ERROR ANALYSIS OF A STEWART PLATFORM

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ABSTRACT

The use of parallel structures, such as a Stewart platform (SP), for machine tools is a current trend. Some powerful analytical tools are needed to analyse the effect of errors in parallel structures. A model of a SP was analysed to include all the sources of errors [1]. We developed a similar error model to a Stewart platform and a Modified Stewart platform (MSP). Both SP and MSP designs have the same error models. The model developed here is intended for the design of a controller to reduce the effects of the errors due to temperature and stress state of the material.

I. INTRODUCTION

Stewart [2] developed SP about 40 years ago. In this early model was developed as a flight simulator for pilot training and used hydraulic actuators. However, in the early correspondences it was clear that Gough developed the structure in 1948 for tire testing. Gough [3] did not publish his work until 1962. Many researchers study the SP due to its simple structure, large payload capacity, high stiffness and accuracy.

The interference constraints between the legs is a disadvantage of the original SP. Due to this limitation, legs cannot go far from the z-axis and as a result the applied torque about the z-axis is limited contrary to xy-plane. However, the force in z-direction is higher than xy-plane since the force applied by each leg should act along the axis of the leg. (See Fig.1) In case of a Modified Stewart Platform, the more torque can be applied about the z-direction because the MSP has less interference constraints on the legs. The basic idea of MSP design is to place the legs on two different concentric circles [4] as shown in Fig.1.

In recent years, the machine tool industry uses a design that is completely different from the multi-axis machine design. The new structure has six degree of freedom (DoF) and it is based on the Stewart Platform. The parallel nature of SP is expected to be more rigid and accurate from the conventional designs.

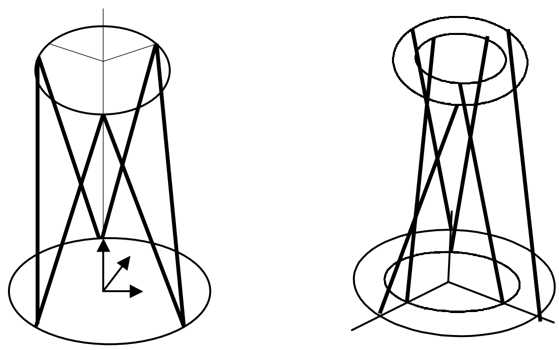

Figure 1 Stewart Platform (left) and MSP (right)

Error models developed for conventional designs may not be suitable for the new design structure. Wang and Masory [5] presented an error model for SP by modelling the legs as a serial kinematic chain using the DH-convention to investigate the effects of the manufacturing errors. Ropponen and Arai [6] created an error model using the differentiation of kinematic equations, while Wang and Ehmann [7] developed another model for SP employing differential changes in leg lengths. Patel and Ehmann [1] developed an error model that can address all the possible error sources, such as manufacturing errors, control error, thermal and stress strains.

In this paper, we first present the nominal and accurate models of SP and MSP that are similarly formulated as in [1], and then the differential error model is developed in section IV. Finally, we present some concluding remarks.

II. NOMINAL MODELS

A Stewart Platform consists of six variable length legs that connect a stationary base (we will call it 'base') to a moving platform by spherical or universal joints. For the nominal model of the Stewart Platform, the following assumptions are made:

i. Each joint has a center about which perfect rotation can occur,
ii. The positions of joint centers are known precisely,

iii. The actuators have only 1-DoF and their motion pass through the joint centers and
iv. The length of each leg can be measured without error.

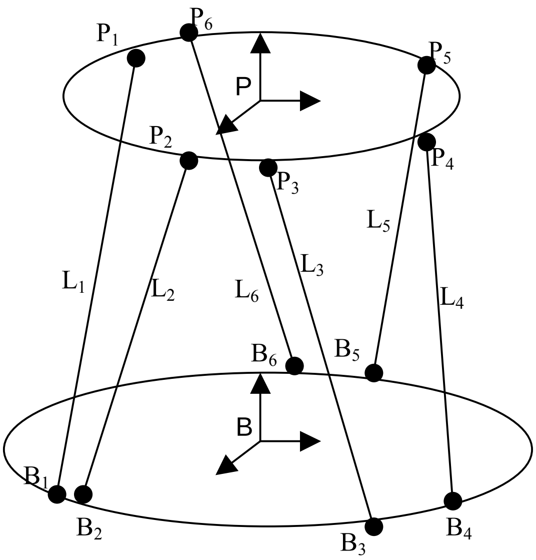

Figure 2 Nominal Model of a Stewart Platform

Two coordinate frames denoted by *B* and P are arbitrarily placed in the base and the platform, respectively (See Fig.2). Fig.2 and Fig.3 illustrate the following definitions. The vectors are denoted in bold case letters. The origin of a coordinate frame, B, is denoted by *0*B. The left

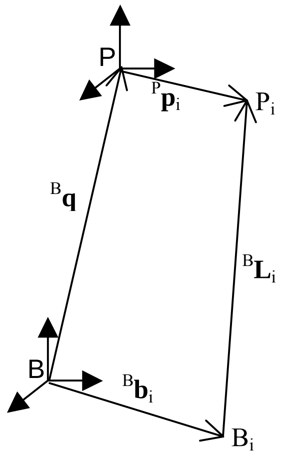

Figure 3 Vector Relations for the i^{th} Leg

superscripts, such as $^{B}\mathbf{L}_i$, denote the reference coordinate frame for the corresponding vector. The joints on the base and the platform are denoted by B_i and P_i, respectively, where the subscript $i \in \{1,2,3,4,5,6\}$. Script B_i and P_i denote the joint coordinate frames that are placed at joints B_i and P_i, respectively. The vectors $^{B}\mathbf{b}_i$ denote the positions of the base joint centers with respect to B and the positions of the platform joint centers are given by the vectors $^{P}\mathbf{p}_i$. A leg vector $^{Bi}\mathbf{L}_i$ that is defined as a vector from B_i to P_i with length L_i is assigned for the i^{th} leg. The lengths L_i are measurable.

The vector, $\mathbf{q}={}^{B}\mathbf{q}$, shows the position of the origin *0*P of the platform with respect to the base, and the rotation matrix $^{B}\mathbf{R}_P$, shows the orientation of the platform with respect to the base.

Following Patel and Ehmann [1], $^{B}R_P$ is expressed in the roll-pitch-yaw (RPY) angles, ϕ, θ, and ψ. Since the RPY angles completely determine the orientation matrix $^{B}R_P$, one can simply determines the position and orientation of the platform with respect to the base, provided that the generalized position vector $\wp=[\mathbf{q}^T, \Omega^T]^T$, where $\Omega=[\phi,\theta,\psi]^T$, is given. To control the operation of SP (and MSP), the generalized position $\wp$ and the leg lengths $\mathbf{L}=[L_1, L_2, L_3, L_4, L_5, L_6]^T$ should be known, in general.

The two cases arise:

1. When L is known, find the generalized position $\wp$ (forward kinematics problem).
2. When $\wp$ is known, find the leg lengths (inverse kinematics problem).

The inverse kinematics problem yields a closed form solution, while the forward kinematics problem generally does not lead to a closed form solution, as this is the case for parallel structures contrary to the serial manipulators.

From Fig.3, the i^{th} leg vector $^{Bi}\mathbf{L}_i$ can be written as,

$$^{Bi}\mathbf{L}_i=L_i{}^{Bi}\mathbf{a}_i={}^{Bi}R_B({}^{B}R_P{}^{P}\mathbf{p}_i+\mathbf{q}-{}^{B}\mathbf{b}_i) \tag{1}$$

where $^{Bi}\mathbf{a}_i$ is a unit vector in the direction of the i^{th} leg and $^{B}R_{Bi}$ is the rotation matrix of the i^{th} joint frame with respect to the base. By taking the Euclidian norm of both sides of equation (1) and using the fact that $^{Bi}\mathbf{a}_i$ is a unit vector, the inverse kinematics solution is obtained as

$$L_i=\|{}^{Bi}\mathbf{L}_i\|=\|{}^{B}R_P{}^{P}\mathbf{p}_i+\mathbf{q}-{}^{B}\mathbf{b}_i\| \tag{2}$$

Up to this point, what was done is to derive the nominal model of a Stewart platform. It was not mentioned to the model of a MSP at all. However, if we look at Fig.2, it is clear that the nominal model of MSP is included in the model of SP, because the joints are shown at different points on the borders of the base and the platform (for a SP, three joint positions is enough for each plane as it can easily be seen from Fig.1). In case of a MSP, the only difference is that two concentric circles each containing three joints should be placed both on the base and the platform. The definitions of vectors and coordinate frames will be same. Therefore, the nominal model for SP is directly applicable to MSP without any modification. Similar discussion is valid for the accurate models.

III. ACCURATE MODELS

In general, the assumptions made for the nominal model are not valid. The position of the tool unavoidably differs from the nominal model. Some of the error sources are faults in manufacturing and assembly, kinematic errors in the actuators, thermal and elastic deformations of the material, errors introduced by the control system, measurement errors, etc.

Now, we introduce an accurate model based on the Patel-Ehmann (PE) model given in [1]. The actual model

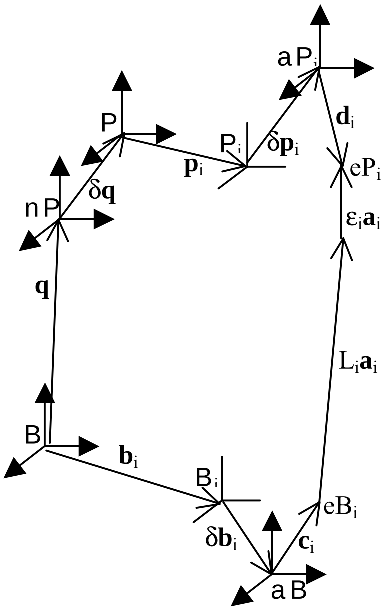

Figure 4 Accurate Vector Relations for the i^{th} Leg

introduced here differs from PE model by the introduction of the generalized position error $\Delta\wp$ on the platform coordinates.

If the assumptions of the nominal model are not valid, then the vector relations shown in Fig.3 considerably changes. With the introduction of error vectors, the vector relations become as shown in Fig.4. At the base, a coordinate frame B_i is placed in the nominal location of the i^{th} joint. At the same time, a frame aB_i is placed at the actual location of the joint. P_i and aP_i are defined similarly at the platform. In the platform, it is introduced another coordinate frame, denoted by nP, at the nominal position of the platform coordinates in order to include the error in the generalized position $\wp$ of the tool, while the coordinate frame P represents the generalized position $\wp$. The vectors $\mathbf{b}_i$ and $\mathbf{p}_i$ give the nominal positions of the joints. The vectors $\delta\mathbf{b}_i$ and $\delta\mathbf{p}_i$ denote the joint location errors that are simply defined as vectors that give the position of the $0aB_i$ and $0aP_i$ with respect to B_i and P_i, respectively. The points eB_i and eP_i show the end points of the actuators, which are no longer coincident with the joint centers. The offset vectors $\mathbf{c}_i$ and $\mathbf{d}_i$ represent the errors in the joints. The leg vectors, which connect the points eB_i and eP_i, consist of two components $L_i\mathbf{a}_i$ and $\varepsilon_i\mathbf{a}_i$ where $\mathbf{a}_i$ is the unit vector along the i^{th} leg and ε_i is the amount of error for i^{th} leg length L_i.

Accurate determination of the error vectors $\delta\mathbf{b}$, $\delta\mathbf{p}$, $\mathbf{c}$, $\mathbf{d}$, and $\varepsilon\mathbf{a}$ is difficult and the relation of $\Delta\wp$ and $\wp$ is nonlinear due to nonlinearity of the rotation matrix. When the errors are known, $\Delta\wp$ has no effect on the equations, because the actual position $a\mathbf{q}$ of $0P$ can be defined as

$$ {}^{B}a\mathbf{q}={}^{B}\mathbf{q}+{}^{B}R_{nP}{}^{nP}\delta\mathbf{q}. \tag{3}$$

From Fig.4, mathematical relations of the vectors are stated as

$$ {}^{B}\mathbf{b}_i+{}^{B}R_{Bi}\{{}^{Bi}\delta\mathbf{b}_i+{}^{Bi}R_{aBi}{}^{aBi}\mathbf{c}_i+(L_i+\varepsilon_i){}^{B}\mathbf{a}_i\}-{}^{B}\mathbf{q} \\ -{}^{B}R_{nP}({}^{nP}\delta\mathbf{q}+{}^{nP}R_{P}\{{}^{P}\mathbf{p}_i-{}^{P}R_{Pi}[{}^{Pi}\delta\mathbf{p}_i+{}^{Pi}R_{aPi}{}^{aPi}\mathbf{d}_i]\})=\mathbf{0} \tag{4}$$

where the R matrices denote rotation matrices between the frames. Rearranging equation (4) and taking the Euclidian norm of the arranged equation, the inverse kinematic equations can be expressed as

$$ L_i=\|{}^{B}R_{nP}({}^{nP}\delta\mathbf{q}+{}^{nP}R_{P}\{{}^{P}\mathbf{p}_i+{}^{P}R_{Pi}[{}^{Pi}\delta\mathbf{p}_i+{}^{Pi}R_{aPi}{}^{aPi}\mathbf{d}_i]\}) \\ +{}^{B}\mathbf{q}-{}^{B}\mathbf{b}_i-{}^{B}R_{Bi}\{{}^{Bi}\delta\mathbf{b}_i+{}^{Bi}R_{aBi}{}^{aBi}\mathbf{c}_i\}\|-\varepsilon_i. \tag{5}$$

Using equation (5), one can compute the leg lengths of a SP (MSP) provided that the generalized position and the other error vectors are known.

The error vectors $\delta\mathbf{q}$, $\delta\mathbf{b}$, $\delta\mathbf{p}$, $\mathbf{c}$, $\mathbf{d}$, and $\varepsilon\mathbf{a}$ determine the accurate model. Normally, it is necessary to model the errors accurately. In general, the errors are functions of stress F, temperature T, command set S and time t [1]. It is also possible that the errors are functions of the generalized position and may depend each other. This means that although the actuator length L_i has a closed form solution, it is likely that equation (5) does not have an analytic solution. Numerical methods should be employed in order to solve the equations. However, the computational burden is too much.

The direct relations between error sources and the position errors are not well-known. Due to this, the assumption of completely known errors is unrealistic. In general, the error vector is a function of stress state F of the material, the temperature T, the friction forces, time t and other undesired environmental or internal effects.

IV. DIFFERENTIAL ERROR MODEL

To obtain the differential error model, the nominal inverse kinematics model is used. The differential error model formulates the position error $\Delta\wp$ in terms of other error components. With this formulation, it is possible to see how the error vectors affect the accuracy of the SP. First, equation (1) is differentiated to yield

$$dL_i{}^{Bi}\mathbf{a}_i+L_i d({}^{Bi}\mathbf{a}_i)=d({}^{Bi}R_B)({}^{B}R_P{}^{P}\mathbf{p}_i+\mathbf{q}-{}^{B}\mathbf{b}_i)+{}^{Bi}R_B(d({}^{B}R_P){}^{P}\mathbf{p}_i+{}^{B}R_P d({}^{P}\mathbf{p}_i)+d(\mathbf{q}-{}^{B}\mathbf{b}_i)). \qquad (6)$$

Equation (6) includes the differentials of rotation matrices ${}^{Bi}R_B$ and ${}^{B}R_P$. The differential of a rotation matrix R can be written as $dR=(d\Omega)_\times R_d$, where R_d is the desired rotation matrix, $d\Omega=[\omega_1,\omega_2,\omega_3]^T$ is the orientation error vector and the subscript $\times$ defines the cross operator matrix given by

$$\lambda_\times=\begin{bmatrix}0 & -\lambda_3 & \lambda_2\\ \lambda_3 & 0 & -\lambda_1\\ -\lambda_2 & \lambda_1 & 0\end{bmatrix} \qquad (7)$$

where $\lambda=[\lambda_1,\lambda_2,\lambda_3]^T$ is a vector. Applying equation (7) to the differentials of rotation matrices, equation (6) becomes

$$dL_i{}^{Bi}\mathbf{a}_i+L_i d({}^{Bi}\mathbf{a}_i)=(d\Omega_{Bi})_\times{}^{Bi}R_{Bd}({}^{B}R_P{}^{P}\mathbf{p}_i+\mathbf{q}-{}^{B}\mathbf{b}_i)+ \qquad (8)$$
$${}^{Bi}R_B((d\Omega_P)_\times{}^{B}R_{Pd}{}^{P}\mathbf{p}_i+{}^{B}R_P d({}^{P}\mathbf{p}_i)+d(\mathbf{q}-{}^{B}\mathbf{b}_i))$$

where ${}^{Bi}R_{Bd}$ and ${}^{B}R_{Pd}$ are the desired rotation matrices. Finally, multiplying both sides of equation (8) by $({}^{Bi}\mathbf{a}_i)^T$, and using the fact that ${}^{Bi}\mathbf{a}_i$ is a unit vector, the following is obtained

$$dL_i=({}^{Bi}\mathbf{a}_i)^T(d\Omega_{Bi})_\times{}^{Bi}R_{Bd}({}^{B}R_P{}^{P}\mathbf{p}_i+\mathbf{q}-{}^{B}\mathbf{b}_i)+ \qquad (9)$$
$$({}^{Bi}\mathbf{a}_i)^T({}^{Bi}R_B)\{(d\Omega_P)_\times{}^{B}R_{Pd}{}^{P}\mathbf{p}_i+{}^{B}R_P d({}^{P}\mathbf{p}_i)+d(\mathbf{q}-{}^{B}\mathbf{b}_i)\}.$$

Note that if ${}^{Bi}R_{Bd}={}^{Bi}R_B$, then from equation (1), first term in the right hand side of equation (9) becomes

$$({}^{Bi}\mathbf{a}_i)^T(d\Omega_{Bi})_\times{}^{Bi}R_B({}^{B}R_P{}^{P}\mathbf{p}_i+\mathbf{q}-{}^{B}\mathbf{b}_i)=({}^{Bi}\mathbf{a}_i)^T\{(d\Omega_{Bi})\times L_i{}^{Bi}\mathbf{a}_i\}=\mathbf{0}$$

because this is a dot product of two orthogonal vectors. Rearranging equation (9), the following expression is obtained

$$dL_i=({}^{Bi}\mathbf{a}_i)^T\{(d\Omega_{Bi})\times L_i{}^{Bi}\mathbf{a}_{id}\}+({}^{Bi}\mathbf{a}_i)^T({}^{Bi}R_B)d\mathbf{q} \qquad (10)$$
$$+({}^{Bi}\mathbf{a}_i)^T({}^{Bi}R_B)\{(d\Omega_P)\times({}^{B}R_{Pd}{}^{P}\mathbf{p}_i)\}$$
$$({}^{Bi}\mathbf{a}_i)^T({}^{Bi}R_B)\{{}^{B}R_P d({}^{P}\mathbf{p}_i)-d({}^{B}\mathbf{b}_i)\}$$

where $L_i{}^{Bi}\mathbf{a}_{id}={}^{Bi}R_{Bd}({}^{B}R_P{}^{P}\mathbf{p}_i+\mathbf{q}-{}^{B}\mathbf{b}_i)$. Using the vector identity $\mathbf{v}^T(\mathbf{r}\times\mathbf{s})=(\mathbf{s}\times\mathbf{v})^T\mathbf{r}$, equation (10) becomes

$$dL_i=({}^{Bi}\mathbf{a}_i)^T\{(d\Omega_{Bi})\times L_i{}^{Bi}\mathbf{a}_{id}\}+({}^{Bi}\mathbf{a}_i)^T({}^{Bi}R_B)d\mathbf{q} \qquad (11)$$
$$+\{({}^{B}R_{Pd}{}^{P}\mathbf{p}_i)\times({}^{Bi}\mathbf{a}_i)\}^T({}^{Bi}R_B)d\Omega_P$$
$$({}^{Bi}\mathbf{a}_i)^T({}^{Bi}R_B)\{{}^{B}R_P d({}^{P}\mathbf{p}_i)-d({}^{B}\mathbf{b}_i)\}.$$

Equation (11) can be represented in matrix form as

$$d\mathbf{L}=J\Delta\wp+\aleph\, d\mathbf{A}+\Im \qquad (12)$$

where

$$d\mathbf{L}=[dL_1, dL_2, dL_3, dL_4, dL_5, dL_6]^T \qquad (13)$$

$$J=\begin{bmatrix}{}^{B1}a_1^T\,{}^{B1}R_B & [({}^{B}R_{Pd}{}^{P}p_1)\times{}^{B1}a_1]^T\,{}^{B1}R_B\\ \vdots & \vdots\\ {}^{B6}a_6^T\,{}^{B6}R_B & [({}^{B}R_{Pd}{}^{P}p_6)\times{}^{B6}a_6]^T\,{}^{B6}R_B\end{bmatrix}\in\Re^{6x6} \qquad (14)$$

$$\Delta\wp=[d\mathbf{q}^T, d\Omega_P{}^T]^T \qquad (15)$$

$$\aleph=\begin{bmatrix}\aleph_1 & 0 & \cdots & 0\\ 0 & \aleph_2 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \ldots & \aleph_6\end{bmatrix}\in\Re^{6x36} \qquad (16)$$

$$d\mathbf{A}=[d({}^{P}\mathbf{p}_1)^T, d(\mathbf{b}_i)^T, \ldots, d({}^{P}\mathbf{p}_1)^T, d(\mathbf{b}_i)^T]^T\in\Re^{36x1} \qquad (17)$$

$$\Im=[\Im_1, \Im_2, \ldots, \Im_6]^T \qquad (18)$$

where

$$\Im_i=({}^{Bi}\mathbf{a}_i)^T\{(d\Omega_{Bi})\times L_i{}^{Bi}\mathbf{a}_{id}\},\ i=1,2,\ldots,6 \qquad (19)$$

$$\aleph_i=[\{({}^{Bi}a_i{}^T)^{Bi}R_B{}^{B}R_P\}^T, -\{({}^{Bi}a_i{}^T)^{Bi}R_B\}^T],\ i=1,2,\ldots,6. \qquad (20)$$

From equation (12), the generalized position error $\Delta\wp$ can be written as

$$\Delta\wp=J^{-1}(d\mathbf{L}-\aleph\, d\mathbf{A}-\Im). \qquad (21)$$

Equation (21) is similar to the expression in [1], except the vector $\Im$, and the definition of d**A** does not include the actuator errors $\mathbf{c}_i$ and $\mathbf{d}_i$ defined in accurate model. However, we believe that the vector $\Im$ can be related to these quantities, which is present in equation (21) thanks to the definition of the nominal model employed in this paper. However, the vectors $\mathbf{c}_i$ and $\mathbf{d}_i$ should have been inserted artificially in the derivation of the differential model given in [1].

V. CONCLUSION

For Stewart platforms, new formulations of the nominal and accurate models are derived based on the derivation in [1]. The difference is that we employed the global position error $\Delta\wp$ in the derivation of the accurate model, while this is not the case in PE model [1]. Similarly, in PE model, the vectors $\mathbf{c}_i$ and $\mathbf{d}_i$ should artificially be inserted in the differential equations to include the actuator errors in the dynamics. Although it is not completely derived yet, we believe that our approach will not require the somehow artificial insertion of actuator errors into the equations.

We also showed that the formulation given in [1] covers the Modified Stewart Platforms; therefore, there is no need to give a separate formulation for MSP.

The work presented here is still not complete, however, the preliminary results are promising to lead to a more complete formulation of the error dynamics of the Stewart platforms.

The research on the subject is still continuing. Future work will include the effect of temperature and stress state on the accuracy of the Stewart platforms and the control of the error components that leads to a more stable operation of Stewart platform based systems.

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