed Applications." Conference Record of the IEEE IAS Annual Meeting, 30,110- 116, 2002.

List of Variables:

J	:Inertia constant.
i^r_{qs}, i^r_{ds}	stator currents in the rotor reference frame.
i'^r_f	Field current referred to the stator side.
r_f	Resistance of the smoothing reactor.
R'_f	Resistance of field winding,.
R_s	Per phase resistance of stator winding, pu.
T_L	Load torque.
v^r_{qs}, v^r_{ds}	:stator voltages in the rotor reference frame.
v_s	Peak value of the source phase voltage.
l_f	Inductance of the smoothing reactor
L_m	Mutual inductance between stator and field.
L_s	Self inductance of stator winding.
L'_f	Self inductance of field winding.
X_m	Mutual reactance between stator and field.
w_r	Electrical angular rotor speed, rad/sec.