FREQUENCY-DOMAIN INVESTIGATIONS OF PERIODIC DIPOLE ARRAYS

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ABSTRACT

Radiation from line and rectangular arrays of periodic, linearly phased, electric current dipoles is investigated in the frequency domain. The reference solutions are obtained via the classical element-by-element summation approach. High frequency asymptotic formulations are obtained in terms of Floquet Waves and their corresponding tip-diffracted contributions. A synthetic aperture type approach is applied in modeling one-and two-dimensional finite arrays.

I. INTRODUCTION

Periodic large arrays of short radiating elements have been investigated for the last decade or so, since they play an important role in phased-array antennas with flexible beam-steering and laser-beam type very narrow beam-forming capabilities, frequency selective surfaces, etc. [1-8]. The traditional direct method is the application of Method of Moments (MoM) [9] which is based on the frequency domain (FD) Green's function formulation plus an element-by-element summation approach. This, however, becomes too complex and numerically time-consuming for large-size arrays. Felsen and his colleagues have introduced various representations for one-(1D) and two-dimensional (2D) arrays in terms of propagating and evanescent Floquet waves (FW), and FW-modulated edge- and tip-diffracted waves (see, e.g., [5-8] and studies listed in their references). The problem has been solved via a variety of alternative techniques in these studies, which have granted different insights into collective behavior of the dipole-excited wavefields.

In this study, numerical investigation of 1D infinite, semi-infinite, finite line arrays, and 2D rectangular arrays of linearly phased, periodic, infinitesimal axial electric current dipoles is discussed. Element-by-element summation based on the frequency domain Green's function representation of an individual dipole element is reviewed in Section II. High frequency asymptotic formulations, which are constructed in terms of radiating and evanescent FWs as well as their corresponding tip-diffracted contributions are reviewed in Section III. Finite arrays are modeled using a synthetic aperture type approach constructed via properly renormalized Floquet wave contributions of semi-infinite line arrays. Finally, the conclusions are outlined in Section IV.

II. ELEMENT BY ELEMENT SUMMATION

The geometry of the periodic line array of N_z linearly phased dipoles is shown in Fig. 1. The infinitesimal z-directed dipoles with unit current amplitude are phased linearly as $k\eta_z n d_z$, $n=0,1,\ldots,N_z-1$, where $k\eta_z d_z$ denote the inter-element phase increment along z coordinate.

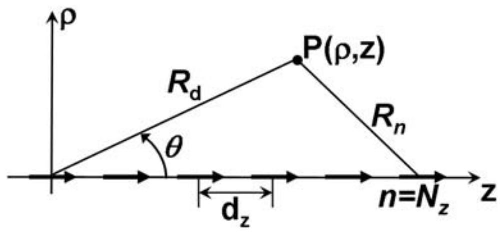

Figure 1. Geometry of the line array

The radiated field can be determined by the z-component of the magnetic vector potential $\vec{A}$ as

$$E_z = -j\omega\mu A_z + \frac{1}{j\omega\varepsilon}\frac{\partial^2 A_z}{\partial z^2}. \tag{1a}$$

Under the assumed *exp(jωt)* time-dependence, A_z can be expressed as

$$A_z = \sum_{n=0}^{N_z-1} \frac{e^{-jkR_n}}{4\pi R_n} e^{-jk\eta_z n d_z}$$

$$R_n = \sqrt{\rho^2 + (z - nd_z)^2}\ . \tag{1b}$$

The geometry of the rectangular array is shown in Fig. 2. The rectangular array is comprised of N_x line arrays shifted by d_x along the x-axis each containing N_z dipoles along the z-axis separated by d_z. The dipoles are phased linearly as $k\eta_x md_x + k\eta_z nd_z$, *m=0,1,...* N_x*-1,* *n=0,1,...,*N_z*-1,* where $k\eta_x d_x$ and $k\eta_z d_z$ denote the inter-element phase increment along x and z coordinates, respectively.

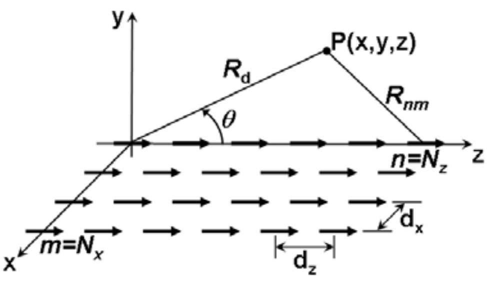

Figure 2. Geometry of the rectangular array

The field radiated by a rectangular array can be computed as,

$$A_z = \sum_{m=0}^{N_x-1}\sum_{n=0}^{N_z-1} \frac{e^{-jkR_{nm}}}{4\pi R_{nm}} e^{-j(k\eta_x md_x + k\eta_z nd_z)}$$

$$R_{nm} = \sqrt{(x-md_x)^2 + y^2 + (z-nd_z)^2}\ . \qquad (2)$$

III. FLOQUET WAVE REPRESENTATION

In what follows, it is demonstrated that numerical efficiency can significantly be improved by accounting for the contributions from individual dipoles in a line array collectively in terms of a small number of FWs [5-8]. The beauty of those representations resides on the fact that all physical wave objects in terms of propagating (PFW) and evanescent Floquet waves (EFW), as well as FW-modulated diffracted waves and their field contributions can be analyzed either individually, or collectively. The starting point will be the FW representation for the field radiated by an infinite line array.

INFINITE LINE ARRAY

The field radiated by an infinite line array of dipoles oriented along the z-axis is the same as (1b) except that the n-sum extends from $-\infty$ to $+\infty$ instead of from 0 to N_z-1. Alternatively, the total wavefields can be expressed in terms of the periodicity-induced FWs. As expressed in [7] this is achieved by using "the equivalence between summation over the contributions from individual elements in an array and their collective treatment via Poisson summation in terms of an infinite series of FWs. Poisson summation converts the effect of the infinite periodic array of individual phased n-indexed dipole radiators collectively into an infinite superposition of linearly smoothly phased q-indexed equivalent FW-modulated line source distributions along the axis of the dipole array."

The total magnetic scalar potential can be expressed as a superposition of q-indexed FWs A_q^{FW} [7] and the *Hankel Function* can be approximated asymptotically as

$$A_z = \sum_{q=-\infty}^{\infty} A_q^{FW}$$

$$A_q^{FW} = \frac{e^{-jk_{zq}z}}{4jd_z} H_0^2(k_{\rho q}\rho) \approx \frac{e^{-j\left(k_{\rho q}\rho + k_{zq}z + \pi/4\right)}}{2d_z\sqrt{2\pi k_{\rho q}\rho}}\ . \qquad (3)$$

The z-domain wavenumber k_{zq} and the radial wavenumber $k_{\rho q}$ are given by

$$k_{zq} = k\eta_z + \frac{2\pi q}{d_z} \qquad q = 0,\ \pm 1,\ \pm 2,\ \ldots \quad (4a)$$

$$k_{\rho q} = \sqrt{k^2 - k_{zq}^2}\ . \qquad (4b)$$

The square-root function in (4b) is defined so that $\mathrm{Im}\{k_{\rho q}\} \leq 0$ on the top Riemann sheet, consistent with the radiation condition when $\rho \to \infty$.

Note that, $|k_{zq}| = |k|$ defines the cutoff condition for the q^{th} FW, which is radially propagating for $|k_{zq}| < |k|$ and evanescent for $|k_{zq}| > |k|$.

Each PFW contributes at the observation point P a ray-asymptotic field lying on a ray cone with semi-angle

$$\beta_q = \cos^{-1}(k_{zq}/k)\ . \qquad (5)$$

Because of the exponential decay of the EFW_q along ρ, a few terms may suffice for an adequate approximation away from the array axis. EFWs become significant only near the array and can be neglected away from the array.

Fig 3 shows the amplitude of E_z versus ρ of an infinite array with inter-element spacing d_z=2λ and η_z=0.5. In this example, the element-by-element summation with N_z=1000 is considered as the reference solution. The array geometry with d_z =2λ permits 3 PFWs to propagate away from the axis those with indexes q= -2, -1, 0. While EFWs can be neglected away from the axis, including more and more EFWs near the array assures a better agreement between the reference solution and the asymptotic solution in terms of FWs.

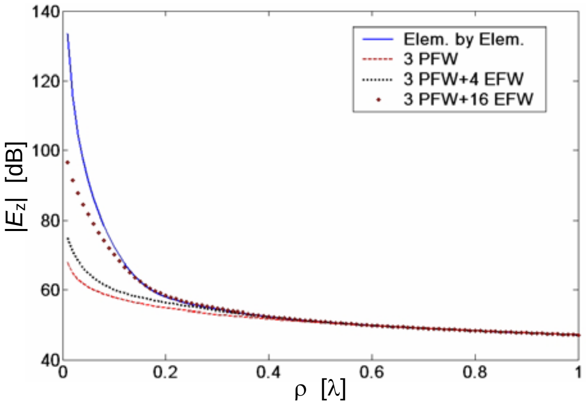

Figure 3. Amplitude of E_z versus ρ at z=0 radiated by an infinite line array.

SEMI-INFINITE LINE ARRAY

If the array is truncated at z=0, finite Poisson summation may be used to convert the individual dipole radiations into collective truncated wavefields -- the Floquet waves -- which can be expressed as [8]

$$A_z = \frac{A_0}{2} + \sum_{q=-\infty}^{\infty} A_q^{FW} U(\beta_q - \theta) + \sum_{q=-\infty}^{\infty} A_q^d \qquad (6)$$

where U is the Heaviside unit function. A_q^{FW} is given in (3) and therefore every FW in (6) is the same as in the infinite array case, but these are confined to the region $\theta < \beta_q$ with β_q locating the conical shadow boundary of the qth PFW. The shadow boundaries for the EFWs are defined as $\cos^{-1}(k/k_{zq})$ and hence the argument of the Heaviside function should be modified accordingly. EFWs can be neglected when observation point is sufficiently far away from the array axis. However, EFWs excite propagating diffracted fields that have to be taken into account.

A_0 in (6) is the individual contribution of the n=0 indexed dipole and equals $A_0 = e^{-jkR_d} / (4\pi R_d)$. The last term A_q^d in (6) is the q[th] diffracted FW which arise from scattering of the PFWs and EFWs at the tip truncation of the array and is computed via the steepest descent path integral through the saddle point θ, and can be expressed as [8]

$$A_q^d = \frac{e^{-jkR_d}}{jkd_z 4\pi R_d} \frac{F(\delta_q^2)}{\cos(\beta_q) - \cos(\theta)} . \qquad (7)$$

The FWs A_q^{FW} in (6) are discontinuous at the shadow boundary $\theta = \beta_q$ and the discontinuity is compensated by an opposite discontinuity of the diffracted fields in (7); so the continuity of A_z is restored. The function *F(.)* in (7) is the transition function of the uniform theory of diffraction (UTD) [5, 8]

$$F(x) = 2j\sqrt{x} e^{jx} \int_{\sqrt{x}}^{\infty} e^{-jt^2} dt . \qquad (8)$$

The argument δ_q of the *F* function in (8) is given by

$$\delta_q = \sqrt{2kR_d} \sin\left(\frac{\beta_q - \theta}{2}\right). \qquad (9)$$

F can be expressed in terms of the complementary error function as follows:

$$F(\delta_q^2) = \pm\sqrt{\pi}\delta_q e^{j\delta_q^2} e^{j\pi/4} erfc(\pm e^{j\pi/4}\delta_q) . \qquad (10)$$

The ± signs apply for $\mathrm{Re}\left(e^{j\pi/4}\delta_q\right)$ *greater/less* than zero.

Fig. 4 shows the amplitude of A_z at a radial distance R_d=2λ for inter-element spacing d_z=2λ and η_z=0.25. In this angular scan example, the element-by-element summation with N_z=1000 is tested to be the reference solution. The array geometry d_z =2λ, permits 4 PFWs to propagate away from the axis those with indexes q= -2, -1, 0, 1. It is clear from the figure that the FWs are discontinuous at these shadow boundaries and that this discontinuity is compensated by adding the diffracted fields. The FWs together with the diffracted waves agree very well with the reference solution.

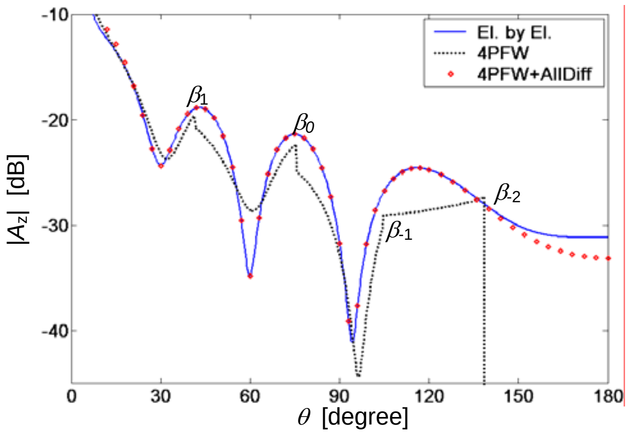

Figure 4. Amplitude of A_z vs. θ radiated by a semi-infinite line array.

FINITE LINE ARRAY

To obtain the field radiated by the finite line array consisting of N_z dipoles, the effects of the truncation on both ends can be accounted for by the superposition of the contributions from two oppositely phased semi-infinite arrays, one of which is shifted along the z-axis by $z=N_z d_z$ and hence can be expressed as

$$A_z^{Finite}(x,y,z) = A_z^{SemiInf}(x,y,z) - e^{-jk\eta_z N_z d_z} A_z^{SemiInf}(x,y,z-N_z d_z) \,. \quad (11)$$

This is illustrated in Fig. 5. For the N_z-element array, the end contributions at the observation point *P* are shown as *1* and *2*. The synthetic aperture approach is based on replacing the contributions *1* and *2* with the contributions *1* and *3*. This is achieved by subtracting contributions of the axially-shifted semi-infinite array from the contributions of the original semi-infinite array as given in (11). Numerically, doing calculations for point *P* with axially-shifted semi-infinite array is equivalent to performing the calculations with the original semi-infinite array for the point *P*'.

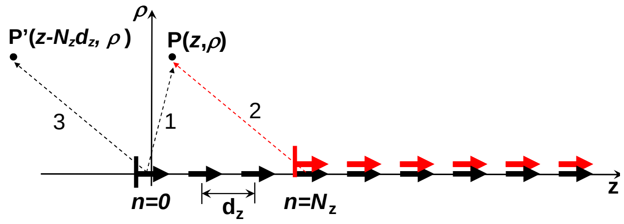

Figure 5. One-dimensional semi-infinite array and the application of synthetic aperture approach.

RECTANGULAR ARRAY

The field radiated by a rectangular array shown in Fig. 2 can now be computed as a superposition of contributions from finite line arrays shifted along the x-axis by $x=m\, d_x$ with a phase increment $k\eta_x d_x$ along x axis and can be represented as

$$A_z^{Rec.}(x,y,z) = \sum_{m=0}^{N_x-1} e^{-jk\eta_x m d_x} A_z^{Fin.}(x-md_x,y,z) \,. \quad (12)$$

Figure 6 shows the amplitude of A_z versus θ at a radial distance $R_d=100\lambda$ at $x=0$ radiated by a rectangular array with $N_z=10$ and $N_x=3$. The interelement distances are $d_x=0.25\lambda$, $d_z=0.25\lambda$ and the interelement phasings are $\eta_x=0$, $\eta_z=1/3$.

V. CONCLUSION

Near-field effects of infinite, semi-infinite, finite periodic line, and rectangular arrays of phased infinitesimal axial electric current dipoles are investigated numerically in the frequency domain. The traditional element-by-element summation approach is compared against high frequency asymptotic formulations in terms of PFWs, EFWs and their corresponding tip-diffracted contributions. The radiated fields from large line and rectangular arrays are constructed using the synthetic aperture approach.

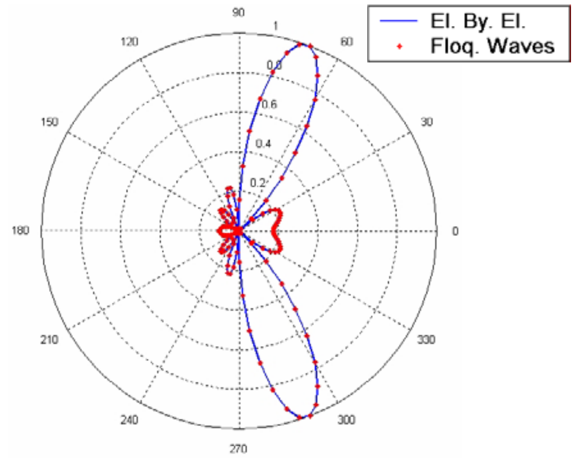

Figure 6. Amplitude of A_z versus θ radiated by a rectangular array.

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