Sliding Mode Control of Switched Uncertain Linear Systems: Designing Robust H_∞ Sliding Surface

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Abstract

The robust H_∞ control problem for a class of uncertain switched linear systems by using the variable structure control is investigated. A robust H_∞ single sliding surface is shown to exist as long as a convex combination of the subsystems of the switched system is robustly stabilizable with disturbance attenuation level γ. A switching law is constructed via the single Lyapunov function technique. Variable structure controllers of subsystems are designed so as the resulting closed-loop system guarantees the robust H_∞ performance. An illustrative example and simulation results are given to demonstrate the effectiveness of the proposed design method.

1. Introduction

Switched systems are an important class of hybrid systems which consist of a family of continuous-time/ discrete-time and logic rules specifying which subsystem is activated along the system trajectory at each instant of time. Many challenging issues in analysis and design of switched systems due to their significance in both theoretical study and engineering applications exist. Switched systems that are linear and do not have uncertainties were extensively studied, e.g., see [1] and references therein. On the other hand, [2] represents the seminal contribution on control of uncertain systems.

Work [3] presented a state-partition-based feedback switching law to study the quadratic stabilization of switched linear systems in stable convex combinations using a single Lyapunov function. In [4], L_2 induced norm of switched systems with external disturbances was considered under the condition of large dwell time. For uncertain discrete-time switched systems, the authors in [5] addressed L_2 gain analysis and control synthesis under arbitrary switching. In [6], the robust H_∞ control and stabilization of uncertain switched linear systems are investigated via the multiple Lyapunov function approach.

It was shown in [6-8] switched systems are a kind of “variable structure” systems [9]. Therefore sliding modes may exist on the switching planes even though not expected to happen [8]. When sliding modes occur, the systems may have rather good properties such as be insensitive to parameter variations and external disturbances while reducing orders of the switched systems. Over the years, many results on variable structure control of uncertain systems without switching have been contributed [8, 9]. However, a few results on sliding mode variable structure control of switched systems appeared up to now [10-15].

Work [10] addressed the sliding mode control for planar switched systems under an arbitrary switching sequence. Only a sliding mode controller was designed in there without investigating the construction of the sliding surface. The sliding motion of switched systems without control input was analyzed in [11] where an approach was proposed to estimate the domain in which the sliding motion may occur. A variable structure controller with a sliding mode sector was presented in [12] for a hybrid system. A sliding mode sector is defined as subspace in which some norm of state decreases for each subsystem of the hybrid system, and a variable structure control law is designed to switch among subsystems so as to ensure the quadratic stability of the hybrid system. In [13] the stabilization of hybrid systems with unstable subsystem was solved

To the authors’ awareness, except for [14-15], on the problem of robust H_∞ sliding mode variable structure control for switched systems results on simultaneous design of sliding surface, switching law and variable structure control law have not been reported by now. Following the same underlying idea and proving argument now we solve the robust H_∞ variable structure control problem for a class of uncertain switched linear systems. A sufficient condition for the existence of a robust H_∞ single sliding surface is derived in terms of Riccati inequality associated with a convex combination of switched systems. First a coordinate transformation matrix is defined to put the switched system into the regular form. A switching law is constructed for the $n-m$ dimensional equivalent sliding mode dynamic system so as to reinforce a single robust H_∞ sliding surface such that the switched system is robustly stabilizable on the sliding surface with disturbance attenuation γ. Variable structure controllers are designed to drive the state of the switched system on the robust H_∞ sliding surface in finite time. Further the paper is organized in the standard form.

2. Problem Formulation and Preliminaries

The following class of uncertain switched linear system

$$\begin{aligned}\dot{x}(t) &= (A_\sigma + \Delta A_\sigma)x(t) + Bu_\sigma + B_1\omega(t),\\ z(t) &= Cx(t),\end{aligned} \tag{1}$$

is considered. Here, $x(t)\in R^n$ is the system state, $\sigma(t):[0,\infty)\to \Xi=\{1,2,\ldots,l\}$ is the piecewise constant switching signal that may depend on t or x, $u_i\in R^m$ is the control input of the $i-th$ subsystem, $z(t)$ is the controlled output, $\omega(t)\in L_2[0,\infty)$ is the external disturbance, and B, B_1, C and $A_i, i\in\Xi$ are constant matrices of appropriate dimensions. The $\Delta A_i, i\in\Xi$ do represent the system parameter uncertainties.

The following relevant assumptions are used:

A1/. The parameter uncertainties can be composed as follows

$$\Delta A_i = E\Sigma_i(t)F, i\in\Xi,$$

where $E\in R^{n\times i}$ and $F\in R^{j\times n}$ are known constant matrices, $\Sigma_i(t)\in R^{i\times j}, i\in\Xi$ are unknown matrices with Lebesgue measurable elements and satisfy $\Sigma_i^{\mathrm{T}}(t)\Sigma_i(t)\le I, i\in\Xi$.

A2/. There exists a known nonnegative constant ϖ such that $\|\omega(t)\|\le\varpi$ for all t.

A3/. The input matrix B has full rank $m<n$.

The single sliding surface is defined as

$$\zeta(t) = Sx(t) = 0. \tag{2}$$

The S is the single sliding matrix to be determined later.

The objective of this paper is to design the sliding matrix S, the switching law $\sigma(t)$, and the variable structure controllers $u_i, i\in\Xi$ such that:

(1). SB is non-singular;

(2). The reduced-order equivalent sliding mode dynamics restricted to the single sliding surface are robustly stabilizable with disturbance attenuation level γ under the designed switching law $\sigma(t)$;

(3). the state of the closed-loop system can in a finite time enter into the single sliding surface (2) and subsequently remains on it.

The concept of asymptotic stability with a desired H_∞ disturbance attenuation level given by a real-valued γ.

Definition 1 [14]. Consider the uncertain switched system

$$\begin{aligned}\dot{x} &= A_\sigma x + B\omega\\ z &= Cx\end{aligned}. \tag{3}$$

For a given positive constant $\gamma>0$, if there exists a switching law $\sigma=\sigma(x)$ and a positive definite matrix P, such that

$$x^{\mathrm{T}}(A_\sigma^{\mathrm{T}}P + PA_\sigma + \gamma^{-2}PBB^{\mathrm{T}}P + C^{\mathrm{T}}C)x < 0 \tag{4}$$

holds, then system (3) is said to be asymptotically stable and satisfy H_∞ disturbance attenuation level γ.

Lemma 1 [15]. Given real matrices R_1 and R_2 with appropriate dimensions and an unknown matrix $\Sigma(t)$ with Lebesgue measurable elements such that $\Sigma^{\mathrm{T}}(t)\Sigma(t)\le I$, then we have

$$R_1\Sigma R_2 + R_1^{\mathrm{T}}\Sigma^{\mathrm{T}}R_2^{\mathrm{T}} \le \beta R_1R_1^{\mathrm{T}} + \beta^{-1}R_2^{\mathrm{T}}R_2$$

where $\beta>0$.

A convex combination of the system (1) is the system

$$\begin{aligned}x(t) &= (\overline{A}+\Delta\overline{A})x(t) + Bu + B_1\omega(t),\\ z(t) &= Cx(t),\end{aligned} \tag{5}$$

$$\overline{A} = \sum\nolimits_{i=1}^{l}\alpha_iA_i,\ \Delta\overline{A} = \sum\nolimits_{i=1}^{l}\alpha_i\Delta A_i,\ \sum\nolimits_{i=1}^{l}\alpha_i = 1, \alpha_i\ge 0.$$

Lemma 2. If there exist constant matrices $P>0$, $K\in R^{m\times n}$, constant scalars $\gamma>0, \lambda>0$ satisfying

$$\begin{aligned}(\overline{A}-BK)^{\mathrm{T}}P + P(\overline{A}-BK) + P(\lambda^2EE^{\mathrm{T}} + \gamma^{-2}B_1B_1^{\mathrm{T}})P +\\ +\frac{1}{\lambda^2}F^{\mathrm{T}}F + C^{\mathrm{T}}C < 0\end{aligned} \tag{6}$$

then system (5) is robustly stabilizable with disturbance attenuation level γ.

Proof: Design the state feedback controller by $u=-Kx$ and let

$$\begin{aligned}Q &= (\overline{A}+\Delta\overline{A}-BK)^{\mathrm{T}}P + P(\overline{A}+\Delta\overline{A}-BK) + \gamma^{-2}PB_1B_1^{\mathrm{T}}P + C^{\mathrm{T}}C\\ &= (\overline{A}-BK)^{\mathrm{T}}P + P(\overline{A}-BK) + \gamma^{-2}PB_1B_1^{\mathrm{T}}P + C^{\mathrm{T}}C + \Delta\overline{A}^{\mathrm{T}}P + P\Delta\overline{A}.\end{aligned}$$

Using Lemma 1, one obtains

$$\begin{aligned}\Delta\overline{A}^{\mathrm{T}}P + P\Delta\overline{A} &= (\sum\nolimits_{i=1}^{l}\alpha_i\Delta A_i)^{\mathrm{T}}P + P(\sum\nolimits_{i=1}^{l}\alpha_i\Delta A_i)\\ &= [E(\sum\nolimits_{i=1}^{l}\alpha_i\Sigma_i(t))F]^{\mathrm{T}}P + P[E(\sum\nolimits_{i=1}^{l}\alpha_i\Sigma_i(t))F]\\ &\le \lambda^2PEE^{\mathrm{T}}P + \lambda^{-2}F^{\mathrm{T}}F.\end{aligned}$$

and

$$\begin{aligned}Q \le (\overline{A}-BK)^{\mathrm{T}}P + P(\overline{A}-BK) + P(\lambda^2EE + \gamma^{-2}B_1B_1^{\mathrm{T}})P +\\ +\frac{1}{\lambda^2}F^{\mathrm{T}}F + C^{\mathrm{T}}C < 0,\end{aligned}$$

These imply the system (5) is robustly stabilizable with disturbance attenuation level γ.

3. Main Novel Results

In order to obtain a regular form of system (1), a nonsingular matrix and the associated vector ξ is defined:

$$T = \begin{bmatrix} \widetilde{B}^{\mathrm{T}} \\ B^{\mathrm{T}} \end{bmatrix}, \tag{7}$$

$$\xi(t) = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = Tx(t) = \begin{bmatrix} \widetilde{B}^{\mathrm{T}} \\ B^{\mathrm{T}} \end{bmatrix} x(t), \tag{8}$$

with $\xi_1 \in R^{n-m}$, $\xi_2 \in R^m$, where $\widetilde{B}$ is an orthogonal complement of the matrix B. It is easy to show

$$T^{-1} = \left[\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1} \quad B(B^{\mathrm{T}}B)^{-1} \right]. \tag{9}$$

By the state transformation $\xi(t) = Tx(t)$, the system (1) is transformed into the regular form

$$\begin{aligned} \dot{\xi} &= (\widehat{A}_\sigma + \Delta\widehat{A}_\sigma)\xi + \widehat{B}u_\sigma + \widehat{B}_1\omega(t), \\ z(t) &= \widehat{C}x(t), \end{aligned} \tag{10}$$

where $\widehat{A}_\sigma = TA_\sigma T^{-1}$, $\Delta\widehat{A}_\sigma = T\Delta A_\sigma T^{-1}$, $\widehat{B} = TB$, $\widehat{B}_1 = TB_1$, $\widehat{C} = CT^{-1}$. The system (10) is apparently equivalent to the system

$$\begin{aligned} \begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} &= \begin{bmatrix} \widehat{A}_{\sigma 11} & \widehat{A}_{\sigma 12} \\ \widehat{A}_{\sigma 21} & \widehat{A}_{\sigma 22} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} 0 \\ B^{\mathrm{T}}B \end{bmatrix} u_\sigma(t) + \begin{bmatrix} \widetilde{B}^{\mathrm{T}}B_1 \\ B^{\mathrm{T}}B_1 \end{bmatrix} \omega, \\ z(t) &= C\left[\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1} \quad B(B^{\mathrm{T}}B)^{-1} \right] \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}, \end{aligned} \tag{11}$$

where

$$\widehat{A}_{\sigma 11} = \widetilde{B}^{\mathrm{T}}A_\sigma \widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1} + \widetilde{B}^{\mathrm{T}}E\Sigma_\sigma(t)F\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1},$$

$$\widehat{A}_{\sigma 12} = \widetilde{B}^{\mathrm{T}}A_\sigma B(B^{\mathrm{T}}B)^{-1} + \widetilde{B}^{\mathrm{T}}E\Sigma_\sigma(t)FB(B^{\mathrm{T}}B)^{-1},$$

$$\widehat{A}_{\sigma 21} = B^{\mathrm{T}}A_\sigma \widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1} + B^{\mathrm{T}}E\Sigma_\sigma(t)F\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1},$$

$$\widehat{A}_{\sigma 12} = B^{\mathrm{T}}A_\sigma B(B^{\mathrm{T}}B)^{-1} + B^{\mathrm{T}}E\Sigma_\sigma(t)FB(B^{\mathrm{T}}B)^{-1}.$$

Without loss of generality, a sliding surface is supposed to be

$$\zeta(t) = M\xi_1 + \xi_2 = 0, M \in R^{n\times(n-m)}, \tag{12}$$

where M is a matrix to be chosen. Then, $S = M\widetilde{B}^{\mathrm{T}} + B^{\mathrm{T}}$. Substituting $\xi_2 = -M\xi_1$ in to (11) yields the following sliding motion:

$$\begin{aligned} \dot{\xi}_1 &= (\widehat{A}_{\sigma 11} - \widehat{A}_{\sigma 12}M)\xi_1 + \widetilde{B}^{\mathrm{T}}B_1\omega, \\ z(t) &= C\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1}\xi_1 - CB(B^{\mathrm{T}}B)^{-1}M\xi_1, \end{aligned} \tag{13}$$

Remark. We can see that the n dimensional switched system (1) reduces m dimensional on the sliding surface (12). Therefore, only the study of the $n-m$ dimensional equivalent sliding mode dynamic system (13) is needed.

Theorem 1. Suppose that there exist a positive definite matrix P, a matrix $K \in R^{m\times n}$, constant scalars $\gamma > 0, \lambda > 0$ satisfying inequality (6). Then there exist a switching law $\sigma(x)$ such that the system (13) is robustly stabilizable and satisfies H_∞ disturbance attenuation level γ, where M in (12) is given by $M = [(B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}}PB(B^{\mathrm{T}}B)^{-1}]^{-1}(B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}}P\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1}$. In this case, the sliding surface is

$$\begin{aligned} \zeta(t) = Sx(t) = \{&[(B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}}PB(B^{\mathrm{T}}B)^{-1}]^{-1}(B^{\mathrm{T}}B)^{-1} \\ &B^{\mathrm{T}}P\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1}\widetilde{B}^{\mathrm{T}} + B^{\mathrm{T}}\}x(t) = 0. \end{aligned} \tag{14}$$

Proof: The system (13) can be equivalently rewritten

$$\begin{aligned} \dot{\xi}_1 &= (\hat{A}_{\sigma 11} - \hat{A}_{\sigma 12}M + \hat{E}\Sigma_\sigma(t)\hat{F})\xi_1 + \widetilde{B}^{\mathrm{T}}B_1\omega, \\ z(t) &= \left[C\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1} - CB(B^{\mathrm{T}}B)^{-1}M \right]\xi_1, \end{aligned} \tag{15}$$

where $\hat{A}_{\sigma 11} = \widetilde{B}^{\mathrm{T}}A_\sigma \widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1}$, $\hat{A}_{\sigma 12} = \widetilde{B}^{\mathrm{T}}A_\sigma B(B^{\mathrm{T}}B)^{-1}$, $\hat{E} = \widetilde{B}^{\mathrm{T}}E$, $\hat{F} = F\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1} - FB(B^{\mathrm{T}}B)^{-1}M$.

Denote

$$\begin{aligned} \overline{A}_c &= T(\overline{A} - BK)T^{-1} \\ &= \begin{bmatrix} \overline{A}_{11} & \overline{A}_{12} \\ \overline{A}_{21} - B^{\mathrm{T}}BK\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1} & \overline{A}_{22} - B^{\mathrm{T}}BKB(B^{\mathrm{T}}B)^{-1} \end{bmatrix} \end{aligned} \tag{16}$$

with $\overline{A}_{11} = \widetilde{B}^{\mathrm{T}}\overline{A}\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1}$, $\overline{A}_{12} = \widetilde{B}^{\mathrm{T}}\overline{A}B(B^{\mathrm{T}}B)^{-1}$; and

$$\begin{aligned} \overline{P} &= T^{-\mathrm{T}}PT^{-1} \\ &= \begin{bmatrix} (\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1}\widetilde{B}^{\mathrm{T}}P\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1} & (\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1}\widetilde{B}^{\mathrm{T}}PB(B^{\mathrm{T}}B)^{-1} \\ (B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}}P\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1} & (B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}}PB(B^{\mathrm{T}}B)^{-1} \end{bmatrix} \\ &= \begin{bmatrix} \overline{P}_{11} & \overline{P}_{12} \\ \overline{P}_{21} & \overline{P}_{22} \end{bmatrix}. \end{aligned} \tag{17}$$

Then, (6) can be rewritten as

$$\begin{aligned} &\overline{A}_c^{\mathrm{T}}\overline{P} + \overline{P}A_c + \overline{P}T(\lambda^2 EE^{\mathrm{T}} + \gamma^{-2}B_1B_1^{\mathrm{T}})T^{\mathrm{T}}\overline{P} + \\ &+ T^{-\mathrm{T}}(\frac{1}{\lambda^2}F^{\mathrm{T}}F + C^{\mathrm{T}}C)T^{-1} < 0. \end{aligned} \tag{18}$$

Pre- and post-multiply (18) by $\left[I_{n-m} \quad -\overline{P}_{12}\overline{P}_{22}^{-1} \right]$ $\left[I_{n-m} \quad -\overline{P}_{12}\overline{P}_{22}^{-1} \right]^{\mathrm{T}}$ to givr

$$\begin{aligned} &(\overline{A}_{11} - \overline{A}_{12}\overline{P}_{22}^{-1}\overline{P}_{12}^{\mathrm{T}})^{\mathrm{T}}\overline{P}_r + \overline{P}_r(\overline{A}_{11} - \overline{A}_{12}\overline{P}_{22}^{-1}\overline{P}_{12}^{\mathrm{T}}) + \\ &+ \overline{P}_r(\lambda^2 EE^{\mathrm{T}} + \gamma^{-2}B_1B_1^{\mathrm{T}})\overline{P}_r + \\ &[\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1} - B(B^{\mathrm{T}}B)^{-1}\overline{P}_{22}^{-1}\overline{P}_{12}^{\mathrm{T}}]^{\mathrm{T}}(\frac{1}{\lambda^2}F^{\mathrm{T}}F + C^{\mathrm{T}}C) \\ &[\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1} - B(B^{\mathrm{T}}B)^{-1}\overline{P}_{22}^{-1}\overline{P}_{12}^{\mathrm{T}}] < 0, \end{aligned} \tag{19}$$

where $\overline{P}_r = \overline{P}_{11} - \overline{P}_{12}\overline{P}_{22}^{-1}\overline{P}_{12}^{\mathrm{T}}$. Note that $\overline{P}_r > 0$ because $\overline{P} > 0$. Therefore, by setting $M = \overline{P}_{22}^{-1}\overline{P}_{12}^{\mathrm{T}} = [(B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}}PB(B^{\mathrm{T}}B)^{-1}]^{-1}(B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}}P\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1}$, (19) becomes

$$\begin{aligned}&(\overline{A}_{11} - \overline{A}_{12}M)^{\mathrm{T}}\overline{P}_r + \overline{P}_r(\overline{A}_{11} - \overline{A}_{12}M) + \\ &+ \overline{P}_r(\lambda^2 EE^{\mathrm{T}} + \gamma^{-2}B_1B_1^{\mathrm{T}})\overline{P}_r + \\ &[\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1} - B(B^{\mathrm{T}}B)^{-1}M]^{\mathrm{T}}(\frac{1}{\lambda^2}F^{\mathrm{T}}F + C^{\mathrm{T}}C) \\ &[\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1} - B(B^{\mathrm{T}}B)^{-1}M] < 0.\end{aligned} \tag{20}$$

Let

$$\begin{aligned}Q_i =& (\widetilde{B}^{\mathrm{T}}A_i\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1} - \widetilde{B}^{\mathrm{T}}A_iB(B^{\mathrm{T}}B)^{-1}M)^{\mathrm{T}}\overline{P}_r + \\ &+ \overline{P}_r(\widetilde{B}^{\mathrm{T}}A_i\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1} - \widetilde{B}^{\mathrm{T}}A_iB(B^{\mathrm{T}}B)^{-1}M) + \\ &+ \overline{P}_r(\lambda^2 EE^{\mathrm{T}} + \gamma^{-2}B_1B_1^{\mathrm{T}})\overline{P}_r + [\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1} - B(B^{\mathrm{T}}B)^{-1}M]^{\mathrm{T}} \\ &(\frac{1}{\lambda^2}F^{\mathrm{T}}F + C^{\mathrm{T}}C)[\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1} - B(B^{\mathrm{T}}B)^{-1}M].\end{aligned}$$

Substituting $\overline{A}_{11} = \widetilde{B}^{\mathrm{T}}\overline{A}\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1}$, $\overline{A}_{12} = \widetilde{B}^{\mathrm{T}}\overline{A}B(B^{\mathrm{T}}B)^{-1}$ and $\overline{A} = \sum_{i=1}^{l}\alpha_iA_i$ into (6) gives

$$\alpha_1Q_1 + \alpha_2Q_2 + \ldots + \alpha_lQ_l < 0. \tag{21}$$

Now, the switching law is defined as

$$\sigma(\xi_1) = \arg\min_{i\in\Xi} \xi_1^{\mathrm{T}}Q_i\xi_1. \tag{22}$$

By Definition 1, we conclude that system (13) is robustly stabilizable with disturbance attenuation level γ under the switching law (22).

The next is the design of variable structure control laws using the reachability condition of sliding surface.

Theorem 2: Assume that the conditions of Theorem 1 are satisfied and the sliding surface of system (1) is given by (14). Then under the control laws

$$\begin{aligned}u_\sigma =& -(SB)^{-1}SA_\sigma x - (SB)^{-1}(\|SE\|\|Fx\| \\ &+ \varpi\|SB_1\| + \mu)sign(\zeta),\end{aligned} \tag{23}$$

the state of the system (1) can in finite time enters and subsequently remains on the sliding surface, where μ is a positive scalar to adjust the convergent rate.

Proof: It is straightforward using A2 hence omitted.

4. An Illustrative Example

umerical and simulation results are presented. Consider the following uncertain switched linear system:

$$\begin{aligned}&\dot{x}(t) = (A_i + \Delta A_i)x(t) + Bu_i + B_1\omega(t), \\ &z(t) = Cx(t), \qquad i = 1,2\end{aligned} \tag{25}$$

where $\Delta A_i = E\Sigma_i(t)F, i = 1,2$,

$$A_1 = \begin{bmatrix}-4 & 1 & 0\\ 2 & 1 & 1\\ 1 & -1 & 0\end{bmatrix},\ A_2 = \begin{bmatrix}-2 & -1 & 2\\ 0 & -3 & -5\\ 1 & -1 & -4\end{bmatrix},\ B = \begin{bmatrix}0\\1\\1\end{bmatrix},\ B_1 = \begin{bmatrix}1\\0\\1\end{bmatrix},$$

$$C = \begin{bmatrix}1\\0\\0\end{bmatrix}^{\mathrm{T}},\ E = \begin{bmatrix}1\\0\\-1\end{bmatrix},\ F = \begin{bmatrix}0\\1\\1\end{bmatrix}$$

and the uncertain parameter $\Sigma_1(t) = \Sigma_2(t) = \eta \in [-1,1]$. We choose the convex combination coefficients $\alpha_1 = \alpha_2 = 0.5$ and the constant $\lambda = 1/\sqrt{2}$. The initial state is $x_0 = [1\ 2\ -1]^{\mathrm{T}}$. The state responses of the two subsystems alone are shown Figure 1 and Figure 2, respectively; both subsystems are unstable.

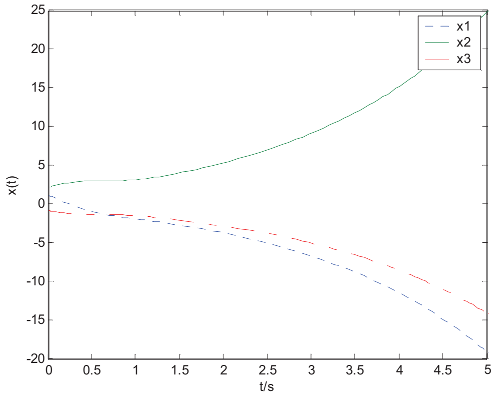

Fig. 1 The state response of the subsystem 1

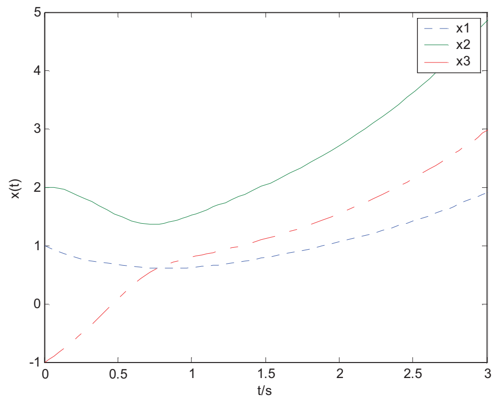

Fig. 2 The state response of the subsystem 2

By solving Riccati inequality (6), one can obtain the following solution

$$P = \begin{bmatrix}1.4659 & 0.1896 & -0.2403\\ 0.1896 & 0.8446 & 0.7391\\ -0.2403 & 0.7391 & 1.1459\end{bmatrix}.$$

By means of (17) and

$$M = [(B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}}PB(B^{\mathrm{T}}B)^{-1}]^{-1}(B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}}P\widetilde{B}(\widetilde{B}^{\mathrm{T}}\widetilde{B})^{-1},$$

we get the matrix M :

$$M = [-0.5983, 1.7754].$$

Therefore, we have

$$Q_1 = \begin{bmatrix} -6.4562 & 3.9582 & 3.9582 \\ 3.9582 & 2.374 & 2.374 \\ 3.9582 & 2.374 & 2.374 \end{bmatrix} Q_2 = \begin{bmatrix} -1.2476 & -1.2896 & -1.2896 \\ 2.1284 & -2.0525 & -2.0525 \\ 2.1284 & -2.0525 & -2.0525 \end{bmatrix}.$$

The robust H_∞ single sliding surface is found as

$$\zeta = Sx = [-1.6786, -0.4116, 1.5884]^T x$$

and the control laws are given by:

$$\begin{aligned} u_1 = &-0.5(7.4796x_1 - 3.6786x_2 - 0.4116x_3) - \\ &-(1.6335\|x_2 + x_3\| + 1.5) \\ &\times \tanh(-1.6786x_1 - 0.4116x_2 + 1.5884x_3), \end{aligned}$$

$$\begin{aligned} u_2 = &-0.5(4.9456x_1 - 1.325x_2 - 7.6529x_3) - \\ &-(1.6335\|x_2 + x_3\| + 1.5) \\ &\times \tanh(-1.6786x_1 - 0.4116x_2 + 1.5884x_3). \end{aligned}$$

The simulation results for the closed-loop system are shown in Figure 4. It can be clearly seen that the closed-loop system of the switched system (25) is robust asymptotically stable.

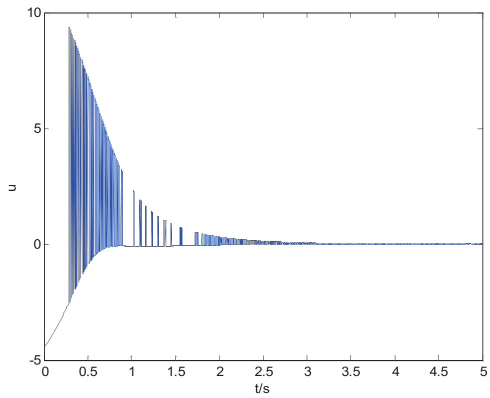

Fig. 3 The input signal of switched system (25)

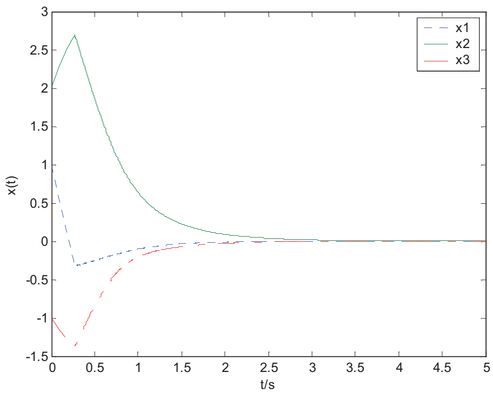

Fig. 4 The state response of switched system (25)

5. Conclusions

The problem of variable structure control for a class of switched linear systems with mismatched parametric uncertainties and external disturbances solved. The robust H_∞ single sliding surface is constructed based on the Riccati inequality associated with the convex combination of the switched system such that the motion of the switched system along the sliding surface is robustly stabilizable with disturbance attenuation γ under the proposed standard switching law.

6. References

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