Combination of l_1 and l_2 Norms for Image Deconvolution Problems

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Abstract

Maintaining a balance between fidelity, over-smoothing and stability in iterative restoration of noisy and blurred images is a difficult task that researchers need to handle. We proposed a very simple yet effective method to combine l_1 and l_2 norms in order to both reduce over-blurring around edges and perform effective denoising within flat areas in images simultaneously and generate visually pleasing results. Experiments performed with tuned parameters for all compared approaches shown that the proposed combination is also the fastest to converge. A strategy for updating feedback parameters within iterations is also experimentally developed and used.

Keywords : norm, deconvolution, debluring, inverse problem

1. Introduction

Recent developments in technology brought digital images and image processing into everyday use. On the other hand, even in the images obtained using a modern imaging device, there are differences from the actual scene. There are many reasons for distortion in images and the most common are the out of focus blur, motion blur and sensor noise along with the defects resulting from lower resolution than the ideal. Atmospheric disturbance on the light path is another blur source, especially in astronomical imaging. Images recorded by image sensors are commonly degraded by a combination of such distortions. Many image restoration applications attempt to undo these blur degradations and restore the expected original image back.

The main plan in image deblurring is to restore the original sharp image from its blurred version or versions without intensifying noise or even reducing noise. A sharp estimate can be obtained when blurring process or most of it is known. Blur process is commonly named as point spread function (PSF) as it is the function of distribution image intensity when the source is a single bright point within a black background. With the assumption of shift-invariant blur the problem reduces to that of image deconvolution with a known PSF. In the literature, many image debluring approaches assume that either the PSF is fully known or calculated prior to debluring process. However, in some cases, such as atmospheric imaging, it is almost impossible to predict characteristics of atmospheric turbulence. In such cases, the mathematical model is practically impossible to know a priori. Therefore, it is most of the time necessary to estimate PSF and restore the image at the same time (blind deconvolution) [1].

Ringing (ripple-like artifacts around edges) and noise amplification are the most dominant artifacts in image deconvolution. Traditional and popular non-blind deconvolution methods such as Wiener Filtering [2] and Richardson-Lucy deconvolution [3] are still widely used in many image restoration methods as they are simple, fast and give good results in case of the relatively small blur. On the other hand in case of kernel errors and relatively high noise, these methods generate unpleasant ringing artifacts around strong edges. These and other for image deconvolution techniques are much reviewed in the literature[4-5].

The blurring is commonly modeled as a convolution of the desired image and PSF with additive noise as

$$Y = H * X + \eta \qquad (1)$$

where Y is a degraded (observed) image, X is the desired image, H is a shift-invariant blur kernel (PSF) and η is the additive noise. $*$ denotes the convolution operation.

The most obvious solution to deblurring is convolution with inverse filter. Among deblurring methods, non-iterative methods such as adaptive unsharp masking [6], use of blur-space for deblurring [7], frequency-wavelet domain approach [8], gradient inverse weighted smoothing [9] can be counted in the literature.

One of the popular approaches to restoration of noisy and blurred images is to iteratively update the estimate based on some penalties measured over original and estimated images and expectations. Recovering the undistorted image using the observed image is an ill-conditioned problem as a small perturbation of data leads to a great divergence in the solution space. Therefore, basic idea of regularization is a solution for balancing fidelity and smoothness of the solution[10]. In almost all approaches, regularization has been used as a way to remedy the state of being ill-conditioned. Regularization is used as a function of the penalty term to prevent unwanted results. However, the choice of regularization plays a critical role in obtaining good results. Regularization term is not easy to design and inappropriate choices often lead to serious side effects. Penalty term is usually the difference in value between neighboring pixels, that is, the transition is penalized by some quadratic function [11]. As a result, the restored image is overly smooth and edges in the image are the most affected. In [12-13] a regularization function is proposed for preserving the edges. Jalobeanu et al. [14] have proposed a method that employs adaptive regularization term according to local characteristics of the image.

Along with the objective criteria such as signal-to-noise-ratio (SNR) for comparisons of various algorithms, researchers also use visual satisfaction of human observers, since SNR is hardly a good measure for degree of blur in images. Nevertheless, SNR still is the most widely used distortion measure when original images are at hand. Since the original image is not at hand, SNR and other measures that give closeness to the original can not be calculated. Li et al. used the irregularity of the distribution as a measure and tried to minimize this for the optimal image[15].

2. Problem formulation

Let the images in interest have $m \times n$ pixels, then Y , X and η are vectors of length mn , and H is a matrix of $mn \times mn$, (1) can be written as follows

$$Y = HX + \eta . \tag{2}$$

using matrix-vector formulation. One usually attempts the reconstruct X by solving

$$\begin{aligned} &mimimize \quad f(X) \\ &subject\,to \quad \|Y - HX\|_p \le \varepsilon \end{aligned} \tag{3}$$

where ε is an estimated upper bound on the noise power and p denotes the p^{th} norm of difference between observed image and estimated image. For solving (3), a linear least squares problem in the form of

$$\hat{X} = \arg\min_X \left\{ \|Y - HX\|_2^2 + \alpha \|f(X)\|_2^2 \right\}. \tag{4}$$

is constructed [16].

This is approximately equivalent to several different formulations available for optimization problems. In the literature, there is a growing interest in using l_1 norm for solving these types of problems. It is claimed that having regularization terms in l_1 norm as

$$\hat{X} = \arg\min_X \left\{ \|Y - HX\|_2^2 + \alpha \|f(X)\|_1^1 \right\}. \tag{5}$$

prevents smoothing of edges [17].

Some researchers [19-20] argued that both outlier processing and impulse noise removal are performed better when both terms in the minimization are in l_1 form as

$$\hat{X} = \arg\min_X \left\{ \|Y - HX\|_1^1 + \alpha \|f(X)\|_1^1 \right\}. \tag{6}$$

We propose a robust solution by combining l_1 and l_2 norms in both terms of minimization as

$$\hat{X} = \arg\min_X \left\{ \begin{array}{l} \lambda_1 \|Y - HX\|_1^1 + \lambda_2 \|Y - HX\|_2^2 \\ + \alpha_1 \|f(X)\|_1^1 + \alpha_2 \|f(X)\|_2^2 \end{array} \right\}. \tag{7}$$

Applying steepest descent formulation to (7), we get

$$\begin{aligned} X^{k+1} = X^k - \lambda_1 \|Y - HX^k\|_1^1 - \lambda_2 \|Y - HX^k\|_2^2 \\ - \alpha_1^k \|f(X^k)\|_1^1 - \alpha_2^k \|f(X^k)\|_2^2 \end{aligned} . \tag{8}$$

Although, for reasonable parameters that create convex systems, it seems that all four optimizations (4,5,6,7) approach to acceptable solutions, the difference between them is in their speed of convergence and the quality of the resulting images after certain number of iterations.

We used bilaterally filtered current image estimate as the regularization term $f(X^k)$ similar to the work of Farsiu et al. [20] Selection and update of weight parameters λ_1, λ_2, α_1^k and α_2^k are described in the following.

3. Parameter Selection

Controlling the amounts of residual norm $\|Y - HX\|_2^2$ and the term $\|f(X)\|_2^2$ representing a priori information about the true image, the selection and adjustments of regularization parameters and Lagrange multipliers in (7) provides a tradeoff between stability and fidelity. A choice with relatively large λ and small α usually results in higher protection of details and less smoothing. Conversely, smaller λ and larger α would generate a smoother result with consequently reduced noise. Literature is rich of studies on how these parameters should be selected. It is intuitive that λ should be selected as a function of the residual norm, α should be related to the smoothing function. In any parameter set choice, it is obvious that entire system must satisfy the convexity requirement. In our tests, based on [21], the values of 1 and 0.5 are found to be adequate for λ_1 and λ_2 respectively. Similarly, α_1^k and α_2^k are updated using

$$\alpha_1^{k+1} = \frac{\|Y - HX^k\|_2^2}{\|Y\|_2^2 - \|f(X^k)\|_1^1} \quad \text{and} \tag{9}$$

$$\alpha_2^{k+1} = \frac{\|Y - HX^k\|_2^2}{\|Y\|_2^2 - \|f(X^k)\|_2^2}. \tag{10}$$

4. Experimental results

Performances of all four optimization approaches are recorded for a fair comparison. Algorithms are tested on several images that are blurred by a Gaussian PSF with standard deviation of 4 and window size of 9x9. Four of those images are shown in Fig. 1 since they contain illustrative examples of edgy and flat regions. These images are used to create three separate sets as with no additional noise, with Gaussian noise and with impulse (salt&pepper) noise. Both Gaussian and impulsive noises had variances of 10^{-3}.

Evaluation of PSNRs for up to 270 iterations is shown in Fig. 2 for cameraman image with no additional noise. Types 1-4 correspond to the algorithms based on equations (4-7) respectively. Stopping condition for iteration is normalized difference as $D^k \le \varepsilon$ where $D^k = \left|X^{k+1} - X^k\right| / \left|X^k\right|$ and ε is 10^{-5} for no-noise cases and 5×10^{-4} for cases with additive noise.

Fig. 2 shows improvements versus iteration number for blurred cameraman image without noise. Best results for both the rate of convergence and the improvement in PSNR values were obtained by the proposed method. Fig. 3 shows normalized difference value D^k as an indication of convergence

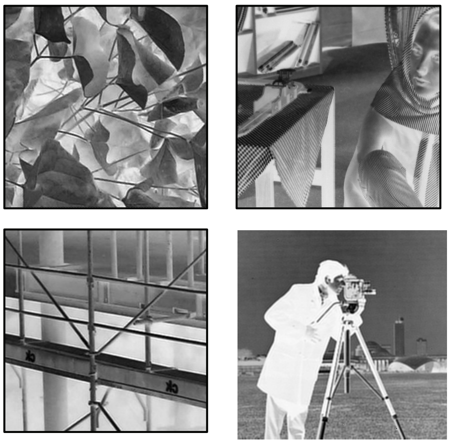

Fig.1. Some test images; leaves, Barbara, construction and cameraman. Images contain various amount of edges, details and flat areas.

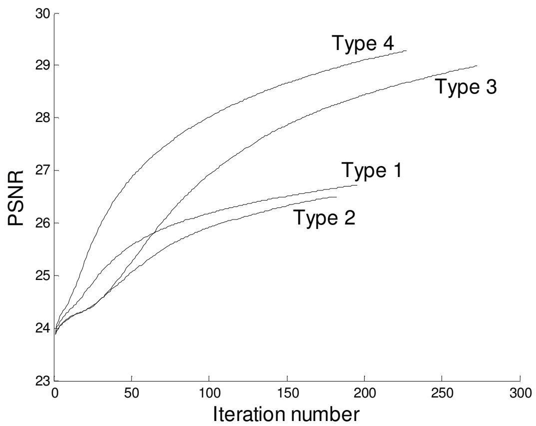

Fig.2. Cameraman test results show that proposed method is the fastest to converge to stopping criterion. Curves are monotonic up to 10^5 iterations for types 3,4 and unstable for types 1,2.

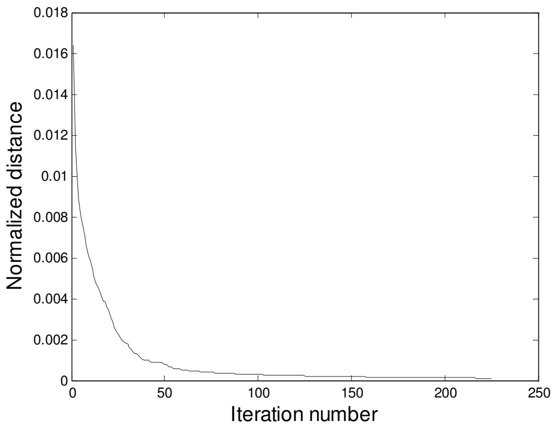

Fig.3. Convergence curve for proposed method illustrates normalized difference between previous and current estimates.

Quantitative comparisons of four tested algorithms using PSNR values are given in Tables 1-3 for test images without noise, with Gaussian noise and with impulse noise respectively. Keeping in mind that the numbers in the tables are PSNRs and iteration numbers when the stopping criteria is reached (relative difference is less than 10-5 or 10-4). It is possible to obtain better results by running the algorithms indefinitely. In order to see the behavior of the algorithms at higher iteration numbers we let the algorithms run up to 105 iterations. In that case PSNRs reached 40dB for the algorithms given by (3) and (4) (types 3,4) and 30dB for the other algorithms (types 1,2). For noisy cases, behavior of algorithm types 1 and 2 were unstable after about 40k iterations.

Table 1. Test results for images without noise

		Type 1	Type 2	Type 3	Type 4
Leaves	Iter No	169	151	232	188
	PSNR	28.46	27.90	30.01	30.70
Barbara	Iter No	138	96	289	278
	PSNR	25.00	24.76	27.97	28.00
building	Iter No	198	351	364	211
	PSNR	29.41	28.58	30.36	31.27
cameraman	Iter No	195	182	273	227
	PSNR	26.72	26.50	28.99	29.28

Table 2. Test results for images with Gaussian noise

		Type 1	Type 2	Type 3	Type 4
Leaves	Iter No	97	103	58	118
	PSNR	26.01	25.87	25.53	26.31
Barbara	Iter No	38	39	26	49
	PSNR	24.07	24.06	23.95	24.26
building	Iter No	156	158	151	182
	PSNR	26.17	25.85	25.91	26.12
cameraman	Iter No	111	116	125	135
	PSNR	24.36	24.25	23.89	24.78

Table 3. Test results for images with impulse noise

		Type 1	Type 2	Type 3	Type 4
Leaves	Iter No	60	62	33	74
	PSNR	25.93	25.81	25.36	26.15
Barbara	Iter No	23	23	19	29
	PSNR	24.08	24.07	23.92	24.22
building	Iter No	88	94	112	114
	PSNR	26.20	26.00	25.86	26.31
cameraman	Iter No	60	62	32	86
	PSNR	24.23	24.17	23.82	24.53

5. Conclusion

It is experimentally shown that the proposed algorithm using a combination of l_1 and l_2 norms for deconvolution/deblurring problems is superior to other compared algorithms that employ only l_1 and l_2 terms. By using l_2 terms we improved the robustness against noise and by inserting l_1 terms we enhanced behavior around the edges and prevented over-smoothing. The convergence rate of the proposed algorithm is also found to be better than others. In this experimental study we assumed that the blur operator is known a priori. One of the future works would be, starting with an initial operator estimate, improve the operator and enhance feedback parameters conveniently.

6. References

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