# *Orbiting Small Flexible Satellites: Switching Based Conrols Relevance in Attitude Stabilization **


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***Abstract*—Earth is being orbited by a number of man-made satellites for various purposes, a considerable number of which are small satellites such as the micro- and pico- ones for civilian purpose. Their main application purposes are communications, image scanning, and Earth exploration scanning. This work explores the attitude control of symmetric, constant-mas, pico-satellite and introduces sliding-mode, switching based nonlinear control to confine the potential attitude oscillation to arbitrary small limit cycle in the vicinity of desired zero steady state equilibrium. An example of symmetric pico-satellite, along with a phase-plane portrait and some simulation samples, is investigated to illustrate the improved performance that may be achieved. Further, a mathematical model for a general case of real-world symmetric flexible spacecraft for future research is also given.**



***Keywords—attitude control; nonlinear control; sliding-mode control; pico-sattelites; special gyroscopes; thuster actuators***


## I. INTRODUCTION

Centuries ago Jean-Jacque Rousseau, famous philosopher of French Enlightenment, has stated: "Who dares to say this far man can go but not a step further!" Indeed advances of sciences and arts, as the foundations of cultures, and technologies, as the living needs of Mankind, demonstrated unprecedented developments during the last two centuries. In particular, in this work the word is about engineering sciences and the arts of systems engineering and technologies. For, these address important rational (and sometimes irrational) aspects of human societies since can be turned not only to technological systems supporting the quality of life but also into the ever existing human desire for expansion beyond real-world limits. By and large such is the case of developments in applied control designs needed for aerospace engineering applications. The present paper is a follow-up, improved re-elaboration on a previous paper by the first three authors [17].

It is well known that there are orbiting the Earth a number of artificial satellites [1], [3], [6], [7] for various purposes (e.g. see Fig. 1) among which there are very many small satellites for civilian purpose, in particular some micro- and pico-satellites [3], [5], [6], [7], [16]. Typically, besides espionage and surveillance, the main civilian areas of applications are communication purposes [5], [9], [14]. The overall motion dynamics of any space structure on the orbit appears as a compatible, compound 3D motion relative to the base frame of gravity centre in space (e.g., planet Earth) and relative to its own frame at its mass centre [3], [21], [24]. In general, functions of communications, image scanning, and remote scanning exploration of Earth [3], [5] are assigned to a range of various pico-satellites. In addition, to these are used for various ad-hock applications since as they are of low cost, and even dispensable as a rule. Thus these represent small space structures that may or may not exhibit flexible deviations [3]. [9], [23], [26].

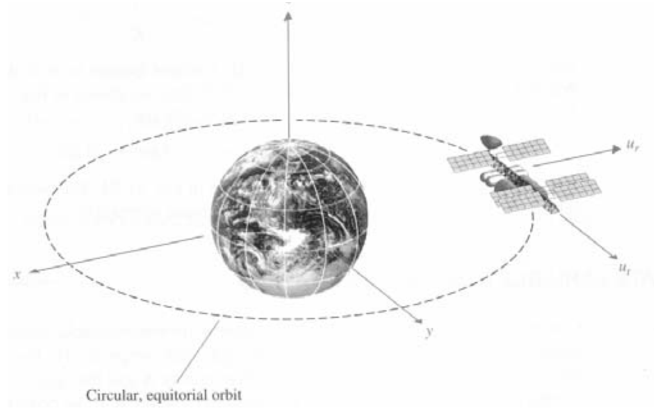


Fig. 1 A schematic illustration of a small satellite on orbit possessing autonomous power supply by solar panel arrays [7].

The rest of this paper is written as follows. The next section is focused on introducing the most important features of the inherently nonlinear dynamics of satellite's oscillations about its vertical axis and the troublesome issues involved in their attitude control. A reasonably accurate nonlinear model of the system dynamics a symmetric constant-mass pico-satellite on the equatorial orbit is presented in Section III along with one classical state-feedback attitude control solution based on an linear approximation of the dynamics of satellite's oscillations. Then in Section IV, there is discussed a potential application of switching based control law synthesis in addition to the traditional pole-placement state feedback design. Conclusion and references are given thereafter.


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## II. On Implications Of Mechanical Oscillation And Vibrations

Deep observation of the movement and its association with the inner dynamics of any mechanical structure constitute one of the keystones of understanding phenomena of oscillations, shocks, vibrations and their consequences [3], [9], [19], [21]. One may well recall, for instance, the horrifying consequences following an earthquake stroke (see Fig. 2) such as the one that hit Skopje about half a century ago. In front of our eyes, it has changed the entire life not only in the city and its environs but also within the entire little Republic of Macedonia. This is one of the reasons why so many research efforts have been devoted worldwide to the control of nonlinear oscillations and vibrations, in general, and in space structures, in particular [3], [6], [9], [18] for a long time by now; for instance see [1], [10].

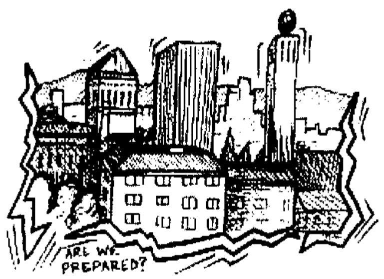


Fig. 2 Danger from vibrations [5], [16]: The case of an earthquake stroke as the devastating one that hit Skopje on July 26, 1963.

Indeed of understanding both spatial and temporal phenomena of nonlinear oscillations, shocks, vibrations and their consequences emerges as a most significant matter in attitude control of real-world satellites and flexible spacecraft structures. In addition, since temporal and spatial compliance with the mission tasks while orbiting Earth are in the foreground of satellites the attitude control must be tight [3].

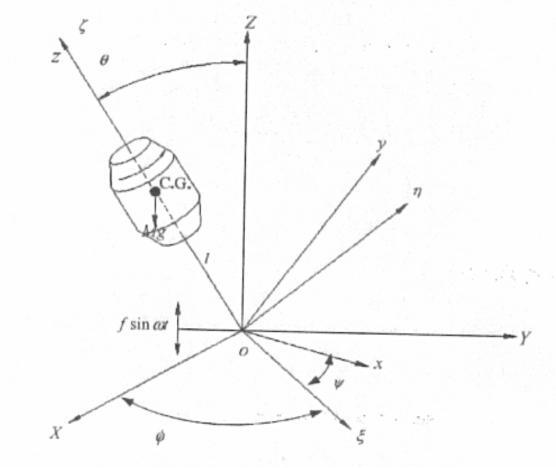


Fig. 3 A schematic diagram along with the local unit-vector frame for orientation reference [3], [21], [23].

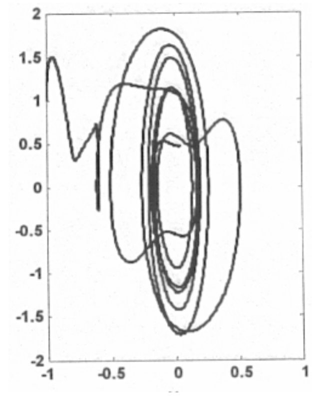


Fig. 4 Phase-plane portrait of the system dynamics that ought to be suppressed to the feasible minimum magnitude limit cycle [1], [3], [16], [19] in order calm orbiting and avoiding danger from local vibrations to be guaranteed.

It should be also noted that controlled synchronization too emerges as a significant issue to in the case of deploying multiple satellites to perform some specific mission on orbit. Naturally, in such a case matter spatial and temporal and spatial dynamic phenomena of orbiting satellites become rather complex since critical systems are created out there.

## III. On Nonlinear System Dynamics, Linear Approximation And Classical Control Designs

B. H. Chen (2002) has sown in [2], when the background base is vibration affected a reasonable model of the angle of controller attitude can be found as follows:

$$\ddot{\theta}(t)+\{[\alpha(1-\theta(t)]^2/[\sin\theta(t)]^3-\beta\sin\theta(t)\}+ \\ +c_1\dot{\theta}(t)+c_2\dot{\theta}(t)^3 = f\,\sin(\omega t)\sin\theta(t). \quad (1)$$

In here the individual terms represent as follows. The term

$$[\alpha(1-\theta(t)]^2/[\sin\theta(t)]^3-\beta\sin\theta(t)= \\ =\alpha^2 g[\theta(t)]-\beta\sin\theta(t)$$

is representing the nonlinear resilience force; term $f\,\sin(\omega t)$ describes an induced excitation; and terms $c_1\dot{\theta}(t)$ and $c_2\dot{\theta}(t)^3$, respectively, model the effects of linear and nonlinear damping due to dissipative force. Naturally, the desired equilibrium steady-state is characterized by zero vertical oscillations $\theta=0$ and $\dot{\theta}=0$. Due to symmetric construction the precession and the spin angles exhibit motions the momentum integrals of which are mutually approximate equal and counter-balanced. Therefore equation (1) describes the governing the attitude motion reasonably well. Dynamic model of a symmetric pico-satellite in the phase plane $x_1Ox_2$, $x_1 \triangleq \theta$, $x_2 \triangleq \dot{\theta}$, $\vec{x}=[x_1\ x_2]^T \in \aleph^2$ and $\vec{x}(t)\in\mathbb{R}^2$ at $\forall t$ fixed (i.e. state-space representation of plant dynamics) thus appears as

$$\dot{x}_1(t)=x_2(t),$$

$$\dot{x}_2(t) = [\beta + f\sin(\omega t)]\sin[x_1(t)] - \alpha^2 g[x_1(t)] - \\ - c_1 x_1(t) - c_2 x_2^3(t) + M\,u(t), \quad (2)$$
$$x_i(0) = x_i^0, i = 1,2.$$

Notice that term $M\,u(t)$ represents resulting torque-momentum generated by satellite's actuators, control $u = u(t;\vec{x}_0)$ acting on its mass $M$ effectively. Apparently, (1) and (2) describe an inherently essential nonlinear dynamics that exhibits a delicate nature.

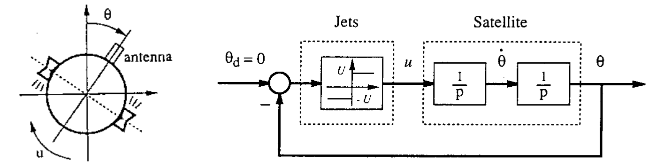


Fig. 5 Conventional sattellite attitude control system [1], [10], [17] , [18] for a symmetric, constant mass, sattelite with jet thrusters on orbit based on ideally linearized model of the satelite's attitude dynamics.

Upon the assumption of small magnitude angular motion of the operating satellite, $\theta \approx \delta\theta = x_1$ and $\dot{\theta} \approx \delta\dot{\theta} = x_2$, nonlinear dynamics can be can be linearized to

$$\dot{\vec{x}}(t) \cong [\partial\vec{f}(x_1,x_2)/\partial\vec{x}]_{\vec{x}=\vec{x}_e}\vec{x}(t) + [\partial\vec{f}(x_1,x_2)/\partial u]_{\vec{x}=\vec{x}_e}u(t) = \\ = A(\vec{x})_e\vec{x}(t) + B(\vec{x})_e u(t), \quad (3)$$

with

$$[\partial\vec{f}(x_1,x_2)/\partial\vec{x}]_{\vec{x}(0)=\vec{x}_0} = \begin{bmatrix} 0 & 1 \\ \partial f_2/\partial x_1(\vec{0}) & \partial f_2/\partial x_2(\vec{0}) \end{bmatrix}. \quad (4)$$

The latter yields further approximated model in the vicinity of equilibrium state vector $\vec{x}_e = \vec{0}$ in terms of equations

$$\dot{x}_1(t) = x_2(t)\ , \quad \dot{x}_2(t) = M\,u(t). \quad (5)$$

It is well known from the classical research monographs [1], [17] and advanced textbooks [6], [10], [14] that one possible solution is the bang-bang (i.e., the ideal relay), also called ideal on-off control. It was shown in [1], [18] that the bang-bang control represents the optimal control with respect to the time since it enforces the fastest transition from the given initial to the desired final state; e.g., such as desired equilibrium steady state of satellite's attitude in fact is. This somewhat idealized but more elaborate qualitative solution is well illustrated in Figure 6, constructed by using the method of isoclines. The phase portrait of the satellite sates indicated the system main remain operating on a rather narrow limit cycle according to magnitudes of the initial states. Thus it has sensitive dependence on the initial states $x_i(0) = x_i^0, i = 1,2$ that may be caused by possible disturbances.

Nonetheless, it can be also shown that by appropriate system engineering re-design of the plant system model, given in Fig. 5, a theoretically a simple bang-bang control can be designed that enforces the zero-state equilibrium in closed loop. This modified solution but effective time-optimal attitude control, which is found using the method of phase-plane isoclines and vector field of systems dynamics [4], [6], [18], is illustrated in Fig. 6.

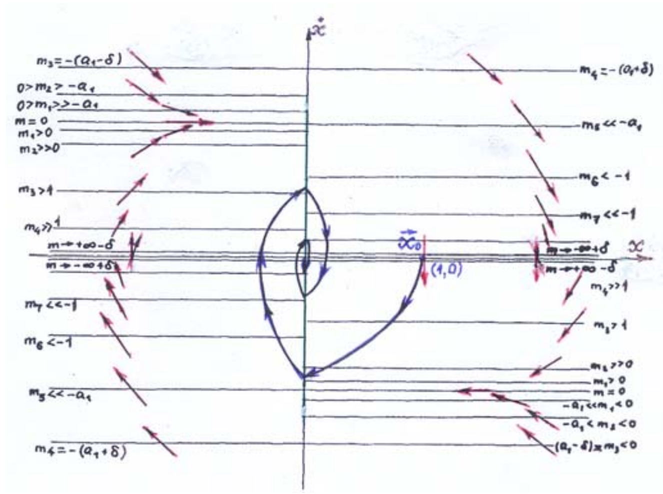

Figure 6. An elaborated quailtative solution to the bang-bang control of satellite atttude via the method of isoclines [1], [4] so as to achieve asymptotic stability of the zero-state equilibrium by finite number of switchings in finite time.

It should be noted, however, for this purpose some slight modification on the satellite body is needed too so as to make the unit coefficients in its transfer function model parameters amenable to minor adjustments. This design is demonstrated by the appropriate distribution of the isoclines as well as the vector field of system dynamics in system's phase plane portrait (Fig. 6) so as to make effective the use of the standard bang-bang control. Notice that this control law is the most elementary switching based control.

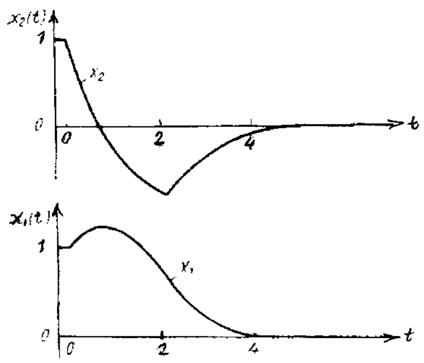


Fig. 7 State-transition trajectries in closed loop with a feedback state-regulator design LQ-optimal solution Pontryagin's Maximum Principle [18].

Also, several variants of state-feedback regulator designs for LQ-optimal [1], [9], [18] control gain matrix (via either solving Lyapunov or Riccati matrix equations) are known in the literature. The state transition trajectories of the first such LQ-optimal control design solution via Pontryagin's

Maximum Principle [17] are given in Figure 7 above. As seen these too demonstrate asymptotic stability of the zero-state equilibrium, which is only asymptotically achievable hence cannot be guaranteed in finite time.

## IV. An Application Of Swittching Based Controls To Sattellite Attitude Stabilization

In here the focus is placed on employing switching based control synthesis [13], [20] within the context of state-feedback regulator stabilization [1], [6], [10]. When subject to an especially synthesized, advanced switching control, the switching-based activation/de-activation sequence of its jet-thrusters, i.e. controlling actuators, then it is possible to reduce successfully to an insignificant measure the limit cycle of local oscillations and thus ensure the desired operation of the satellite on its orbit. Yet, synthesis of the needed switching law is not a straightforward task despite the great scientific advances in switched systems and control theory during the last couple of decades, e.g. see [10] as well as [11], [12] and [15].

The present contribution is a further elaboration on switching-based satellite attitude control based on previous work [5], [16] by the authors on the grounds of the ideas and theoretical results in [11], [12] and [15] as well as [6]. For this purpose, nonlinear state equations (2) of satellite attitude dynamics

$$\begin{cases} \dot{x}_1(t) = f_1(x_1, x_2) = 1\cdot x_2(t), \\ \dot{x}_2(t) = f_2(x_1, x_2; g, \alpha, \beta, c_1, c_2) + Mu(t), \end{cases} \tag{5}$$

where $f_1 : \mathbb{R}_0^+ \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ and $f_2 : \mathbb{R}_0^+ \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ are transformed to

$$\begin{cases} \dot{x}_1^{\sigma(t)}(t) = x_2^{\sigma(t)}(t) + f_1^{\sigma(t)}(x_1^{\sigma(t)}, x_2^{\sigma(t)}), \\ \dot{x}_2^{\sigma(t)}(t) = K\, u_{\sigma(t)}(t) + f_2^{\sigma(t)}(x_1^{\sigma(t)}, x_2^{\sigma(t)}), \end{cases} \tag{6}$$

with $f_1^{\sigma(t)} : \mathbb{Z}_0^+ \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ and $f_2^{\sigma(t)} : \mathbb{Z}_0^+ \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ switched functions according to the discrete-events generated by switching function $\sigma = \sigma(t) : [0, \infty) \to S = \{1, 2\}$. It should be also noted, the control gain in the second equation of (6) is determined on the grounds of state-feedback regulator design

$$u(t) = -K\vec{x}(t) \quad \text{hence} \quad \dot{\vec{x}}(t) = [A(\vec{x})_e - B(\vec{x})_e K]\vec{x}(t), \tag{7}$$

using the state-dependent linearized model (3) via the LQ-optimal control theory [ ], namely

$$\frac{\partial\{\frac{1}{2}x^T(0)Px(0)\}}{\partial k_{jk}} = 0, \; j,k = 1,2, \tag{8}$$

along with Lypunov matrix equation

$$[A(\vec{x}) - B(\vec{x})K]^T P + P[A(\vec{x}) - B(\vec{x})K]^T + K^T RK + Q = O, \tag{9}$$

so as to satisfy both closed-loop stability and sufficient minimization conditions. Here, $P$ is a positive definite matrix whose elements are functions of gains $k_{jk}$ in $K$, $R$ is a chosen positive definite matrix while $Q$ is a chosen positive semi-definite matrix.

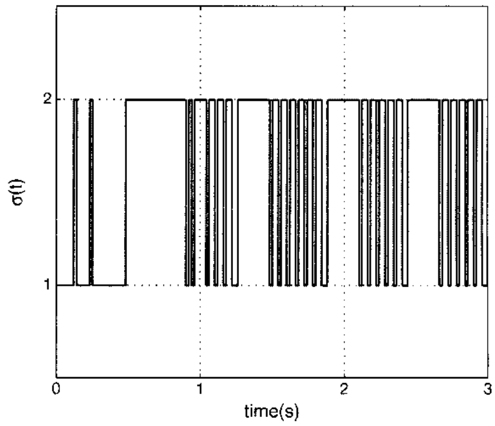


Fig. 8 Novel switching-based control design to facilitate the attitudes control via state-regulator stabilization the gain of wwhich is computed throuh Lyapunov matrix equation [1], [9], [17].

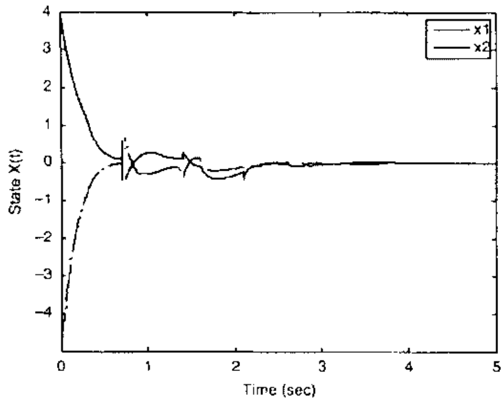


Fig. 9 State-transition trajectories in closed loop under a switching-based feedback state-regulator design (6)-(10): In 3 seconds all the transeints have disappeared.

Then a hysteresis-type, switching law is synthesized over the overlapping regions $\Omega_i$, $\Omega_j$ in phase plane $x_1 O x_2$ of the satellite, $x_1 \triangleq \theta$ and $x_2 \triangleq \dot{\theta}$ delimited by switching surfaces $S_{i,j}$, $i = 1,2$ and $j = 1,2$ [12]. In such a solution, $\sigma = \sigma(t)$ caused discrete event drives the control sequence continuous from the right everywhere in the phase plane. In formal mathematical terms, the designed switching law [12] is given as follows:

$$\textit{Whenever } t = 0, \; \sigma(0) = \min\arg\{\Omega_i, \Omega_j\};$$

$$\textit{Whenever } t > 0,$$

$$\sigma(t) = i,\ if\ \vec{x}(t) \notin \Omega_i\ and\ \sigma(t^-) = i,$$
$$if\ \vec{x}(t) \in \Omega_i\ and\ \sigma(t^-) = i. \tag{10}$$

*Whenever* $t > 0$,

$$\sigma(t) = \min\arg\{\Omega_i, \Omega_j : \vec{x}(t) \in \Omega_j\},$$
$$if\ \vec{x}(t) \notin \Omega_i\ and\ \sigma(t^-) = i.$$

It should be noted though, the implementations of such a switching law based satellite attitude control requires a high-precision, especially balance-calibrated, compound gyroscope sensing-measuring feedback [3], [9], [14]. The schematic diagram of such compound gyroscope employing three single gimbal control moment gyros is depicted in Figure 10.

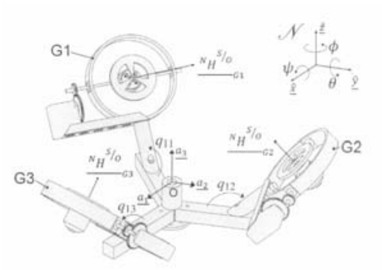


Fig. 10 Schematic diagram of a high-precision, compound gyroscope sensor-transducer [3].

Still, it is believed rather important to give an outline for the most important future research application. Namely, the real challenging task for future research is to explore and find switching based sliding-mode and state-feedback solution to the attitude control of a flexible satellite [3], [9], [25], [27], which is illustrated by Figure 11 and the outline given in the sequel. With right-handed body reference frame $O_{x_b y_b z_b}$ at zero attitude angles $O_{y_b}$ is normal to the orbital plane, $O_{z_b}$ is aligned with the vector from the spacecraft to Earth hence $\vec{g}$, $O_{x_b}$ is parallel to the velocity vector; in this reference frame the two solar arrays are aligned with plane $O_{x_b y_b}$ hence their installation axes are both parallel to the pitch axis.

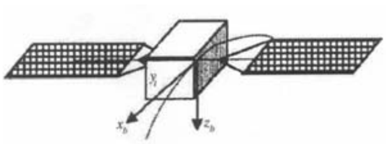

Figure 11. For future research challenge: A more realistic schematic of a symmetric, constant mass pico-satellite on orbit around Earth [27] .

From the analytical mechanics, for the attitude dynamics of a flexible spacecraft (Fig. 10) in Lagrange formalism

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\vec{\omega}}}\right) - \hat{\omega}\frac{\partial L}{\partial \vec{\omega}} = M(\vec{u}),\quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\vec{\eta}}}\right) - \frac{\partial L}{\partial \vec{\eta}} = 0, \tag{11}$$

via using the analysis methodology in [24], it may well be derived the rigorous 3D model of any orbiting satellite motion. Namely, the model in question, for all $i = 1, 2, ..., m$, in given [25] as follows:

$$\begin{cases} I_x\ddot{\varphi} + [(I_y - I_z - I_x)\omega_0 - h_y]\dot{\psi} + \\ \quad +[(I_z - I_y)\omega_0\varphi + h_z]\dot{\theta} + (I_z - I_y)\dot{\psi}\dot{\theta} + \\ \quad +[4(I_y - I_z)\omega_0^2 - h_y\omega_0]\varphi - h_z\omega_0 + \dot{h}_x + \\ \quad +\sum_{i=1}^{i=n} F_{sxi}\ddot{\eta}_{pi} = M_x^e(\vec{v}), \end{cases} \tag{12}$$

$$\begin{cases} I_y\ddot{\theta} + [(I_x - I_z)\dot{\varphi} + (I_z - I_x)\omega_0\psi + h_x]\dot{\psi} + \\ \quad +[(I_x - I_z)\omega_0\dot{\varphi} + (I_z - I_x)\omega_0^2\psi + h_x\omega_0]\varphi + \\ \quad +h_z\dot{\varphi} + h_z\omega_0\psi + 3\omega_0^2(I_x - I_z)\theta + \dot{h}_y + \\ \quad +\sum_{i=1}^{i=n} F_{syi}\ddot{\eta}_{pi} = M_y^e(\vec{v}), \end{cases} \tag{13}$$

$$\begin{cases} I_z\ddot{\psi} + [(I_x + I_z - I_y)\omega_0 + h_y]\dot{\varphi} + \\ \quad +[(I_x - I_y)\omega_0\psi + h_x]\dot{\theta} + (I_y - I_x)\dot{\varphi}\dot{\theta} + \\ \quad +[(I_y - I_x)\omega_0^2 - h_y\omega_0]\psi + h_x\omega_0 + \dot{h}_z + \\ \quad +\sum_{i=1}^{i=n} F_{szi}\ddot{\eta}_{pi} = M_z^e(\vec{v}), \end{cases} \tag{14}$$

$$\ddot{\eta}_{pi} + 2\xi_{pi}\omega_{pi}\dot{\eta}_{pi} + \omega_{pi}^2\eta_{pi} + F_{sxi}^T(\ddot{\varphi} - \omega_0\psi) + \\ + F_{syi}^T\ddot{\theta} + F_{szi}^T(\ddot{\psi} - \omega_0\dot{\varphi}) = 0. \tag{15}$$

since the environmental torque can be decomposed into Earth gravitational, aerodynamic, magnetic, solar-radiation pressure, and solar and lunar gravitation torques. In addition to Lagrange function, notice that: $\omega = [\omega_x\ \omega_y\ \omega_z]^T$ is the angular rate; $\hat{\omega}$ is the anti-symmetric matrix of $\omega$; $M(\vec{u})$ represents generalized torque which included both actuation control toques and environmental torques (acting disturbances); $I_x$, $I_y$; $I_z$ are the principle moments of inertia about roll, pitch and yaw axes; respectively $\varphi$, $\theta$ and $\psi$ are the roll, pitch and yaw attitude angles; $M_x^e$, $M_y^e$ and $M_z^e$ are the acting torques excluding the gravitational torque; $F_{sxi}$, $M_{syi}$ and $M_{szi}$ are the coupling matrices between the attitude and vibration modes of the two solar arrays; $\omega_{pi}$ is the i-th modal frequency of the two solar arrays; $\eta_{pi}$ is the i-th vibration mode coordinate of the two solar arrays; $\xi_{pi}$ is the i-th vibration damping coefficients of the two solar arrays; $\omega_0$ is the orbital angular velocity; $h_x$, $h_y$ and $h_z$ are the angular momentums of fly-wheels installed along the $O_{y_b} -$, $O_{z_b} -$ $O_{x_b} -$ axes; $\vec{u} = [u_{c_1}\ u_{c_2}\ u_{c_3}]^T$ is vector of the inner control torque that along with the required attitude maneuver angles, which is generated by installed motors in the spacecraft. The only assumption adopted is about the zero cross-products inertia among roll, pitch and yaw axes, $\varphi$, $\theta$ and $\psi$.

## V. Concluding Remarks And Future Research

It has been shown in this paper how effective the sophisticated switching based control theory may be in the foreseen applications to inherently nonlinear, oscillatory, dynamic systems such as aircrafts and space-crafts in motion. It has thus provided some tangible insights towards practical implementations. Also it has been demonstrated how much it is necessary nowadays to carry out theoretical research studies on sophisticated advances is systems and control and find out how to transcend them into engineering application domains. Thus, it is believed not only it contributed new highlights on such sophisticated theories but also to techniques of such applications oriented studies.

The illustrative example has provided for the needed evidence about achievable closed-loop performance by employing the relevant synthesis of switching based controls [5], [12], [17], [20]. One category of the advanced switching based controls is the celebrated sliding-mode control synthesis within the framework of variable structure control systems [22], [23], which is a topic for a future research.

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