A Fuzzy Decision Model for Optimal Shunt Allocation on Unbalance Radial Distribution with Nonlinear Loads
Part I : The Solution Method

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Abstract: A fuzzy decision model to solve the optimal location and size problem of shunt capacitors for unbalanced distribution system with harmonic sources is presented this paper. The overall optimization problem is formulated as a fuzzy dynamic programming problem of the minimization of three-phase power loss and capacitor cost under the constraints of voltage limits and total voltage harmonic distortion per phase. So much the function objective as the constraints sets of the overall problem is modeled by fuzzy sets based on knowledge of planer about system and whose membership functions values are obtained from the results of three-phase harmonic load flow analysis. The harmonic load flow modeling can takes into account the skin effect of conductors at higher frequencies, coupling mutual between phases and the presence of multiple and different harmonic sources. The fuzzy programming algorithm developed has the advantage to reduce greatly the effort computational that is demanded in the process of search of the optimal solution and further it's provides good prospects for application in practical distribution system in an environment uncertainty and vagueness.

Keywords: Capacitor placement, harmonic, unbalanced network, fuzzy logic decision, and reactive compensation

I. INTRODUCTION

Economic benefits resulting from energy and peak loss reduction when applying shunt capacitors in radial distribution feeder can be optimized by properly selecting the location and sizes of the capacitor installations along the feeders and laterals. Up to now, most of the methods derived for optimal shunt capacitor placement consider the power system operated at fundamental frequency only, because it's just designed for the linear portion of electric loads [1-5], and the effect of nonlinear portion are completely ignored. In practice, a portion of electric load is nonlinear due to the widespread use of energy-efficient fluorescent lamps and solid-state devices in most electrical home appliances and office equipment.

These nonlinear loads tend to introduce current and voltage harmonics into the power and control voltages due the possible resonance condition which could lead to potentially dangerous magnitudes of harmonic signals, additional stress on equipment insulation, increased capacitor failure, and interference with communication systems. Furthermore, a concern with capacitor sizing and location is the increase in voltages due the possible resonance condition. One assumption that is still present in major the capacitor placement problem is that the inherent unbalanced nature of the system is not taken to account in the optimization model.

The methodology proposed in this paper for the optimal reactive compensation of distribution systems will provide planning strategies which satisfies security, save and energy quality criterions, gets subsidies for the final decision of the planners/operators about the location and sizing of the capacitors on network. In this sense, we used a routine of three-phase harmonic load flow to evaluate the system operational state included in a decision model based in fuzzy set theory. This approach intends to incorporate to the problem the characteristic of inherent uncertainty to whole the process of decision centered in the judgment human, that is, the choice on the part of the planner of a satisfactory solution in agreement with conflicts criterions.

The proposed decision model obtains a solution that assists to a specific load demand. This solution satisfies the engagement between the optimality of the power losses (objective) and of two operational constraint sets: voltage regulation and voltage harmonic distortion. For so much, the optimality of the objective and the attendance of the two restrictions are relaxed, considering the opinions of the planner, which are modeled by fuzzy sets. A fuzzy dynamic programming is then developed to yield the optimal locations by considering the effect of the reactive power source to the voltage variation, three-phase power losses and voltage harmonic distortion. The proper size of shunt capacitors to be installed at each feasible location is solved by simple heuristic optimization routine whose steps are based on the method of local variations proposed in [7].

II. PROBLEM DESCRIPTION AND FORMULATION

In the harmonic load flow model, the concept of load composition is used, that is the loads are classified as either linear (passive loads) or nonlinear (any harmonic source). In determining harmonic voltages caused by nonlinear loads, a three-phase network model to take to account the unbalanced loads, mutual coupling effect of conductors and the effect skin on conductors resistance is used. The current injection method based on admittance matrix presented in [8] was reformulated to evaluate the harmonic voltages profile at each bus system on three-phase basis. It is assumed that the entire distribution system is modeled as a combination of passive elements (branch and load impedances) represented by its impedance matrix and harmonic current sources. Therefore, the principle of superposition may be applied to enable each harmonic to be considered independently. Because the elements have different admittance values for each harmonic order, is necessary to modify the admittance matrix for each harmonic order of frequency to be studied.

In this paper the following notation is used to definition of admittances network of m buses and nr branches: $yl_{i\phi,h}$: linear load ($Pl_{i\phi}$, $Ql_{i\phi}$) admittance on bus i (phase ϕ) at h-th harmonic frequency; $yc_{i\phi}$: capacitor admittance on bus i (phase ϕ) at h-th harmonic frequency; $y_{k\phi,h}$: branch k admittance (phase ϕ) at h-th harmonic frequency; ($r_{k\phi}$, $x_{k\phi}$): resistance (reactance) em ohm per length unit of branch k related to phase ϕ.

If skin effect on resistance of conductors is considered at higher frequencies, the resulting h-th harmonic frequency load admittance, shunt capacitor admittances and branches admittances are respectively given by

$$yl_{i\phi,h} = Pl_{i\phi}/|v_{i\phi,1}|^2 - jQl_{i\phi}/\left(h*|v_{i\phi,1}|^2\right) \tag{1}$$

$$yc_{i\phi,h} = h*yc_{i\phi,1} \tag{2}$$

$$y_{k\phi,h} = \frac{1}{r_{k\phi}*\left(1+\dfrac{0.6468*h^2}{192+0.518*h^2}\right)+jh*x_{k\phi}} \tag{3}$$

where $v_{i\phi,1}$ is the fundamental voltage at bus i (phase ϕ) and the term between parentheses on denominator in (3) corresponds an estimated correction factor for skin effect.

All nonlinear loads are treated as harmonic current sources which h-th current injected on phase ϕ ($\phi = a, b ,c$) of bus i is derived as follows:

$$i_{i\phi,h} = \frac{i_{i\phi,1}}{h} \tag{4}$$

$$i_{i\phi,1} = \left(\frac{Pn_{i\phi}+jQn_{i\phi}}{v_{i\phi,1}}\right)^* \tag{5}$$

where ($Pn_{i\phi}$, $Qn_{i\phi}$) is the nonlinear active (reactive) load on phase ϕ of bus i obtained from their nominal data's. The harmonic current injected can be always obtained directly by field tests or measurements.

Therefore, once the injected harmonic currents are previously known or calculated from the data of the nonlinear loads, the bus voltages for the h-th harmonic are calculated as:

$$\mathbf{V}_{abc,h} = \mathbf{Y}^{-1}{}_{abc,h}\,\mathbf{I}_{abc,h} \tag{6}$$

where:

$$\mathbf{V}_{abc,h} = [\ v_{1a,h}\ v_{1b,h}\ v_{1c,h}\ \ v_{2a,h}\ v_{2b,h}\ v_{2c,h} \ldots\ldots v_{ma,h}\ v_{mb,h}\ v_{mc,h}]^T$$

$$\mathbf{I}_{abc,h} = [\ i_{1a,h}\ i_{1b,h}\ i_{1c,h}\ \ i_{2a,h}\ i_{2b,h}\ i_{2c,h} \ldots\ldots i_{ma,h}\ i_{mb,h}\ i_{mc,h}]^T$$

and $\mathbf{Y}_{abc,h}$ is the three phase admittance for the h-th order harmonic. In [9], the authors shown a generalized technique for the development of the three-phase admittance for unbalanced distribution systems.

Once the all harmonic voltages and fundamental voltages per phase are known, the total peak power including losses at harmonic frequencies, is expressed by

$$P_{loss} = \sum_{k=1}^{nr} LP_k \tag{7}$$

where

$$LP_k = \sum_{h=1}^{N} \sum_{\substack{\phi=a,b,c \\ i,j=1,m \\ i\neq j}} \{r_{k\phi}[|v_{i\phi,h} - v_{j\phi,h}|*|y_{k\phi,h}| \ ,\quad k = 1,..nr \tag{8}$$

where nr is the total branches number, N is an upper limit of the harmonic orders being considered, and i and j are terminal bus indices of branch k.

At any bus i, the r.m.s voltage per phase ϕ ($\phi = a,b,c$) is defined by

$$|V_{i\phi}| = \sqrt{\sum_{h=1}^{N} |v_{i\phi,h}|^2} \tag{9}$$

Finally, the total harmonic voltage distortion factor ($HDF_{i\phi}$) per phase ϕ of each bus i, which is used to describe harmonic pollution is calculated as follows:

$$HDF_{i\phi}(\%) = \frac{\sqrt{\sum_{h=2}^{N} |v_{i\phi,h}|^2}}{|v_{i\phi,1}|} \times 100 \tag{10}$$

This factor is generally required to be lower than the accepted maximum value.

In this paper, the objective of placing shunt capacitors along distribution feeders is to reduce the total power loss and bring the bus voltages and total harmonic voltage distortion factor within prescribed limits, while the cost of capacitors into account. Being like this, the total annual cost function due to capacitor placement and power loss is written as:

$$C = k_p * P_{loss} + \sum_{j \in \Omega_c} k_j^c * Q_j^c \qquad (11)$$

where k_p the equivalent annual cost per unit of power loss (in \$/kW/year), Ω_c represents the shunt capacitors buses, Q_j is capacitor size on bus j and k^c_j is their corresponding equivalent annual cost per kVAr.

Therefore, the objective of the compensation reactive problem is to minimize (11) subject to

$$HDF_{i\phi} \le HDF_{max} \qquad (12)$$

and

$$V_{min} \le \left|V_{i\phi}\right| \le V_{max} \qquad (13)$$

where $i = 1,2,...m$, $\phi = a,b,c$; V_{min} , V_{max} , and HDF_{max} correspond to the permissible minimum and maximum r.m.s voltage per phase, and maximum total voltage harmonic distortion per phase, respectively.

III. THE FUZZY SOLUTION METHODOLOGY

The above problem equivalent to nonlinear integer programming problem; If all possible combinations of the capacitor locations and sizes are considered it will becomes a very exhaustive search problem. To reduce the efforts to finding the optimal solution, the capacitor placement problem was solved based on fuzzy sets decision methodology. The inherent uncertainty to the solution of the problem is owed exclusively to the element human decider, that is to say, the application of a model based on fuzzy sets is made herself on the stage of the problem related to the process of decision. In this proposed approach, a fuzzy objective function characterized by the fuzzy set $\widetilde{P}$ related to three-phase power losses is used. The constraints sets imposed on the system are divided into two groups. The bus voltage limits per phase and the total voltage harmonic distortion factor per phase are treated as fuzzy constraints since they are related to the imprecise (fuzzy) load profiles. The other constraints, maximum allowable capacitor size to be placed at any feeder location, are considered to be crisp values. The possible voltages, losses and distortion harmonics states are crisp state variables. However, the fuzzy decision model contains a fuzzy objective function $\widetilde{P}$ and two fuzzy sets constraints $\widetilde{V}_{i\phi}$ (bus voltage) and $\widetilde{H}_{i\phi}$ (voltage harmonic distortion factor).

It is obvious that the fuzzy decision (optimal placement) must take both the objective function and two constraints sets into account. In fuzzy set notations, the fuzzy set decision $\widetilde{D}$ is the confluence (intersection) of the objective function $\widetilde{P}$ and the per phase constraints sets $\widetilde{V}_{i\phi}$ and $\widetilde{H}_{i\phi}$. In other words, we have

$$\widetilde{D}_i = \widetilde{P} \cap \widetilde{V}_{ia} \cap \widetilde{V}_{ib} \cap \widetilde{V}_{ic} \cap \widetilde{H}_{ia} \cap \widetilde{H}_{ib} \cap \widetilde{H}_{ic} \qquad (14)$$

In proposed approach, the possible locations for installation of capacitors are selected after harmonic load flow solution for those bus that violates the r.m.s voltages and harmonic distortion limits. At each stage of network (initial or after partial compensation), such buses are denominated *candidates buses*. To selection of number of capacitors units at each candidate bus, we consider a minimum standard size that is commercially available and the maximum reactive power at referred buses.

It should be observed in (14) that fuzzy sets operations such as intersection operation could only be achieved through membership functions of the fuzzy sets. Thus, to reach an optimal capacitor placement using fuzzy decision model, it is essential to define the membership function for the sets $\widetilde{P}, \widetilde{V}_a\, \widetilde{V}_b, \widetilde{V}_c, \widetilde{H}_a, \widetilde{H}_b$ and $\widetilde{H}_c$.

Before we do that, let's recall that our aim is to reach an optimal decision such that the total cost is minimum with the seven constraints satisfied. In linguist terms, (14) can be rewritten as:

IF (total cost is very low) and
(the bus i r.m.s. voltage limits at phase ϕ is satisfied)and
(the bus i harmonic distortion factor at phase ϕ is satisfied) THEN
(the resultant capacitor placement is desirable) (15)

A. *Membership function for the fuzzy objective function* $\widetilde{P}$

Since our purpose is to have a reactive compensation with minimum cost, we can reduce the total branch i loss by defining a membership function for the three phase power loss such that a high loss branch is given a low membership value. A good fuzzy decision is then defined to be one that with a high membership value. Therefore, the membership function adopted for the three-phase power loss is written as:

$$\mu_{\widetilde{P}_i} = e^{-k_l * \frac{LP_i}{P_{loss}}} \qquad i \in \{\text{buses candidates}\} \qquad (16)$$

where k_l is an coefficient that can be choose arbitrarily by planner, in agreement with its knowledge on the system, LP_i is the total power loss in branch that finishes in the bar i and P_{loss} is the total three-phase power loss.

B. Membership function for voltage deviation $\widetilde{V}_{i\phi}$

Let us consider i as a possible candidate bus for capacitor location that the minimum voltage limit is violate. Let V_{min} (V_{max}) the minimum (maximum) voltage acceptable in phase ϕ of bus i. With these two values we can define the membership function for fuzzy constraint $\widetilde{V}_{i\phi}$ such that the bus with high voltage deviation per phase is given a low membership value. Consequently, a good fuzzy decision is then defined to be one those with a high membership value. Therefore, the membership function chosen to represent voltage deviation per phase constraints is defined as

$$\mu_{\widetilde{V}_{i\phi}} = \begin{cases} 1 \quad \text{if} \quad V_{i\phi} \geq V_{min} \\ \left[1 + k_v * \left(\dfrac{V_{i\phi} - V_{min}}{V_{max} - V_{min}}\right)\right]^{-1} \quad \text{if } V_{i\phi} \leq V_{min} \end{cases}$$

$$i \in \{\text{buses candidates}\} \qquad (17)$$

where the coefficient k_v can causes more or less fuzzyfication these membership function which is chosen in agreement with the knowledge the planner about the voltage profile system.

C. Membership function for constraint voltage harmonic distortion $\widetilde{H}_{i\phi}$

Another constraint is to have minimum voltage harmonic distortion in each phase after capacitors placement. If $N_{\phi i}$ is the percent of nonlinear load of bus i at phase ϕ, and HDF_{max} is maximum limit of harmonic distortion factor per phase, we can to reduce the total voltage harmonic distortion by defining a membership function for $\widetilde{\boldsymbol{H}}_{\phi}$ such that a bus i with high voltage harmonic distortion per phase is given a low membership function value and the bus i with a low voltage harmonic distortion per phase is given a high value. The membership function chosen is defined by (18).

$$\mu_{\widetilde{H}_{i\phi}} = \begin{cases} 1 \quad \text{if} \quad HDF_{i\phi} \leq HDF_{max} \\ e^{k_h * N\phi i * \frac{HDF_{max} - HDF_{i\phi}}{HDF_{max}}} \quad \text{if} \quad HDF_{i\phi} > HDF_{max} \end{cases}$$

$$i \in \{\text{set buses candidates}\} \qquad (18)$$

IV. THE OVERALL FUZZY DYNAMIC PROGRAMMING ALGORITH

With the membership functions calculated by (16), (17) and (18), the membership function value of fuzzy decision, $\mu_{\widetilde{D}i}$ which is the intersection of the fuzzy sets can be determined. An optimal decision in the fuzzy dynamic programming formulation is one that has lowest membership value in $\widetilde{\boldsymbol{D}}_i$. Thus, the overall optimization problem can be formulated as finding the lowest $\mu_{\widetilde{D}i}$ in (19):

$$\mu_{\widetilde{\boldsymbol{D}}i} = \textbf{Min}\,\{\mu_{\widetilde{P}_i}, \mu_{\widetilde{V}_{ia}}, \mu_{\widetilde{V}_{ib}}, \mu_{\widetilde{V}_{ic}}, \mu_{\widetilde{H}_{ia}}, \mu_{\widetilde{H}_{ib}}, \mu_{\widetilde{H}_{ic}}\}$$

$$i \in \{\text{candidates buses}\} \qquad (19)$$

If $\textbf{Min}\,\{\mu_{\widetilde{\boldsymbol{D}}i}\} = \mu_{\widetilde{\boldsymbol{D}}i}$, $i \in \{\text{candidates buses}\}$, then i is an optimal bus location. Fig. 1 depicts the fuzzy decision as the intersection among the fuzzy objective and fuzzy constraints defined previously.

After we decide the optimal location of shunt capacitor, the method of local variations is used to find the optimal capacitor sizes that will provide minimum annual losses cost

The overall computational algorithm for the fuzzy programming model can be described in the following steps:

Step 1. Perform the three-phase harmonic load flow to calculate the bus voltage r.m.s per phase and total power losses.

Step 2. Identify the bus with voltage violations (*candidates buses*) and calculate the membership functions $\mu_{\widetilde{P}_i}, \mu_{\widetilde{V}_{i\phi}}$, $\mu_{\widetilde{H}_{i\phi}}$ and $\mu_{\widetilde{D}_i}$ for those buses as in (16), (17) (18) and (19).

Step 3. Choose the optimal location of shunt capacitor as the bus with the lowest membership value $\mu_{\widetilde{D}_i}$ (bus i).

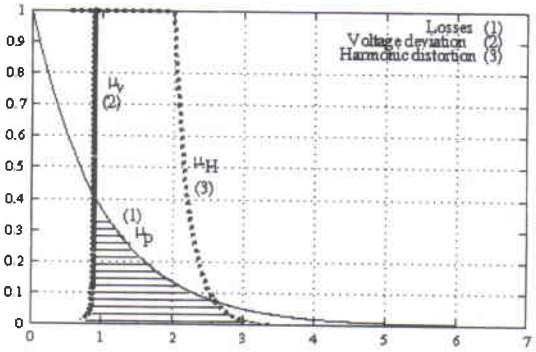

Fig. 1 Fuzzy decision as intersection among fuzzy objective and constraints.

Step 4. Try the capacitor placement at bus i with various discrete sizes of minimum standard size. Choose the optimal size Q^c, which will result in the lowest cost function without violating the r.m.s voltage and harmonic distortion constraints in three phases.

Step 5. Install Q^c at bus i and simulate again the harmonic load flow to calculate if there is new bus voltage violation. If still exists, return to step 2; else stop.

VII. CONCLUSIONS

In this paper, a fuzzy decision model has been developed to determine the optimal location of compensation shunt capacitor for the unbalanced distribution systems with harmonic current sources. For take the uncertainties or imprecision in profile load, fundamental and harmonics voltages, membership functions are derived for three-phase losses, r.m.s deviation voltage and harmonic distortion factor at each phase using fuzzy set notations. With these membership functions, a fuzzy decision model is defined to decide the optimal bus location under the fuzzy dynamic programming environment. The form of membership functions defined for objective and constraints can be easily adapted in agreement with the decision-maker needs providing more flexibility to the decision model. Another very important feature of the proposed fuzzy approach is a more realistic modeling of three performance indices: losses, deviation voltage and harmonic distortion factor, which are treated on three-phase basis. Furthermore, the skin effect of resistance of conductors and cables at higher frequencies can be take into account in the harmonic load flow solution. A simple heuristic algorithm that is based on the method of local variations is incorporated to fuzzy programming to find the optimal sizes of capacitors at each optimal location. The numerical results and analysis on a typical distribution systems will be given in the future paper, Part – II.

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